This assignment is due at 9:00AM on the due date. To receive full credit, all your answers should be carefully justified. Unless otherwise specified, your answers must be expressed in closed form where required.

This assignment must be typeset in \LaTeX{} and turned in through Gradescope as a PDF file. Submissions that are not turned in through Gradescope, handwritten homework, or homework that is prepared with other tools (e.g. MS Word) will not be accepted. There will be a 5 point deduction each for incorrectly selecting pages on Gradescope or failing to use the provided template.

Each solution must be written independently by yourself - please see Piazza for the updated collaboration policy.

1. [6 pts] At Tashweena’s truffle ranch, she employs both dogs and pigs to sniff out precious truffles. She has at most 8 animals in total and each animal is distinct. One day, while Gautam is out foraging for mushrooms, he happens to spot two of her pigs. When he tells Tashweena about this sighting, she tells him that the chance of any two of her animals, randomly chosen, both being pigs is exactly $1/2$. How many animals does Tashweena employ?

2. [8 pts] Shannon has discovered a breakthrough in mushroom hunting. Her new technique maximizes efficiency, and she wants to share her knowledge with other mushroom farmers. She goes to a local meeting of mushroom enthusiasts, where she counts $n \geq 2$ people in attendance, including herself. Billy, the host of the meeting, claims that each of the $n$ people in the meeting is friends with at least $n/2$ of the other mushroom enthusiasts. Note that each of these friendships is mutual (that is, if Person A is friends with Person B, then Person B is friends with Person A). Shannon then tells all of her friends about the new technique; these friends tell all of their friends, and so on. Prove that all $n$ mushroom enthusiasts will eventually learn Shannon’s technique.

3. [8 pts] Tony has agreed to help Tashweena train a new truffle pig. As a part of the training, he will take the pig to Mt. Mushroom to forage for truffles. Each time the pig smells what may be a truffle, it will attempt to root the truffle out from underground. Since the pig is only a novice, it will find a truffle or be fooled by a decoy with equal and independent probabilities on each attempt. As soon as the pig finds 8 truffles, Tony will take the pig home to celebrate. However, if the pig is fooled by 8 decoys first, Tony will take the pig home to prevent it from
getting too discouraged. What is the probability that the pig makes exactly 14 attempts before going home? You may assume that on each attempt, the pig either finds a truffle or a decoy.

4. [8 pts] Steph hires Belinda to protect her mushroom farm. As a busy Penn student, Belinda can only work 9 shifts in a week but there are 35 possible shifts — 5 on each day of the week. None of the shifts are overlapping.

To decide Belinda’s schedule, Steph randomly chooses 9 of the 35 possible shifts. Each shift is equally likely, and Steph doesn’t pick any shift twice. What is the probability that Belinda will protect the mushrooms at least once on every day of the week?