This assignment is due at the beginning of the class on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - please see Piazza for the course collaboration policy.

Also, please remember to double check that you have submitted the correct version of your homework onto Gradescope by re-downloading it.

1. **[12 pts]** The newly self-aware CIS 160 staff decides to create their own tongue-in-cheek satirical newspaper called *Proving the Contradiction* (PTC). First, they need to split themselves up into at least \( k \) non-overlapping pairs of writers to write the stories (no writer can be working on more than one story at a time). In order for a pairing to work, the writers in the pair must share a common experience (so they can write about it). Luckily, each writer has shared a unique experience with each of at least \( 2k \) other writers on staff. Prove that the \( n > 0 \) staff members can separate themselves into at least \( k \) disjoint writing pairs.

2. **[15 pts]** PTC has a set \( J \) of junior writers and a set \( S \) of senior writers. None of the writers are friends with themselves. No two junior writers are friends, and no two senior writers are friends. There is at least one pair of friends on the staff, though. Every senior writer has at least as many friends as every junior writer. We have \(|S|\) stories to assign. Every story must be assigned to a pair of writers who are friends, and no writer can be assigned to more than one story. Prove that PTC can assign stories in such a way that every senior writer gets a story.

3. **[15 pts]** Amit, editor-in-chief of PTC (and the H homeworks), is going over the headlines for the first edition of the paper when he realizes that the staff has a very limited sense of humor. As a result, some of the “joke headlines” proposed by the staff might actually be true! To help him out, prove or disprove each of the headlines below.\(^1\)

   (a) “BREAKING: Markov’s inequality holds for all random variables!”

   (b) “NEWS FLASH: If \( Z \leq 20 \) is a random variable with \( \mathbb{E}[Z] = 10 \) and \( \Pr[Z < 0] \neq 0 \), Markov’s inequality can still be used to show that \( \Pr[Z \leq 5] \leq \frac{2}{3} \).”

4. **[14 pts]** It is well-known that, on average, 212 people read each paper published by PTC. Dhruv, one of PTC’s most talented journalists, has been tasked with writing the Thanksgiving edition of the paper.

\(^1\)We love a good laugh, so feel free to include your own joke headlines in your submission.
(a) Dhruv wants to write the very best paper (like no paper ever was). He decides that least 265 people have to read his paper for it to indeed be the very best. Using Markov’s inequality, help Dhruv find an upper bound on the probability that a PTC paper was read by at least 265 people.

(b) Satisfied with this upper bound, Dhruv then asks you to show him a distribution of paper readership where the probability that a PTC paper was read by at least 265 people is exactly equal to the upper bound you provided in part (a).

(c) During his meeting with the editor-in-chief Amit, Dhruv learns that no paper published by PTC was read by fewer than 53 people! Happy to be working for such a successful newspaper, he then tries to use this information to improve his upper bound on the probability that a PTC paper was read by at least 265 people. Can he improve this bound? If so, explain how and provide a new distribution as you did in part (b). If not, explain why the new information does not help.

(d) It is also well-known that the standard deviation of the number of people who read each PTC paper is equal to 2. Using Chebyshev’s inequality, help Dhruv find an upper bound on the probability that a PTC paper was read by at least 265 people.

5. [14 pts] Senior writers Monal and Alexandra are filming a prank for PTC Video. They leave a copy of the most recent 160 lecture notes out on Locust Walk and livestream a video of it until someone picks it up and reads it. Every person who encounters the notes independently has a \( \frac{1}{6} \) chance of reading them, and Monal and Alexandra will stop filming when someone reads the notes or 5 people have encountered them, whichever comes first. If exactly one person encounters the notes every 15 seconds, what is the expectation and variance of the number of seconds of livestreamed video? You can assume that the livestream starts 15 seconds before the first person encounters the notes and you do not need to consider any time spent encountering or reading the notes.