Lecture 16

CIS 341: COMPILERS

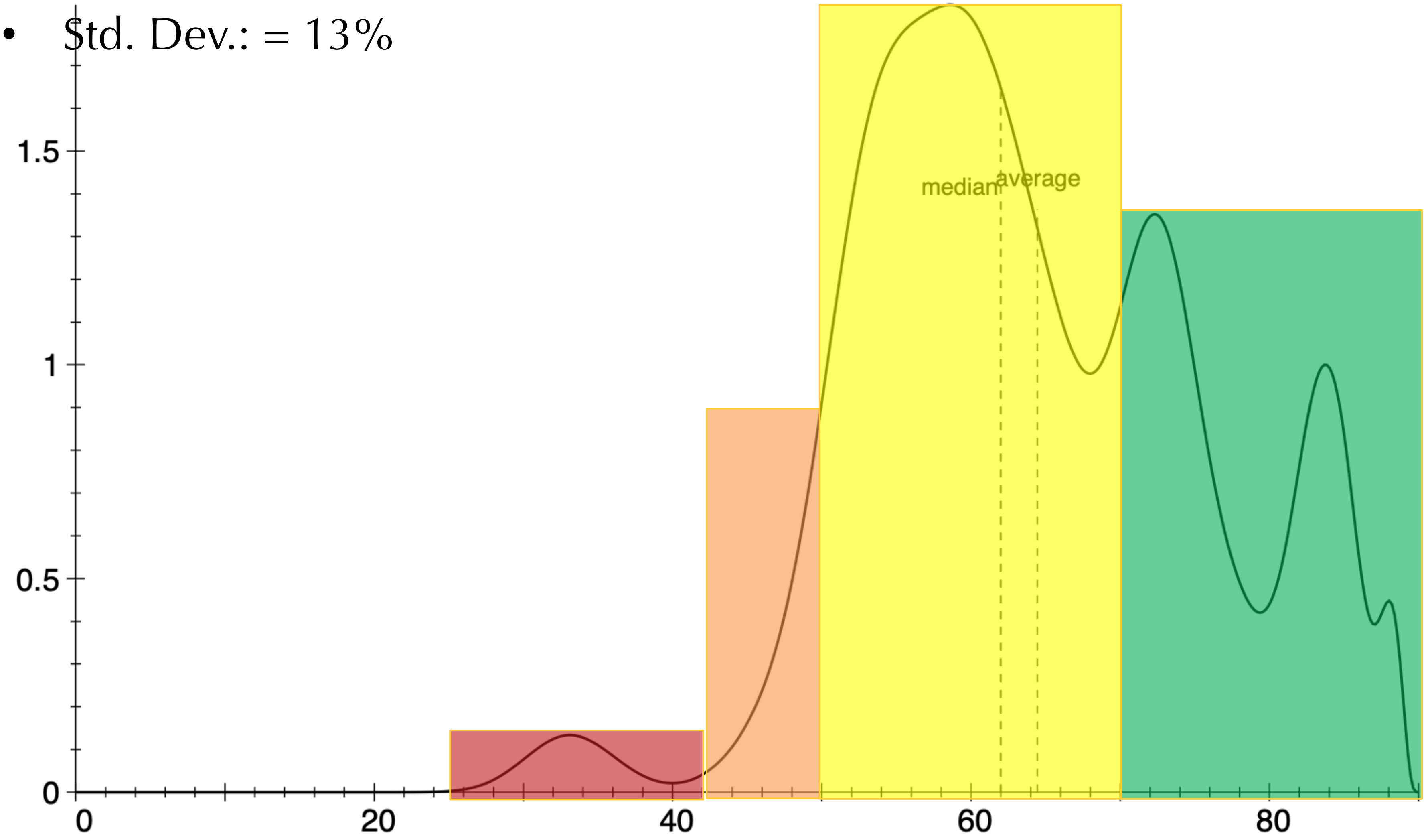
Announcements

- HW4: OAT v. 1.0
  - Parsing & basic code generation
  - **Due: Wednesday, March 23rd**

- HW5: OAT v. 2.0
  - Available Thursday or Friday
  - Records, function pointers, type checking, array-bounds checks, etc.
  - Due: Wednesday, April 13th

Midterm

- Average: 64/90 = 71.6%
- Median: 62/90 = 69%
- Std. Dev.: = 13%



See cc.ml

CLOSURE CONVERSION

Closure Conversion Summary

- A ***closure*** is a pair of an environment and a code pointer
  - the environment is a map data structure binding variables to values
  - environment could just be a list of the values (with known indices)

- Building a closure value:
  - code pointer is a function that takes an extra argument for the environment: A → B becomes (Env * A → B)
  - body of the closure "projects out" then variables from the environment
  - creates the environment map by bundling the free variables

- Applying a closure:
  - project out the environment, invoke the function (pointer) with the environment and its "real" argument

- Hoisting:
  - Once closure converted, all functions can be lifted to the top level

Scope, Types, and Context

STATIC ANALYSIS

Variable Scoping

- Consider the problem of determining whether a programmer-declared variable is in scope.
- Issues:
  - Which variables are available at a given point in the program?
  - Shadowing – is it permissible to re-use the same identifier, or is it an error?

