Rounding, Logical Ops
Introduction to Computer Systems, Fall 2022

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How are you feeling about binary representation?

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Logistics Part 1

- HW00 Binary Quiz: **This Friday** 9/16 @ 11:59 pm
  - Quiz On Canvas
  - Should have everything you need

- Recitations Starting this week!
  - Optional, but can be very useful
  - Increasingly useful as the semester goes on
  - More info on Ed

Logistics Part 2

- HW01 bits.c: to be released sometime this week
  - Will require VM setup (also to be released soon)
  - Has you “program” in C
  - Today’s lecture is very relevant for it
- Starting to count PollEverywhere
- More OH posted on the course website
  - (including mine)

Any questions on anything before I begin?

Lecture Outline

- **Floats Continued**
- Logical Operators
  - Shifting
- Boolean Algebra

Lecture 2 Take-aways

- We can represent Negative integers with 2C

- We can represent fractional numbers with Floats

- C/Java data types like int and float are limited by their number of bits
  - A data type of N bits has 2^N unique bit patterns
  - More on this later in lecture

Binary Scientific Notation

- Scientific notation in Binary: `-1.1001 * 2`^{4}
  - Sign: whether we are negative or positive
  - Ones place: Always starts with a non-zero 'bit'
    - (unless overall expression is 0)

A non-zero bit
must be **1**!
This 1 can be implicit

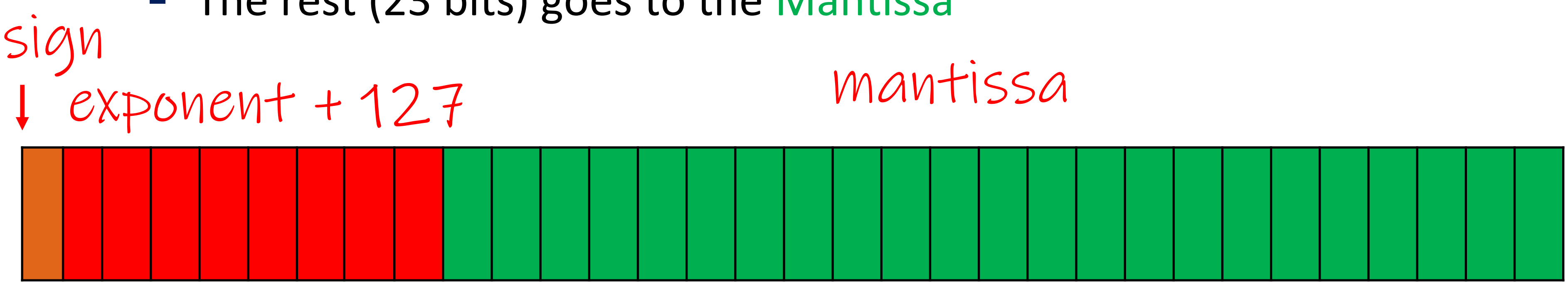
  - Mantissa: Everything after the binary point
  - Exponent: We are in base 2, so we raise 2 to this value

- We can represent a scientific notation binary number with only the Sign, Mantissa, and Exponent

IEEE Floating Point Notation

- We can represent a scientific notation binary number with only the Sign, Mantissa, and Exponent

- Allocate 32 bits, with
  - First bit goes to the Sign (1 for negative, 0 for non-negative)
  - The next 8 bits go to the Exponent + 127 (as an unsigned 8-bit int)
    - This means the exponent must fall between -127 and 128
  - The rest (23 bits) goes to the Mantissa



Special Numbers

- There are some special values to IEEE floating point representation

Value	Sign	Exponent	Mantissa
0	0	All 0's	All 0's
-0	1	All 0's	All 0's
NaN	1 or 0	All 1's	Not 0
∞	0	All 1's	All 0's
-∞	1	All 1's	All 0's

<- e.g. 0b00000...

Not a Number ->

At least one of the 23 bits in mantissa must be a 1

All 1's mean
0b11111111 or 0xFF

- There are also "Subnormal" values, but we won't talk about that

Floating Point: Finite Size Issues

- Float's are only 32 bits, and computers are finite
  - there is a limit to representable numbers
- DEMO
  - 1.1 + 2.2 != 3.3 ? (float_add.c)
  - 240000001 != 240000001 ? (int_float.c)
- "Underflow" can also be an issue
  - When a result is too small in magnitude to be representable
  - (Common issue with Bayesian computations)

Takeaway: Finite Resources

- Computers are physical machines, and limited by being physical machines
  - Many numbers are stored as approximations
  - Overflow or underflow can occur

- These errors can be catastrophic:

Ariane flight V88

Boeing 787

Data Representation Work Arounds

- There are “Workarounds” to data types with limited bits:
  - Choose data types with more bits (C examples)
    - `int128_t` (128-bit integer)
    - `double` (64-bit floating-point number)
  - Use custom data types that are only bound by memory size
    - Python has `integer` and `decimal`
    - Java has `BigInteger` and `BigDecimal`
  - Rigorous testing of software ☺

