1. **[18 pts]** Give answers to the following questions. You do not have to show your work for this question.

   (a) Write the following sets explicitly, i.e. list the members of these sets.
   
   i. \( \{ x \mid x \text{ is a square of an integer, } x \text{ is even, and } x < 150 \} \)
   
   ii. \( \{ x \mid x \text{ is an integer such that } x^2 = 7 \} \)
   
   iii. \( \{ x \mid x \text{ is a real number such that } x^2 - 63 = 1 \} \)
   
   iv. \( \{ x \mid 3x \text{ is a positive integer less than 32 and } 4|x \} \)

   (b) Use the set builder notation to give a nontrivial description of each of these sets (nontrivial means that your solution should not simply enumerate every element).
   
   i. \( \{ 7, 10, 14, 20, 21, 28, 30, 35, 40 \} \)
   
   ii. \( \{ 3, 15, 35, 63, 99 \} \)
   
   iii. \( \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6 \} \)

   (c) What is the cardinality of each of the following sets?
   
   i. \( \{ a, \{ a \} \} \)
   
   ii. \( \{ \{ p, m \} \} \)
iii. \( \{d, \{d\}, \{d, \{d\}\}\} \)

(d) Determine whether each of the following is true or false.

i. \( \emptyset \subseteq \{x\} \)

ii. \( \emptyset \in \{x\} \)

iii. \( x \subseteq \{x\} \)

iv. \( \{x\} \in \{x\} \)

v. \( \{x\} \subseteq \{x\} \)

vi. \( \{x\} \in \{\{x\}\} \)

(e) What is the power set of \( \{x, y\} \), where \( x \) and \( y \) are distinct elements? What is the cardinality of the powerset?

(f) Find two sets \( A \) and \( B \) such that \( A \in B \) and \( A \subseteq B \).

2. [8 pts] Kenneth, Simona, Olivia, Stephanie, and Billy are lined up on Locust Walk in hopes of driving an NSO golf cart. A volunteer lines them up in alphabetical order and tells them the following: the first two people in line must play rock, paper, scissors to decide who gets to ride the first available golf cart. The winner of the match gets to ride away; the loser of the match must stay to play the next person in line. This process continues until there is one TA left, who will wait for the next golf cart to arrive. In how many orders can the 5 TAs receive golf carts?

3. [10 pts] To advertise his candidacy for student government, Shaya decides to color a row of \( n \) consecutive cobblestones in the Quad, where \( n \) is a positive integer. At home, he has a pack of chalk with 28 different colors. To stand out, he decides that he’d like the colors of the bricks to be symmetric about the middle of the \( n \) cobblestones. For example, if \( n = 4 \), then Shaya might paint the stones Red-Blue-Blue-Red; if \( n = 5 \), he could choose Red-Blue-Green-Blue-Red.

Unfortunately, he’s too busy getting signatures to do any calculations, so he needs your help. How many different ways are there to color the \( n \) cobblestones with 28 colors such that the colors are symmetric about the middle?

4. [6 pts] There are \( n \geq 2 \) students in the Hill dining hall and they are all enthusiastically networking. Given that there is at least one student who has not shaken hands with every other student in the dining hall, what is the maximum number of students who could have shaken hands with everyone else?
5. **[10 pts]** Zach is lost in the basement of DRL and stumbles upon a hoard of energy drinks. In the hoard, there are 10 identical Red Bulls, 15 identical Monsters, and 40 identical Five Hour Energies. He is strong enough to carry all the drinks in his bag, but also wants to leave at least one drink at DRL for his 9 AM class tomorrow. How many different ways can Zach bring drinks back in his bag?

6. **[6 pts]** Tashweena is decorating her new dorm room. To demonstrate her school spirit, she wants to arrange blue and red lights in a circle on her wall. She wants to use at least one blue and at least one red light; additionally, for each light, she wants both of its neighbors to be the same color. If there are 75 red lights in her final design, how many lights did she use to decorate in total?

7. **[4 pts]** Gautam, the most intrepid member of the CIS 160 staff, challenges Rajiv to a game. If Gautam wins, there will be no final exam. If he loses, the class will proceed as usual.

Rajiv proposes the following game: to begin, 2019 cards numbered consecutively from 1 to 2019 are placed inside a Wawa plastic bag. Gautam’s score is initialized to be 0. Rajiv will draw cards, one by one, and read the card value to Gautam. When he hears the card value, Gautam can choose to either add or subtract the card value from his current score. If his score is 1 after all 2019 cards are drawn, he wins!

If possible, help Gautam find a winning strategy. If it’s not possible, explain why.

8. **[4 pts]** You are given a $8 \times 8$ board. The 4 corner squares are initially black, while the rest are initially white. An *operation* is to flip the colors of all squares in a single row or a single column. Does there exist a sequence of operations to make the entire board black? If so, give an example; if no such sequence exists, explain why.

9. **[4 pts]** Kunaal and Tien find a secret room in Huntsman Hall while looking for a place to study. They open the door and see a dusty table with a row of 100 coins on top. The coins are of various denominations. Competitive and eager to make money, they start playing a game: Kunaal removes a coin from either end of the row and puts it in his pocket; next, Tien picks a coin from one of the two ends of the remaining row of 99 coins and puts it in his pocket. They repeat this process until there are no more coins on the table.

Kunaal needs your advice: find a strategy that Kunaal can implement to guarantee that he will win at least as much money as Tien. If no such strategy exists, explain why.