- Example: The following program is syntactically correct but not well-formed. (y and q are used without being defined anywhere)

```
int fact(int x) {
  var acc = 1;
  while (x > 0) {
    acc = acc * y;
    x = q - 1;
  }
  return acc;
}
```

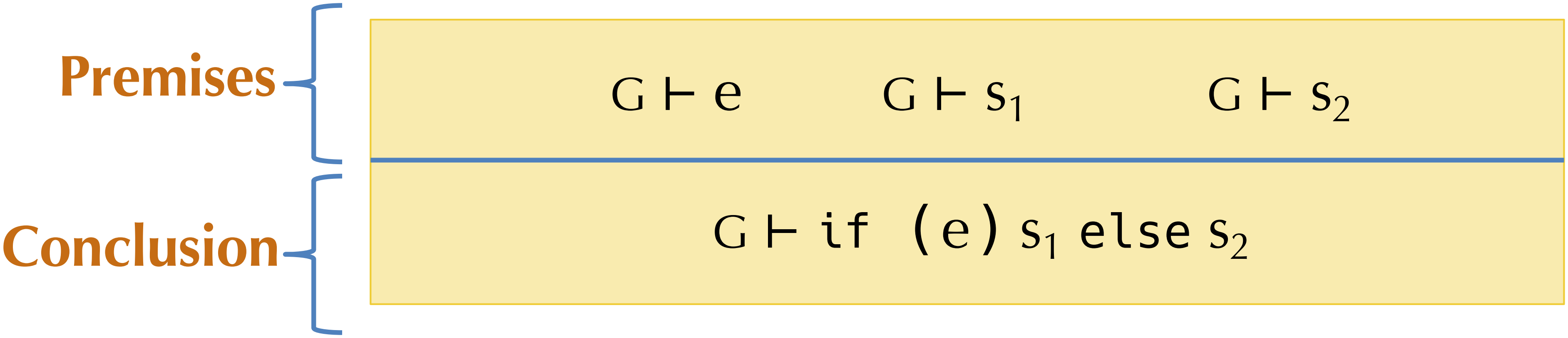
Q: Can we solve this problem by changing the parser to rule out such programs?

Inference Rules

- We can read a judgment $G \vdash e$ as
  "the expression e is well scoped and has free variables in G"
- For any environment G, expression e, and statements s_1, s_2.

$$G \vdash \texttt{if}\ (e)\ s_1\ \texttt{else}\ s_2$$

  holds if $G \vdash e$ and $G \vdash s_1$ and $G \vdash s_2$ all hold.
- More succinctly: we summarize these constraints as an ***inference rule***:



Premises

Conclusion

$$\frac{G \vdash e \qquad G \vdash s_1 \qquad G \vdash s_2}{G \vdash \texttt{if}\ (e)\ s_1\ \texttt{else}\ s_2}$$

- Such a rule can be used for *any* substitution of the syntactic metavariables G, e, s_1 and s_2.

Judgments

- A ***judgment*** is a (meta-syntactic) notation that *names* a relation among one or more sets.
  - The sets are usually built from object-language syntax elements and other "math" sets (*e.g.*, integers, natural numbers, *etc.*)
  - We usually describe them using metavariables that range over the sets.
  - Often use domain-specific notation to ease reading.
  - The meaning of judgments, *i.e.*, which sets they represent, is defined by (collections of) inference rules

- Example: When we say "G ⊢ e is a judgment where G is a context of variables and e is a term, defined by these […] inference rules" that is shorthand for this "math speak":
  - Let Var be the set of all (syntactic) variables
  - Let Exp be the set {e | e is a term of the untyped lambda calculus}
  - Let $\mathcal{P}$(Var) be the (finite) powerset of variables (set of all finite sets)
  - Define *well-scoped* ⊆ ($\mathcal{P}$(Var), Exp) to be a relation satisfying the properties defined by the associated inference rules […]
  - Then "G ⊢ e" is notation that means that (G, e) ∈ *well-scoped*

Scope-Checking Lambda Calculus

- Consider how to identify "well-scoped" lambda calculus terms
  - Given: G, a *set* of variable identifiers, e, a term of the lambda calculus
  - *Judgment*: $G \vdash e$ "the free variables of e are included in G"

$$\frac{x \in G}{G \vdash x}$$

"the variable x is free, but in scope"

$$\frac{G \vdash e_1 \qquad G \vdash e_2}{G \vdash e_1\ e_2}$$

"G contains the free variables of e_1 and e_2"

$$\frac{G \cup \{x\} \vdash e}{G \vdash \texttt{fun}\ x \rightarrow e}$$

"x is available in the function body e"

