CIS 500 Software Foundations

Homework Assignment 1

Induction; Operational Semantics

Due: Monday, September 19, 2005, by noon

Submission instructions:

This assignment should be done together with your *study group*. Only one assignment should be submitted per group, and it does not matter which group member submits the assignment. However, make sure that all group members' names and emails are listed on the submission. These members will be in your group for the entire semester, unless there are special circumstances. If you receive help from someone outside of your study group, excluding the course staff, you must also acknowledge them on your assignment.

Solutions must be submitted electronically (in ascii, postscript, or PDF format). Follow the instructions at `http://www.seas.upenn.edu/~cis500/homework.html`.

1 Exercise Consider this simple grammar:

```
t ::= ⋆
      □
      t && t
      t || t
```

State the structural induction principle for terms `t`.

2 Exercise Using the terms from above, define the function *countstars* that returns the number of stars ($\star$) that appear in the term.

3 Exercise Consider the following relation, defined on terms from the previous exercises.

$$\frac{}{\star \sim \square}\ \textsc{Ax1} \qquad \frac{}{\square \sim \star}\ \textsc{Ax2}$$

$$\frac{\mathtt{t}_1 \sim \mathtt{t}_4 \qquad \mathtt{t}_2 \sim \mathtt{t}_3}{(\mathtt{t}_1\ \mathtt{\&\&}\ \mathtt{t}_2) \sim (\mathtt{t}_3\ \mathtt{\&\&}\ \mathtt{t}_4)}\ \textsc{Amp1} \qquad \frac{(\mathtt{t}_2\ \mathtt{\&\&}\ \mathtt{t}_1) \sim (\mathtt{t}_4\ \mathtt{\&\&}\ \mathtt{t}_3)}{(\mathtt{t}_1\ \mathtt{\&\&}\ \mathtt{t}_2) \sim (\mathtt{t}_3\ \mathtt{\&\&}\ \mathtt{t}_4)}\ \textsc{Amp2} \qquad \frac{\mathtt{t}_1 \sim \mathtt{t}_3 \qquad \mathtt{t}_2 \sim \mathtt{t}_4}{(\mathtt{t}_1\ \mathtt{||}\ \mathtt{t}_2) \sim (\mathtt{t}_3\ \mathtt{||}\ \mathtt{t}_4)}\ \textsc{Bars}$$

Are the following judgments derivable? If so, give a derivation. If not, briefly justify why.

(a) $(\star\ \mathtt{\&\&}\ \square) \sim (\star\ \mathtt{\&\&}\ \star)$

(b) $\big((\star\ \mathtt{||}\ \star)\ \mathtt{\&\&}\ (\star\ \mathtt{\&\&}\ \square)\big) \sim \big((\star\ \mathtt{\&\&}\ \square)\ \mathtt{\&\&}\ (\square\ \mathtt{||}\ \square)\big)$

4 Exercise What is the induction principle for the $\sim$ relation defined above?

5 Exercise Prove that the $\sim$ relation defined above is symmetric, using induction on the $\sim$ relation. Recall that the relation is symmetric if $\mathtt{t} \sim \mathtt{s}$ implies that $\mathtt{s} \sim \mathtt{t}$.

6 Exercise Prove that if $\mathtt{t} \sim \mathtt{s}$ is derivable, then it is derivable without using the rule Amp2.

7 Exercise Give a definition of multistep evaluation $\mathtt{t} \rightarrow^* \mathtt{t}'$ (Definition 3.5.9 in TAPL) using three inference rules.

8 Exercise The multistep evaluation rule can also be defined with two inference rules.

$$\frac{}{\mathtt{t} \Rightarrow \mathtt{t}}\ \textsc{Refl} \qquad \frac{\mathtt{t} \to \mathtt{t}' \quad \mathtt{t}' \Rightarrow \mathtt{t}''}{\mathtt{t} \Rightarrow \mathtt{t}''}\ \textsc{Trans}$$

(We use $\mathtt{t} \Rightarrow \mathtt{t}'$ for multi-step evaluation defined by the above rules to keep it clear which definition we mean.) Prove that this definition is equivalent to the one as in Definition 3.5.9. In other words a judgment $\mathtt{t} \to^* \mathtt{t}'$ is derivable if and only if the judgment $\mathtt{t} \Rightarrow \mathtt{t}'$ is derivable.

9 Exercise Exercise 3.5.16 in TAPL.

10 Exercise Exercise 3.5.17 in TAPL.

11 Exercise Suppose we remove the rule E-Succ from Figure 3-2 and replace E-PredSucc with the following:

$$\frac{}{\mathtt{pred\ (succ\ t)} \to \mathtt{t}}\ \textsc{E-PredSucc2}$$

Note the difference: E-PredSucc requires that the argument to `pred` to be a numeric value `nv` whereas E-PredSucc2 requires only that the argument look like `(succ t)`. Which of the following theorems are true of the language now? Explain why or why not.

(a) Every value is in normal form.

(b) If `t` is in normal form, then `t` is a value.

(c) Uniqueness of normal forms.

(d) Termination of evaluation.

12 Debriefing

1. How many hours did you spend on this assignment?
2. Would you rate it as easy, moderate, or difficult?
3. Did everyone in your study group participate?
4. How deeply do you feel you understand the material it covers (0%–100%)?

If you have any other comments, we would like to hear them; please send them `cis500@cis.upenn.edu`.

13 Final remarks Some solutions are sketched in the back of the book. You should write out your answers to the assignment before looking. Note that we will be looking for more detail in your proofs than is provided by the back of the book.