Warm-up:

A. List the red black tree properties.

Q1: Staircase  Let $P$ be a set of $n$ points in the plane. The staircase of $P$ is the set of all points in the plane that have no points in $P$ both above and to the right. Describe an algorithm to compute the staircase of a set of $n$ points in $O(n \log n)$ time.

Q2: Smallest Subtree  Given a binary tree and an integer $d$, find the smallest subtree with a root at depth $d$.

Q3: Augmenting Trees  We want to maintain a data structure $D$ representing an infinite array of integers under the following operations:

- $\text{INIT}(D)$: Create a data structure for an infinite array with all entries being zero.
- $\text{LOOKUP}(D, x)$: Return the value of integer with index $x$.
- $\text{UPDATE}(D, x, k)$: Change the value of integer with index $x$ to $k$.
- $\text{MAX}(D)$: Return the maximal index for which the corresponding integer is non-zero.
- $\text{SUM}(D)$: Return the sum of all integers in the array.

Q4: Classic Heap Problem  Given an infinite stream of numbers, show how to maintain the $k$ smallest elements at any given time.

Q5: Heap Practice

A. True or False: Every BST is a Heap

B. True or False: A sorted array is a Heap

C. Recall that you can sort elements by inserting them into a BST and then doing an inorder traversal. True or False: there exists an $O(n)$ in-order traversal function for Heaps that allows you to print out the elements in a heap in order in $O(n)$ time.

D. Consider the tree $T$ and array $A$ that define a heap $H$. True or False: a level order traversal of $T$ outputs the elements in $H$ in the same order they appear in $A$. 

Q6: **Heap Operations**  Consider the following heap (drawn as a tree):

```
          10
         /  \
        8    3
       /  \
      1    6
```

A. Is this a Min heap or a Max heap?

B. Draw the array representation of this heap.

C. Show the resulting tree after calling **ExtractMax**

Q7: **Splay Tree Basic Operations**  Insert the following into an initially empty splay tree: 3,6,2,7,4. Show the resulting tree.

Q8: **RB Tree Operations**  Insert the following into a RB tree: 3,1,6,2,9. Show the resulting tree. Then, delete 1 and show the resulting tree.
Some Useful Facts

These will be stapled to the back of your exam for reference (if needed).

1. \( \lg n = \log_2 n \)
   \( \ln n = \log_e n \)

2. Below are some formulas that may come handy.

   - \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \)
   - \( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \)
   - \( (a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i} \)

3. Log rules:
   - \( a^{\log_a b} = b \)
   - \( \log ab = \log a + \log b \)
   - \( \log \frac{a}{b} = \log a - \log b \)
   - \( \log a^b = b \log a \)
   - \( \log_a a^b = b \)
   - \( \log_a a = 1 \)
   - \( \log 1 = 0 \)

4. Expected value:
   - \( \mathbb{E}[X] = \sum_x xP(X = x) \)
   - Linearity of Expectation: for any finite collections of random variables \( X_1, X_2, ..., X_n \), \( \mathbb{E}[\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} \mathbb{E}[X_i] \)