Lecture Outline

- Floats Continued
- **Logical Operators**
  - **Shifting**
- Boolean Algebra

Logical Operations on `bool`

- Operations on Boolean (True/False) values
  - Likely familiar with most of these from Java
  - AND, OR, XOR, NOT

XOR == eXclusive OR

A	NOT A
False	True
True	False

A	B	A AND B	A OR B	A XOR B
False	False	False	False	False
False	True	False	True	True
True	False	False	True	True
True	True	True	True	False

Bits as "`bool`"

- A Boolean value can be represented by a single bit
  - 1 is true, 0 is false
  - We can represent our logical operations as operations on bits

A	NOT A
0	1
1	0

A	B	A AND B	A OR B	A XOR B
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

Bitwise Operators

Useful for HW01

- An individual bit is not a datatype, data types are group of bits. Instead, these operations work on all bits in a type
  - Each operator acts on each bit position independently
  - Consider the following examples on an imaginary 2-bit type
  - (Parenthesis in table contain the C syntax)

A	NOT A (~A)
00	11
01	10
10	01
11	00

A	B	A AND B (A & B)	A OR B (A | B)	A XOR B (A ^ B)
00	00	00	00	00
01	10	00	11	11
10	01	00	11	11
11	10	10	11	01
….	…	…	…	…
11	11	11	11	00

Bitwise Operators in C

Useful for HW01

- These bitwise operators exist in C
- Table below contains descriptions and example of syntax
  - Assume **`A`** and **`B`** are of type **`int`**

Logical Name	C Syntax	Example
AND	`&`	`A & B`
OR	`|`	`A | B`
XOR	`^`	`A ^ B`
NOT	`~`	`~A`

Shifting Bits

Useful for HW01

- Two more bitwise operators, left shift and right shift

Description	C Syntax	Bit pattern
Original x	`--`	`0b01101011`
X left shift by 1	`x << 1`	`0b11010110`
X right shift by 1	`x >> 1`	`0b00110101`
X left shift by 2	`x << 2`	`0b10101100`

  - Still confined to the size of the data type, bits can be shifted off on the left or right side.
  - During left shift, always fill in with 0's from the right
  - During a right shift: (More on these in a second)
    - Either fill with 0's from left (Logical)
    - Duplicate the MSB (Arithmetic)

Arithmetic vs Logical Shift

Useful for HW01

Description	Bit pattern
Original x	`0b10111011`
X >> 1 - X right shift by 1 (logical)	`0b01011101`
X >> 1 - X right shift by 1 (arithmetic)	`0b11011101`

- In C
  - The syntax for both shifts is the same (**`x >> 1`**)
  - the shift type is automatically chosen based on the data type
    - Unsigned types like **`unsigned int`** for logical right shift
    - Signed types like **`int`** or **`signed int`** for arithmetic right shift

Shifts & Powers of 2

Assume ints are 4 bits for examples

- When dealing with binary , Powers of 2 are everywhere
- Note that shifting to the left by one is the same as multiplying by 2

Before	Operation	After
`int x = 2; (0b0010)`	`x = x << 1;`	`x == 4; (0b0100)`

  - This extends to **`x << n`** being the same as **`x * 2`**^{n}

- Similar applies to right shifts for division

Before	Operation	After
`int x = -4; (0b1100)`	`x = x >> 1; (arithmetic)`	`x == -2; (0b1110)`
`unsigned int x = 12; (0b1100)`	`x = x >> 1; (logical)`	`x == 6; (0b0110)`

  - This extends to **`x >> n`** <u>usually</u> being the same as **`x / 2`**^{n}

Getting & Clearing Bits

Useful for HW01

- Can use a combination of shifts, ANDs and ORs to manipulate bits

0 indexed from the right

- Say I wanted to set get the 5th bit from an 8-bit integer 'a'
  - Answer **`(a >> 5) & 0x01`**
  - Walkthrough:

```
a = 0bYYXYYYYY  // X = bit we want
                // Y = bit we don't want
(a >> 5) = 0b*****YYX  // * = bit padded from
                       //     shift
(a >> 5) & 0x01 = 0b*****YYX
                 &0b00000001
                = 0b0000000X
```

At a bit level:

X & 0 = 0

x & 1 = X

Poll Everywhere

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- Which of the following sets the MSB of any unsigned 8-bit int 'a' to 0, and leaves the rest of the bits the same?

A. ((1 << 7) & a) ^ a

B. ~(1<<7) & a

C. ((a >> 7) & 0) << 7

D. I’m not sure

```
a =         0bXYYYYYYY
result =    0b0YYYYYYY
```

Poll Everywhere

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- Which of the following sets the MSB of any unsigned 8-bit int 'a' to 0, and leaves the rest of the bits the same?