Scope-checking Code

- Compare the OCaml code to the inference rules:
  - structural recursion over syntax
  - the check either “succeeds” or ”fails"

```
let rec scope_check (g:VarSet.t) (e:exp) : unit =
    begin match e with
    | Var x -> if VarSet.member x g then () else failwith (x ^ "not in scope")
    | App(e1, e2) -> ignore (scope_check g e1); scope_check g e2
    | Fun(x, e)   -> scope_check (VarSet.union g (VarSet.singleton x)) e
    end
```

$$\frac{x \in G}{G \vdash x}$$

$$\boxed{\text{VAR}}$$

$$\frac{G \vdash e_1 \qquad G \vdash e_2}{G \vdash e_1\ e_2}$$

$$\boxed{\text{APP}}$$

$$\frac{G \cup \{x\} \vdash e}{G \vdash \texttt{fun}\ x \rightarrow e}$$

$$\boxed{\text{FUN}}$$

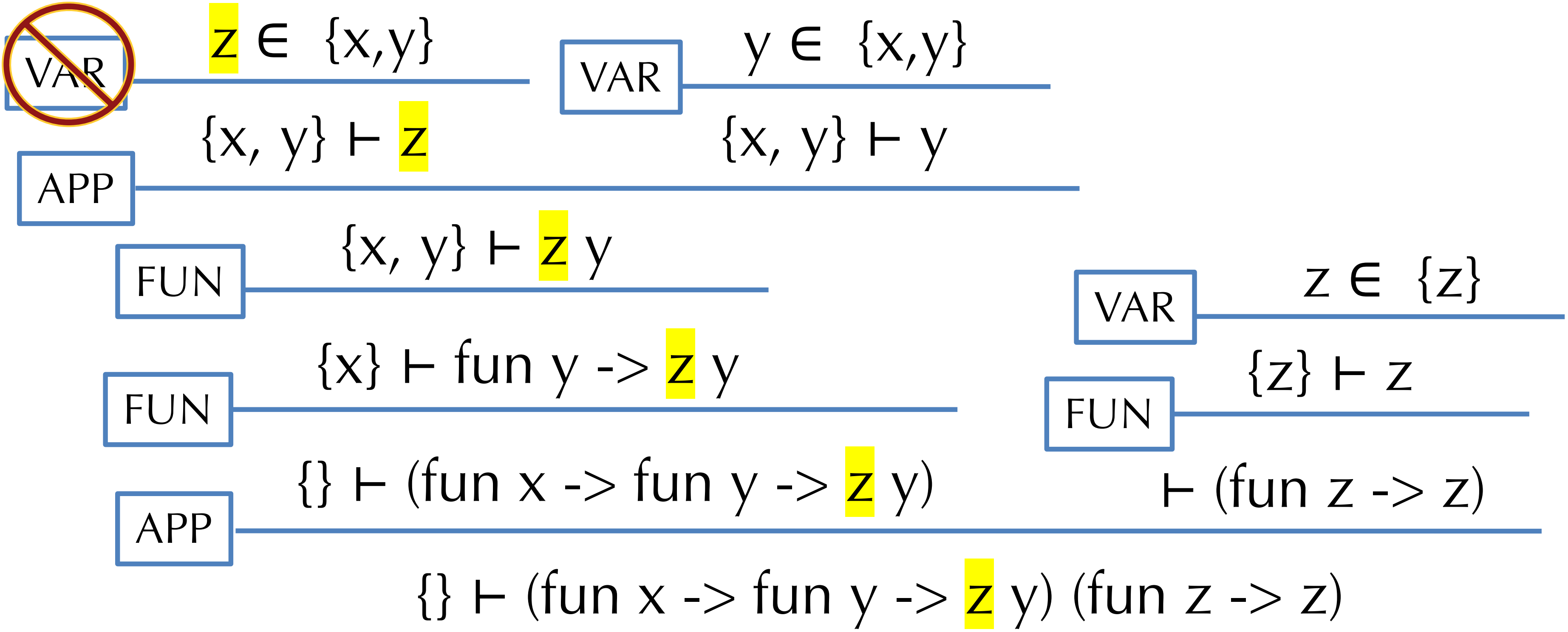
- The inference rules are a *specification* of the intended behavior of this scope checking code.
  - they don’t specify the order in which the premises are checked

Example Derivation Tree

$$\text{APP}\ \dfrac{\text{FUN}\ \dfrac{\text{FUN}\ \dfrac{\text{APP}\ \dfrac{\text{VAR}\ \dfrac{x \in \{x,y\}}{\{x, y\} \vdash x} \qquad \text{VAR}\ \dfrac{y \in \{x,y\}}{\{x, y\} \vdash y}}{\{x, y\} \vdash x\ y}}{\{x\} \vdash \text{fun } y \text{ -> } x\ y}}{\{\} \vdash (\text{fun } x \text{ -> fun } y \text{ -> } x\ y)} \qquad \text{FUN}\ \dfrac{\text{VAR}\ \dfrac{z \in \{z\}}{\{z\} \vdash z}}{\vdash (\text{fun } z \text{ -> } z)}}{\{\} \vdash (\text{fun } x \text{ -> fun } y \text{ -> } x\ y)\ (\text{fun } z \text{ -> } z)}$$

- Note: the OCaml function `scope_check` verifies the existence of this tree. The structure of the recursive calls when running `scope_check` is the same shape as this tree!
- Note that $x \in E$ is implemented by the function `VarSet.mem`

Example Failed Derivation



VAR

z ∈ {x,y}

{x, y} ⊢ z

VAR

y ∈ {x,y}

{x, y} ⊢ y

APP

{x, y} ⊢ z y

FUN

{x} ⊢ fun y -> z y

FUN

{} ⊢ (fun x -> fun y -> z y)

VAR

z ∈ {z}

{z} ⊢ z

FUN

⊢ (fun z -> z)

APP

{} ⊢ (fun x -> fun y -> z y) (fun z -> z)

- This program is *not* well scoped
  - The variable z is not bound in the body of the left function.
  - The typing derivation fails because the VAR rule cannot succeed
  - (The other parts of the derivation are OK, though!)

