This assignment is due at the beginning of the first section on the due date. Unless all problems carry equal weight, the point value of each problem is shown in [ ]. To receive full credit, all your answers should be carefully justified. Each solution must be written independently by yourself — no collaboration is allowed.

For this homework only, there will be no penalty for not using \TeX. You may choose to legibly hand-write and scan the solutions or use another text editor if you wish. However, we strongly advise you to learn \TeX for this homework as it will be required for all other homeworks.

Note that Question 3 is intended more as a brainteaser to get you into the problem solving mindset of CIS 160, and does not directly relate to material in lecture. Such questions generally only appear in the first few homeworks — after that, homework questions will relate more closely with the material seen in lecture and recitations.

1. [12 pts] Let $p$, $q$, and $r$ be the following propositions.

   $p$: The students are all hanging out together.

   $q$: The sun is behind the clouds.

   $r$: The CIS 160 TAs all miss each other.

Express the following propositions using $p$, $q$, $r$ and logical operators.

Including a line or two of explanation for your solution may be helpful in guiding your thought process, but don’t worry too much about providing justification. For this problem specifically, we’ll only be grading your final answer.

(a) The sun is behind the clouds and the students are all hanging out together.

(b) If the students are not all hanging out together or the sun is behind the clouds, then the CIS 160 TAs do not all miss each other.

(c) It is necessary for the sun to be behind the clouds for the students to all be hanging out together. It is also necessary for the sun to be behind the clouds for the CIS 160 TAs to not all miss each other.

(d) The CIS 160 TAs all miss each other if and only if the students are all hanging out together.

(e) The CIS 160 TAs do not all miss each other and the students are all hanging out together.
(f) The students all hanging out together is sufficient for the sun to be behind the clouds and the CIS 160 TAs to not all miss each other.

2. **[10 pts]** Decide if the following proposition forms are tautologies using a truth table. Make sure your truth table shows all intermediate logical expressions — for example, in showing the truth table for \((p \lor \neg q) \land p\), your table should contain separate columns for \(p\), \(q\), \(\neg q\), \(p \lor \neg q\), as well as the final expression. You should also clearly state your final answer to the question.

   (a) \([((p \lor \neg q) \lor (p \land \neg q)) \land (p \implies \neg q)]\)
   (b) \([((p \land q) \implies (p \lor \neg q)) \implies (q \lor \neg q)]\)

3. **[8 pts]** Uh-Bee-Shake, Tomiwa, and Sid are back on campus and want to get back to their Zoom exercise regimen, which this semester has a 70’s theme. As such, Sid wanted everything to be period-accurate and bought some headbands for the group. Wanting it to be a surprise, Sid tells the other TAs that he has bought 3 purple and 2 green headbands and will randomly select and send a headband to each of them. All without looking at the headbands, Sid first randomly selects a headband for himself, then randomly selects and sends headbands to Tomiwa and Uh-Bee-Shake. On their first Zoom workout sesh, the TAs blindly put on their headbands. Being perfect logicians, they look at each other’s headbands and decide to try to deduce what color headbands they have. After thinking for a while, Sid concludes “I don’t know what color headband I have.” After hearing this, Uh-Bee-Shake thinks momentarily and also concludes “I don’t know what color headband I have.” At this point, does Tomiwa know what color headband he is wearing? Please justify your answer.