A. ((1 << 7) & a) ^ a

B. ~(1<<7) & a

C. ((a >> 7) & 0) << 7

D. I'm not sure

```
a =         0bXYYYYYYY
result =    0b0YYYYYYY

a & 0b01111111 = result

a & ~(0b10000000) = result

a & ~(1 << 7) = result
```

Lecture Outline

- Floats Continued
- Logical Operators
  - Shifting
- **Boolean Algebra**

Disclaimer

- We just talked about bit-wise logical operators, and I will be using bit-wise operator syntax for the next section
  - 1 is still equal to TRUE
  - 0 is still equal to FALSE

- It may be easier to think of this next section as applying specifically to Boolean data types
  - (Though this can also be applied to bit-wise operators)
    - Treat True as the "all 1" bit pattern
    - Treat False as the "all 0" bit pattern

Useful for HW01

Boolean rules

- Identity
  - A & 1 = A
  - A & 0 = 0
  - A | 1 = 1
  - A | 0 = A
  - ~~A = NOT NOT A = A
- Associative
  - A & (B & C) = (A & B) & C
  - A | (B | C) = (A | B) | C
- Distributive
  - A & (B | C) = (A & B) | (A & C)
  - A | (B & C) = (A | B) & (A | C)
- More Identity
  - A & A = A
  - A | A = A
  - A & ~A = 0
  - A | ~A = 1

More on De Morgan's later

- De Morgan's Law
  - ~(A & B) = ~A | ~B
  - ~(A | B) = ~A & ~B

Truth Tables

- A table you can write for an expression to represent all possible combinations of input and output for an expression

- Truth Table for (A & (A & ~B)):

A (input)	B (input)	Output
0	0	0
0	1	0
1	0	1
1	1	0

Boolean Simplification

- We can apply rules to simplify Boolean patterns

- Consider the previous example
  - (A & (A & ~B))
  - ((A & A) & ~B) // By associative property
  - (A & ~B) // By distributive Property

- Consider:
  - (A | B) & (A | ~B)

Boolean rules

- Identity
  - A & 1 = A
  - A & 0 = 0
  - A | 1 = 1
  - A | 0 = A
  - ~~A = NOT NOT A = A
- Associative
  - A & (B & C) = (A & B) & C
  - A | (B | C) = (A | B) | C
- Distributive
  - A & (B | C) = (A & B) | (A & C)
  - A | (B & C) = (A | B) & (A | C)
- More Identity
  - A & A = A
  - A | A = A
  - A & ~A = 0
  - A | ~A = 1

More on De Morgan's soon

- De Morgan's Law
  - ~(A & B) = ~A | ~B
  - ~(A | B) = ~A & ~B

Simplify:

(A | B) & (A | ~B)

Boolean Simplification

- We can apply rules to simplify Boolean patterns

- Consider the previous example
  - (A & (A & ~B))
  - ((A & A) & ~B) // By associative property
  - (A & ~B) // By distributive Property

Simplification can have
Multiple correct simplifications

- Consider:
  - (A | B) & (A | ~B)
  - A | (B & ~B) // by distributive property
  - A | 0 // by identity property
  - A // by identity property

De Morgan's Law

- De Morgan's Law
  - ~(A & B) = ~A | ~B
  - ~(A | B) = ~A & ~B
- Provides a way to convert between AND to OR
  - (with some help from NOT)
- Truth Tables for proof:

A	B	~(A | B)	~A & ~B	~(A & B)	~A | ~B
0	0	1	1	1	1
0	1	0	0	1	1
1	0	0	0	1	1
1	1	0	0	0	0

De Morgan’s Law: Demo

- Write a statement equivalent to OR, but without using OR
  - A | B
  - ~~(A | B) // identity property
  - ~(~A & ~B) // De Morgan’s Law

- This still works for multi-bit data and bitwise operations

Useful for HW01

Boolean rules

These apply to multi-bit operations as well!

- Identity
  - A & 1 = A
  - A & 0 = 0
  - A | 1 = 1
  - A | 0 = A
  - ~~A = NOT NOT A = A

Bit-wise operations just follow these N times for N bits

- Associative
  - A & (B & C) = (A & B) & C
  - A | (B | C) = (A | B) | C
- Distributive
  - A & (B | C) = (A & B) | (A & C)
  - A | (B & C) = (A | B) & (A | C)
- More Identity
  - A & A = A
  - A | A = A
  - A & ~A = 0
  - A | ~A = 1
- De Morgan's Law
  - ~(A & B) = ~A | ~B
  - ~(A | B) = ~A & ~B

Next Lecture

- Next Time: We start hardware!
  - Start with Transistors & circuits
  - Booleans & bits will still be necessary
    - Be sure to be familiar with C bitwise ops, Boolean logic & De Morgan's Law

- HW00 Due this Friday!!!!

- HW01 & VM Setup to come out soon