Uses of the inference rules

- We can do proofs by induction on the structure of the derivation.
- For example:

Lemma: If $G \vdash e$ then $fv(e) \subseteq G$.

Proof.

By induction on the derivation that $G \vdash e$.

$$\frac{x \in G}{G \vdash x}$$

- case: VAR then we have $e = x$ (for some variable x) and $x \in G$. But $fv(e) = fv(x) = \{x\}$, but then $\{x\} \subseteq G$.

$$\frac{G \vdash e_1 \quad G \vdash e_2}{G \vdash e_1\ e_2}$$

- case: APP then we have $e = e_1\ e_2$ (for some $e_1\ e_2$) and, by induction, we have $fv(e_1) \subseteq G$ and $fv(e_2) \subseteq G$, so $fv(e_1\ e_2) = fv(e_1) \cup fv(e_2) \subseteq G$

$$\frac{G \cup \{x\} \vdash e_1}{G \vdash \texttt{fun}\ x \rightarrow e_1}$$

- case: FUN then we have $e = (\text{fun } x \text{ -> } e_1)$ for some x, e_1 and, by induction, we have $fv(e_1) \subseteq G \cup \{x\}$, but then we also have $fv(\text{fun } x \text{ -> } e_1) = fv(e_1) \setminus \{x\} \subseteq ((G \cup \{x\}) \setminus \{x\}) \subseteq G$

$fv(x) = \{x\}$

$fv(\texttt{fun}\ x \rightarrow exp) = fv(exp) \setminus \{x\}$ *('x' is a bound in exp)*

$fv(exp_1\ exp_2) = fv(exp_1) \cup fv(exp_2)$

See tc.ml

STATICALLY RULING OUT PARTIALITY: TYPE CHECKING

Adding Integers to Lambda Calculus

exp ::=	
| …	
| n	*constant integers*
| $exp_1 + exp_2$	*binary arithmetic operation*
val ::=	
| `fun` x `->` exp	*functions are values*
| n	*integers are values*
$n\{v/x\} \quad = \quad n$	*constants have no free vars.*
$(e_1 + e_2)\{v/x\} \quad = (e_1\{v/x\} + e_2\{v/x\})$	*substitute everywhere*

$$\frac{exp_1 \Downarrow n_1 \quad exp2 \Downarrow n_2}{exp_1 + exp_2 \ \Downarrow (n1 \ [\![+]\!] \ n2)}$$

Object-level '+'

Meta-level '+'

NOTE: there are no rules for the case where exp1 or exp2 evaluate to functions! The semantics is *undefined* in those cases.

Type Checking / Static Analysis

- Recall the interpreter from the Eval3 module:

```
let rec eval env e =
  match e with
  | …
  | Add (e1, e2) ->
    (match (eval env e1, eval env e2) with
       | (IntV i1, IntV i2) -> IntV (i1 + i2)
       | _ -> failwith "tried to add non-integers")
  | …
```

- The interpreter might fail at runtime.
  - Not all operations are defined for all values (e.g., 3/0, 3 + true, …)
- A compiler can't generate sensible code for this case.
  - A naïve implementation might "add" an integer and a function pointer

Type Judgments

- In the judgment: $E \vdash e : t$
  - E is a ***typing environment*** or a ***type context***
  - E maps variables to types. It is just a set of bindings of the form: $x_1 : t_1, x_2 : t_2, \ldots, x_n : t_n$
- For example: $x : \text{int}, b : \text{bool} \vdash \text{if (b) 3 else } x : \text{int}$
- What do we need to know to decide whether "if (b) 3 else x" has type int in the environment x : int, b : bool?
  - b must be a bool i.e. $x : \text{int}, b : \text{bool} \vdash b : \text{bool}$
  - 3 must be an int i.e. $x : \text{int}, b : \text{bool} \vdash 3 : \text{int}$
  - x must be an int i.e. $x : \text{int}, b : \text{bool} \vdash x : \text{int}$

Simply-typed Lambda Calculus

- For the language in "tc.ml" we have five inference rules:

INT

$$\frac{}{E \vdash i : \text{int}}$$

VAR

$$\frac{x : T \in E}{E \vdash x : T}$$

ADD

$$\frac{E \vdash e_1 : \text{int} \qquad E \vdash e_2 : \text{int}}{E \vdash e_1 + e_2 : \text{int}}$$

FUN

$$\frac{E, x : T \vdash e : S}{E \vdash \text{fun } (x{:}T) \text{ -> } e \ : T \text{ -> } S}$$

APP

$$\frac{E \vdash e_1 : T \text{ -> } S \qquad E \vdash e_2 : T}{E \vdash e_1\ e_2 : S}$$

- Note how these rules correspond to the code.

