CIS 160 Recitation #3

Induction, Multisets, and Combinatorial Proofs
Propositions (brief review)

Sample proposition: Prove that the product of a non-zero rational number and an irrational number is irrational.

Contrapositive: If the product of two numbers is rational, then the two numbers are not a non-zero rational number and an irrational number.

Contradiction: Assume for the sake of contradiction that the product of a non-zero rational number and an irrational number is rational.
**Induction**

Let $P(n)$ be a predicate whose truth depends on $n$.

We want to prove that $P(n)$ is true for all integers $n$ greater than or equal to some integer $n_0$.

**Base Case:** Show $P(n_0)$ is true.

**Induction Hypothesis:** Assume $P(k)$ is true for some integer $k \geq n_0$.

**Induction Step:** Using this assumption, prove that $P(k + 1)$ is true.
Multisets

If we have a multiset $M$ with $n_1$ objects of type $a_1$, $n_2$ objects of type $a_2$, ..., and $n_k$ objects of type $a_k$, such that objects of the same type are indistinguishable from one another, then the number of permutations of the objects in $M$ is:

$$\frac{(n_1 + n_2 + \ldots + n_k)!}{n_1!n_2!\ldots n_k!}$$
Sticks and Crosses

What if we want to take r-combinations with repetition?

Think of “sticks” as dividers between categories of objects.

Think of “crosses” as objects that we assign to each category.

Then, if we have \( n \) categories and we want to select \( r \) objects with repetition, we “permute” \( n - 1 \) sticks and \( r \) crosses:

\[
\binom{n + r - 1}{r}
\]

r-combinations
Combinatorial Proofs

- We can prove that two expressions are equivalent by showing that they are both a solution to the same counting question.
- To do so, we come up with a counting question and two valid counting procedures that answer the question, one that results in one expression, and another that results in the other expression.

- We will do an example today.