CIS 160 Recitation #4
Induction, Binomial Theorem, Pigeonhole Principle
Induction Review

Let $P(n)$ be a predicate whose truth depends on $n$.

We want to prove that $P(n)$ is true for all integers $n$ greater than or equal to some integer $n_0$.

Base Case: Show $P(n_0)$ is true.

Induction Hypothesis: Assume $P(k)$ is true for some integer $k \geq n_0$

Induction Step: Using this assumption, prove that $P(k + 1)$ is true.
Binomial Theorem

For any real numbers $a$ and $b$ and non-negative integer $n$

$$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$
Pigeonhole Principle

- If \( k + 1 \) objects are distributed into \( k \) bins, there will be at least one bin with at least 2 objects
- Generalized PHP: If \( n \) objects are distributed into \( k \) bins, there will be at least one bin with at least \( \left\lceil \frac{n}{k} \right\rceil \) objects

- When using PHP, cite it and describe what your “pigeons” (objects) are and what your “holes” (bins) are
- Tip: often a good technique for problems asking you to prove existence (look for “there exists”)