Type Checking Derivations

- A *derivation* or *proof tree* has (instances of) judgments as its nodes and edges that connect premises to a conclusion according to an inference rule.
- Leaves of the tree are *axioms* (i.e. rules with no premises)
  - Example: the INT rule is an axiom
- Goal of the typechecker: verify that such a tree exists.
- Example: Find a tree for the following program using the inference rules on the previous slide:

⊢ (fun (x:int) -> x + 3) 5 : int

Example Derivation Tree

$$\text{APP}\ \dfrac{\text{FUN}\ \dfrac{\text{ADD}\ \dfrac{\text{VAR}\ \dfrac{x : \text{int} \in x : \text{int}}{x : \text{int} \vdash x : \text{int}} \qquad \text{INT}\ \dfrac{}{x : \text{int} \vdash 3 : \text{int}}}{x : \text{int} \vdash x + 3 : \text{int}}}{\vdash (\text{fun } (x{:}\text{int}) \text{ -> } x + 3) : \text{int -> int}} \qquad \text{INT}\ \dfrac{}{\vdash 5 : \text{int}}}{\vdash (\text{fun } (x{:}\text{int}) \text{ -> } x + 3)\ 5 : \text{int}}$$

- Note: the OCaml function `typecheck` verifies the existence of this tree. The structure of the recursive calls when running `typecheck` is the same shape as this tree!
- Note that $x : \text{int} \in E$ is implemented by the function `lookup`

Notes about this Typechecker

- The interpreter evaluates the body of a function only when it's applied.
- The typechecker always checks the body of the function
  - even if it's never applied
  - We *assume* the input has some type (say t_1) and reflect this in the type of the function (t_1 `->` t_2).

- Dually, at a call site (e_1 e_2), we don't know what *closure* we're going to get.
  - But we can calculate e_1's type, check that e_2 is an argument of the right type, and determine what type e_1 will return.

- Question: Why is this an approximation?
- Question: What if `well_typed` always returns `false`?

oat.pdf

TYPECHECKING OAT

Checking Derivations

- A *derivation* or *proof tree* has (instances of) judgments as its nodes and edges that connect premises to a conclusion according to an inference rule.
- Leaves of the tree are *axioms* (i.e. rules with no premises)
  - Example: the INT rule is an axiom
- Goal of the type checker: verify that such a tree exists.
- Example1: Find a tree for the following program using the inference rules in oat.pdf:

```
var x1 = 0;
var x2 = x1 + x1;
x1 = x1 - x2;
return(x1);
```

Example2: There is no tree for this ill-scoped program:

```
var x2 = x1 + x1;
return(x2);
```

Example Derivation

```
var x1 = 0;
var x2 = x1 + x1;
x1 = x1 - x2;
return(x1);
```

$$\dfrac{\dfrac{\mathcal{D}_1 \quad \mathcal{D}_2 \quad \mathcal{D}_3 \quad \mathcal{D}_4}{G_0\,;\cdot\,;\texttt{int} \vdash \texttt{var}\ x_1 = 0;\ \texttt{var}\ x_2 = x_1 + x_1\,;\ x_1 = x_1 - x_2\,;\ \texttt{return}\ x_1\,;\ \Rightarrow\ \cdot, x_1\!:\!\texttt{int}, x_2\!:\!\texttt{int}}\ [\text{STMTS}]}{\vdash \texttt{var}\ x_1 = 0;\ \texttt{var}\ x_2 = x_1 + x_1\,;\ x_1 = x_1 - x_2\,;\ \texttt{return}\ x_1\,;}\ [\text{PROG}]$$

Example Derivation

$$\mathcal{D}_1 \quad = \quad \dfrac{\dfrac{\dfrac{\dfrac{}{G_0\,;\cdot \vdash 0 : \texttt{int}}\,[\text{INT}]}{G_0\,;\cdot \vdash 0 : \texttt{int}}\,[\text{CONST}]}{G_0\,;\cdot \vdash \texttt{var}\ x_1 = 0 \Rightarrow \cdot, x_1 : \texttt{int}}\,[\text{DECL}]}{G_0\,;\ \cdot\ ;\texttt{int} \vdash \texttt{var}\ x_1 = 0;\ \Rightarrow \cdot, x_1 : \texttt{int}}\,[\text{SDECL}]$$

$$\mathcal{D}_2 \quad = \quad \dfrac{\dfrac{\dfrac{\dfrac{}{\vdash + : (\texttt{int},\texttt{int}) \rightarrow \texttt{int}}\,[\text{ADD}] \quad \dfrac{x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}}{G_0\,;\cdot, x_1 : \texttt{int} \vdash x_1 : \texttt{int}}\,[\text{VAR}] \quad \dfrac{x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}}{G_0\,;\cdot, x_1 : \texttt{int} \vdash x_1 : \texttt{int}}\,[\text{VAR}]}{G_0\,;\cdot, x_1 : \texttt{int} \vdash x_1 + x_1 : \texttt{int}}\,[\text{BOP}]}{G_0\,;\cdot, x_1 : \texttt{int}\,;\texttt{int} \vdash \texttt{var}\ x_2 = x_1 + x_1;\ \Rightarrow \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}\,[\text{DECL}]}{G_0\,;\cdot, x_1 : \texttt{int}\,;\texttt{int} \vdash \texttt{var}\ x_2 = x_1 + x_1;\ \Rightarrow \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}\,[\text{SDECL}]$$

