1. **[10 pts]** Suppose that $d, t \in (\mathbb{R} \setminus \{0\})$. Prove that

$$d < \frac{1}{d} < t < \frac{1}{t} \implies d < -1$$

2. **[16 pts]** Let $S, T$ and $R$ be sets. Prove or disprove the following.

(a) $S \setminus (T \setminus R) = (S \setminus T) \setminus R$

(b) $S \setminus (T \setminus R) \subseteq (S \setminus T) \cup R$.

(c) $S \setminus R = (S \setminus (T \cup R)) \cup ((S \cap T) \setminus R)$

(d) Prove that if $\exists y$ such that $y \in S \times T \land y \in T \times S$, $\exists x$ such that $x \in S \land x \in T$.

3. **[8 pts]** Fearing Tony Stark, Thanos’ half brother “Yonah” Mannos decides he wants to hide 8 of his (distinguishable) infinity stones across Penn’s campus. There are only three hiding spaces that are safe from the clutches of Stark’s minions: the Towne basement, the Button, and the Levine 5th floor bump space. To make things difficult for Stark, Mannos wants to make sure that there is at least one infinity stone in each of the hiding places. How many ways can Mannos hide the infinity stones? Make sure you use the Principle of Inclusion-Exclusion in your answer.

4. **[8 pts]** If the coefficients $a$ and $b$ of the equation of a straight line $y = -\frac{a}{b}x$ are two distinct digits from the set \{1, 2, 3, 6, 7, 8, 9\}, then how many distinct straight lines are possible?

5. **[12 pts]** Jeffrey is aspiring to become the very best Pokémon master. He stops by his friend, John, to get some very special Pokémon for his team. John’s Pokémon are Magikarps, but can learn any type of move in existence. Jeffrey decides he will have a team of 4 Magikarps, each with 2 moves: an attack move and a defense move. Each of these moves can either be water, fire, electric, or normal type, but Jeffrey doesn’t want the two moves to have the same type. For example, Jeffrey will not accept a Pokémon with a water-type attack move and a water-type defense move.

(a) Jeffrey wants to have a team of 4 Pokémon such that he has a Magikarp of each attack move type and one of each defense move type. How many possible teams of Magikarps can Jeffrey form? Assume that the order of the Magikarps does not matter in a team.
(b) John recently expanded his move collection to have moves of 10 different types, and Jeffrey now has enough capacity to have 10 Pokémon on his team! How many different teams can Jeffrey form now? Jeffrey still wants one Magikarp of each attack move type and one of each defense move type.

6. [8 pts] Yonah, Rishab, and AJ are looking to organize the $n$ other CIS 160 staff members into various committees. Each TA can only join one committee. Assume there are at least 15 TAs.

(a) In how many way can the CIS 160 staff be organized into three indistinguishable committees, each of size 5?

(b) Now, the head TAs decide they want to lead the committees. In how many way can the CIS 160 staff be organized if Yonah decides he wants exactly 5 other people on his committee, AJ requires exactly 4 other people, and Rishab needs only exactly 3 other people?

7. [8 pts] Wanting to get to know the students in her recitation better, Sarah decides to begin class with an icebreaker. She has all $n$ of her students stand in a circle and gives each of them thin strings of yarn such that, for any two students, a string of yarn is held tightly between them on either end. Note that this means that each student is holding the ends of $n-1$ different strings of yarn.

Assume that all of the strings are held at the same height. Given this, at how many points do two lengths of yarn intersect? Ignore intersections that occur at the endpoints of the strings, and assume there is no place where three strings of yarn intersect.