CIS 500 Software Foundations

Homework Assignment 5,

Simply-Typed Lambda Calculus; Simple Extensions

Due: Wednesday, October 27, 2004, by noon

Submission instructions: Same as last time.

1 Exercise TAPL 9.2.1

2 Exercise The following derivation trees of typing judgements in the simply typed λ-calculus with booleans are incomplete. If possible, fill in the missing types (marked with a $\_\_$) and complete the derivation tree. For terms that are not typable, prove it.

(a) $\dfrac{}{\emptyset \vdash \lambda x : \mathsf{B}.\lambda y : (\mathsf{B} \to \mathsf{B}).y\ x : \_\_}$ [T-???]

(b) $\dfrac{}{\emptyset \vdash (\lambda x : \_\_.x\ x)\ (\lambda x : \_\_.x\ x) : \_\_}$ [T-???]

(c) $\dfrac{}{x : \mathsf{B} \to \mathsf{B} \vdash (\lambda x : \mathsf{B}.x) : \_\_}$ [T-???]

(d) $\dfrac{}{x : \mathsf{B} \to \mathsf{B} \vdash (\lambda y : \mathsf{B}.x) : \_\_}$ [T-???]

(e) $\dfrac{}{g : \mathsf{B} \to \mathsf{B} \vdash \lambda y : \mathsf{B}.g\ (\mathsf{if}\ y\ \mathsf{then\ false\ else}\ y) : \_\_ \to \_\_}$ [T-???]

(f) $\dfrac{}{\emptyset \vdash \lambda x : \_\_.\lambda y : \_\_.x\ (\lambda f : \mathsf{B} \to \mathsf{B}.f\ y) : \_\_ \to \_\_ \to \mathsf{B}}$ [T-???]

3 Exercise TAPL 9.3.10

4 Exercise Recall the definition of the simply typed λ-calculus with pairs (see TAPL, Figure 11-5).

(a) State and prove the substitution lemma. (N.B., you only have to write down the cases that change when we add pairs.)

(b) Prove the preservation theorem for the same system. You may use any other lemmas (inversion, weakening, etc.) that you need without proof, but you must give their statements.

5 Exercise Prove part (1) of Theorem 9.5.2 in TAPL.

6 Debriefing

1. How many hours (per person) did you spend on this assignment?
2. Would you rate it as easy, moderate, or difficult?
3. Did everyone in your study group participate?
4. How deeply do you feel you understand the material it covers (0%–100%)?

If you have any other comments, we would like to hear them; please send them to `cis500@cis.upenn.edu`.