Example Derivation

$$x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \;;$$

$$\mathcal{D}_3 \quad \dfrac{\dfrac{}{\vdash \texttt{-} : (\texttt{int}, \texttt{int}) \to \texttt{int}}\;[\textsc{add}] \quad \dfrac{x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}{G_0 ; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash x_1 : \texttt{int}}\;[\textsc{var}] \quad \dfrac{x_2 : \texttt{int} \in \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}{G_0 ; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash x_2 : \texttt{int}}\;[\textsc{var}]}{\dfrac{G_0 ; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash x_1 \;\texttt{-}\; x_2 : \texttt{int}}{G_0 ; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} ; \texttt{int} \vdash x_1 \;\texttt{=}\; x_1 \;\texttt{-}\; x_2 \texttt{;} \Rightarrow \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}\;[\textsc{assn}]}\;[\textsc{bop}]$$

$$\mathcal{D}_4 \quad = \quad \dfrac{\dfrac{x_1 : \texttt{int} \in \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}{G_0 ; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} \vdash x_1 : \texttt{int}}\;[\textsc{var}]}{G_0 ; \cdot, x_1 : \texttt{int}, x_2 : \texttt{int} ; \texttt{int} \vdash \texttt{return}\; x_1 \texttt{;} \Rightarrow \cdot, x_1 : \texttt{int}, x_2 : \texttt{int}}\;[\textsc{Ret}]$$

Type Safety

"Well typed programs do not go wrong."

– Robin Milner, 1978

Theorem: (simply typed lambda calculus with integers)

If $\vdash e : t$ then there exists a value v such that $e \Downarrow v$.

- Note: this is a *very* strong property.
  - Well-typed programs cannot "go wrong" by trying to execute undefined code (such as 3 + (fun x -> 2))
  - Simply-typed lambda calculus is guaranteed to terminate! (i.e. it isn't Turing complete)

Type Safety For General Languages

Theorem: (Type Safety)

If $\vdash P : t$ is a well-typed program, then either:

(a) the program terminates in a well-defined way, or

(b) the program continues computing forever

- Well-defined termination could include:
  - halting with a return value
  - raising an exception

- Type safety rules out undefined behaviors:
  - abusing "unsafe" casts: converting pointers to integers, etc.
  - treating non-code values as code (and vice-versa)
  - breaking the type abstractions of the language
- What is "defined" depends on the language semantics…

COMPILING

Compilation As Translating Judgments

- Consider the source typing judgment for source expressions:

$$C \vdash e : t$$

- How do we interpret this information in the target language?

$$[\![C \vdash e : t]\!] = \quad ?$$

- $[\![C]\!]$ translates contexts
- $[\![t]\!]$ is a target type
- $[\![e]\!]$ translates to a (potentially empty) stream of instructions, that, when run, computes the result into some operand

- INVARIANT: if $[\![C \vdash e : t\,]\!]$ = ty, operand , stream
  then the type (at the target level) of the operand is ty=$[\![t]\!]$

Example

- $C \vdash 341 + 5 : \text{int}$ what is $[\![C \vdash 341 + 5 : \text{int}]\!]$?

$[\![\vdash 341 : \text{int}]\!] =$ `(i64, Const 341, [])` $[\![\vdash 5 : \text{int}]\!] =$ `(i64, Const 5, [])`

-- --

$[\![C \vdash 341 : \text{int}]\!] =$ `(i64, Const 341, [])` $[\![C \vdash 5 : \text{int}]\!] =$ `(i64, Const  5, [])`

$[\![C \vdash 341 + 5 : \text{int}]\!] =$ `(i64, %tmp, [%tmp = add i64 (Const 341) (Const 5)])`

What about the Context?

- What is ⟦C⟧?
- Source level C has bindings like: x:int, y:bool
  - We think of it as a finite map from identifiers to types
- What is the interpretation of C at the target level?
- ⟦C⟧ maps source identifiers, "x" to source types and ⟦x⟧
- What is the interpretation of a variable ⟦x⟧ at the target level?
  - How are the variables used in the type system?

$$\frac{x : t \in L}{G ; L \vdash x : t} \quad \textbf{TYP\_VAR}$$

as expressions
(which denote values)

$$\frac{x : t \in L \quad G ; L \vdash exp : t}{G ; L ; rt \vdash x = exp ; \Rightarrow L} \quad \textbf{TYP\_ASSN}$$

as addresses
(which can be assigned)

Interpretation of Contexts

- ⟦C⟧ = a map from source identifiers to types and target identifiers

- INVARIANT:
  x:t ∈ C means that

  (1) lookup ⟦C⟧ x = (t, `%id_x`)
  (2) the (target) type of `%id_x` is ⟦t⟧* (a pointer to ⟦t⟧)

Interpretation of Variables

- Establish invariant for expressions:

$$\left\llbracket \frac{x{:}t \in L}{G;L \vdash x : t}\ \textbf{TYP\_VAR} \right\rrbracket = \texttt{(\%tmp, [\%tmp = load i64* \%id\_x])}$$

as expressions
(which denote values)

where (`i64`, `%id_x`) = lookup ⟦L⟧ x

- What about statements?

$$\left\llbracket \frac{x{:}t \in L \quad G;L \vdash exp : t}{G;L;rt \vdash x = exp;\ \Rightarrow L}\ \textbf{TYP\_ASSN} \right\rrbracket = \text{stream } @ \ \texttt{[store } \llbracket t \rrbracket \texttt{ opn, } \llbracket t \rrbracket\texttt{* \%id\_x]}$$

as addresses
(which can be assigned)

where (t, `%id_x`) = lookup ⟦L⟧ x
and ⟦G;L ⊢ exp : t⟧ = (⟦t⟧, `opn`, stream)

Other Judgments?

- Statement:
  ⟦C; rt ⊢ stmt ⇒ C′⟧ = ⟦C′⟧ , stream

- Declaration:
  ⟦G;L ⊢ t x = exp ⇒ G;L,x:t ⟧ = ⟦G;L,x:t⟧, stream

  INVARIANT: stream is of the form:

  ```
  stream’ @
  [ %id_x = alloca ⟦t⟧;
    store ⟦t⟧ opn, ⟦t⟧* %id_x ]
  ```

  and ⟦G;L ⊢ exp : t ⟧ = (⟦t⟧, `opn`, stream′)

- Rest follow similarly

COMPILING CONTROL

Translating while

- Consider translating "`while(e) s`":
  - Test the conditional, if true jump to the body, else jump to the label after the body.

`⟦C;rt ⊢ while(e) s ⇒ C'⟧ = ⟦C'⟧,`

```
lpre:
    opn = ⟦C ⊢ e : bool⟧
    %test = icmp eq i1 opn, 0
    br %test, label %lpost, label %lbody
lbody:
     ⟦C;rt ⊢ s ⇒ C'⟧
     br %lpre
lpost:
```

- Note: writing `opn = ⟦C ⊢ e : bool⟧` is pun
  - translating ⟦C ⊢ e : bool⟧ generates *code* that puts the result into `opn`
  - In this notation there is implicit collection of the code

Translating if-then-else

- Similar to while except that code is slightly more complicated because if-then-else must reach a merge and the else branch is optional.

$$\llbracket \mathtt{C;rt} \vdash \mathtt{if}\ (\mathtt{e}_1)\ \mathtt{s}_1\ \mathtt{else}\ \mathtt{s}_2 \Rightarrow \mathtt{C'} \rrbracket = \llbracket C' \rrbracket$$

```
    opn = ⟦C ⊢ e : bool⟧
    %test = icmp eq i1 opn, 0
    br %test, label %else, label %then
then:
     ⟦C;rt ⊢ s1 ⇒ C’⟧
     br %merge
else:
    ⟦C; rt s2 ⇒ C’⟧
     br %merge
merge:
```

Connecting this to Code

- Instruction streams:
  - Must include labels, terminators, and "hoisted" global constants

- Must post-process the stream into a control-flow-graph

- See frontend.ml from HW4

OPTIMIZING CONTROL

Standard Evaluation

- Consider compiling the following program fragment:

```
if (x & !y | !w)
  z = 3;
else
  z = 4;
return z;
```

```
    %tmp1 = icmp Eq ⟦y⟧, 0      ; !y
    %tmp2 = and ⟦x⟧ ⟦tmp1⟧
    %tmp3 = icmp Eq ⟦w⟧, 0
    %tmp4 = or %tmp2, %tmp3
    %tmp5 = icmp Eq %tmp4, 0
    br %tmp4, label %else, label %then

then:
    store ⟦z⟧, 3
    br %merge

else:
    store ⟦z⟧, 4
    br %merge

merge:
    %tmp5 = load ⟦z⟧
    ret %tmp5
```

Observation

- Usually, we want the translation $[\![e]\!]$ to produce a value
  - $[\![C \vdash e : t]\!]$ = (ty, operand, stream)
  - e.g. $[\![C \vdash e_1 + e_2 : \text{int}]\!]$ = (i64, `%tmp`, [`%tmp = add` $[\![e_1]\!]$ $[\![e_2]\!]$])

- But when the expression we're compiling appears in a test, the program jumps to one label or another after the comparison but otherwise never uses the value.

- In many cases, we can avoid "materializing" the value (i.e. storing it in a temporary) and thus produce better code.
  - This idea also lets us implement different functionality too:
    e.g. short-circuiting boolean expressions

Idea: Use a different translation for tests

Usual Expression translation:

$$[\![C \vdash e : t]\!] = (\text{ty, operand, stream})$$

Conditional branch translation of booleans, without materializing the value:

$$[\![C \vdash e : \text{bool@}]\!]\ \text{ltrue lfalse} = \text{stream}$$

$$[\![C, rt \vdash \text{if } (e) \text{ then s1 else s2} \Rightarrow C']\!] = [\![C']\!],$$

```
   insns3
then:
    ⟦s1⟧
    br %merge
else:
   ⟦s2⟧
    br %merge
merge:
```

Notes:

- takes two extra arguments: a “true” branch label and a “false” branch label.
- Doesn’t “return a value”

- Aside: this is a form of continuation-passing translation…

where

$[\![C, rt \vdash s_1 \Rightarrow C']\!] = [\![C']\!], insns_1$

$[\![C, rt \vdash s_2 \Rightarrow C'']\!] = [\![C'']\!], insns_2$

$[\![C \vdash e : \text{bool@}]\!]$ `then` `else` = $insns_3$

Short Circuit Compilation: Expressions

- $\llbracket C \vdash e : \text{bool@} \rrbracket\ \text{ltrue lfalse} = \text{insns}$

$$\frac{}{\llbracket C \vdash \text{false} : \text{bool@} \rrbracket\ \text{ltrue lfalse} = \texttt{[br \%lfalse]}}\ \text{FALSE}$$

$$\frac{}{\llbracket C \vdash \text{true} : \text{bool@} \rrbracket\ \text{ltrue lfalse} = \texttt{[br \%ltrue]}}\ \text{TRUE}$$

$$\frac{\llbracket C \vdash e : \text{bool@} \rrbracket\ \text{lfalse ltrue} = \text{insns}}{\llbracket C \vdash !e : \text{bool@} \rrbracket\ \text{ltrue lfalse} = \text{insns}}\ \text{NOT}$$

Short Circuit Evaluation

Idea: build the logic into the translation

$$\frac{[\![C \vdash e1 : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{right} = \mathrm{insns}_1 \qquad [\![C \vdash e2 : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{lfalse} = \mathrm{insns}_2}{[\![C \vdash e1 \,|\, e2 : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{lfalse} = }$$

```
    insns₁
right:
     insn₂
```

$$\frac{[\![C \vdash e1 : \mathrm{bool@}]\!]\ \mathrm{right}\ \mathrm{lfalse} = \mathrm{insns}_1 \qquad [\![C \vdash e2 : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{lfalse} = \mathrm{insns}_2}{[\![C \vdash e1 \& e2 : \mathrm{bool@}]\!]\ \mathrm{ltrue}\ \mathrm{lfalse} = }$$

```
    insns₁
right:
     insn₂
```

where `right` is a fresh label

Short-Circuit Evaluation

- Consider compiling the following program fragment:

```
if (x & !y | !w)
  z = 3;
else
  z = 4;
return z;
```

```
    %tmp1 = icmp Eq ⟦x⟧, 0
    br %tmp1, label %right2, label %right1

right1:
    %tmp2 = icmp Eq ⟦y⟧, 0
    br %tmp2, label %then, label %right2

right2:
    %tmp3 = icmp Eq ⟦w⟧, 0
    br %tmp3, label %then, label %else

then:
    store ⟦z⟧, 3
    br %merge

else:
    store ⟦z⟧, 4
    br %merge

merge:
    %tmp5 = load ⟦z⟧
    ret %tmp5
```

Why Inference Rules?

- They are a compact, precise way of specifying language properties.
  - E.g. ~20 pages for full Java vs. 100's of pages of prose Java Language Spec.

- Inference rules correspond closely to the recursive AST traversal that implements them

- Type checking (and type inference) is nothing more than attempting to prove a different judgment ($E \vdash e : t$) by searching backwards through the rules.

- Compiling in a context is nothing more than a collection of inference rules specifying yet a different judgment ($G \vdash src \Rightarrow target$)
  - Moreover, the compilation rules are very similar in structure to the typechecking rules

- Strong mathematical foundations
  - The "Curry-Howard correspondence": Programming Language ~ Logic, Program ~ Proof, Type ~ Proposition
  - See CIS 500 if you're interested in type systems!

Beyond describing “structure”… describing “properties”

Types as sets

Subsumption

TYPES, MORE GENERALLY

Arrays

- Array constructs are not hard
- First: add a new type constructor: T[]

NEW

$$\frac{E \vdash e_1 : \text{int} \qquad E \vdash e_2 : T}{E \vdash \text{new } T[e_1](e_2) \ : T[]}$$

e_1 is the size of the newly allocated array. e_2 initializes the elements of the array.

INDEX

$$\frac{E \vdash e_1 : T[] \qquad E \vdash e_2 : \text{int}}{E \vdash e_1[e_2] \ : T}$$

UPDATE

$$\frac{E \vdash e_1 : T[] \qquad E \vdash e_2 : \text{int} \qquad E \vdash e_3 : T}{E \vdash e_1[e_2] = e_3 \text{ ok}}$$

Note: These rules don't ensure that the array index is in bounds – that should be checked *dynamically*.

Tuples

- ML-style tuples with statically known number of products:
- First: add a new type constructor: $T_1 * \ldots * T_n$

TUPLE

$$\frac{E \vdash e_1 : T_1 \quad \ldots \quad E \vdash e_n : T_n}{E \vdash (e_1, \ldots, e_n) : T_1 * \ldots * T_n}$$

PROJ

$$\frac{E \vdash e : T_1 * \ldots * T_n \quad 1 \leq i \leq n}{E \vdash \#i\ e\ :\ T_i}$$

References

- ML-style references (note that ML uses only expressions)
- First, add a new type constructor: T ref

REF

$$\frac{E \vdash e : T}{E \vdash \text{ref } e : T \text{ ref}}$$

DEREF

$$\frac{E \vdash e : T \text{ ref}}{E \vdash \,!e \; : T}$$

ASSIGN

$$\frac{E \vdash e_1 : T \text{ ref} \qquad E \vdash e_2 : T}{E \vdash e_1 := e_2 \; : \text{unit}}$$

Note the similarity with the rules for arrays…