CIS 500 — Software Foundations

Homework Assignment 3

Induction and Operational Semantics

Due: Monday, September 25, 2006, by noon

Instructions: Use the same submission procedure as last time, paying attention to the following points:

- Submit this file as `hw3`, for example, using the command:

```
~cis500/bin/cis500submit hw3 hw3.pdf
```

- For this assignment, please write out proofs in detail, even though the arguments we are dealing with are pretty simple (i.e., in a journal paper, the whole proof would probably be "By straightforward induction"!). The proof of Theorem 3.5.4 is a good model.

1 Exercise Suppose we add a new term `flip` to the grammar in Figure 3-2 of TAPL, along with two evaluation rules:

$$\texttt{flip} \longrightarrow \texttt{true} \qquad \text{(E-Flip1)}$$

$$\texttt{flip} \longrightarrow \texttt{false} \qquad \text{(E-Flip2)}$$

Obviously, these additions break the property of determinacy of evaluation: given a term `t`, there may in general be more than one term `t'` such that $\texttt{t} \longrightarrow \texttt{t'}$. Similarly, the uniqueness of normal forms is lost.

Suppose we write $R(\texttt{t})$ for the set of values $\{\texttt{v} \mid \texttt{t} \longrightarrow^* \texttt{v}\}$.

a. Prove that $R(\texttt{t})$ is finite for every `t`.

b. Can we do better than just "finite"? I.e., is there a simple way of calculating, from `t`, an upper bound on the size of $R(\texttt{t})$?

2 Exercise [Required for all groups containing at least one PhD student; optional otherwise] Suppose, instead, that we add a new term `choose` to the grammar in Figure 3-2 of TAPL, along with this evaluation rule:

$$\texttt{choose} \longrightarrow \texttt{nv} \qquad \text{(E-Choose)}$$

That is, `choose` reduces in one step to an arbitrary numeric value.

Again, this addition breaks the properties of determinacy of evaluation and uniqueness of normal forms. Indeed, it even introduces infinite nondeterminism: it is easy to write down terms `t` for which $R(\texttt{t})$ is infinite.

a. For which natural numbers n is it the case that there is some term $\mathtt{t}$ with $|R(\mathtt{t})| = n$. Define a function that, for each n, yields such a term.

b. Prove that evaluation always terminates in the extended system.

3 Exercise TAPL exercise 3.5.16.

Note: Solutions to this exercise and the next one can be found in the back of TAPL. Feel free to compare your solutions with these. However, to get the benefit of the exercise, it is important that you write out your own solutions in full *before* looking at ours.

4 Exercise TAPL exercise 3.5.17.

5 Debriefing

1. Approximately how many hours *per person* (on average) did you spend on this assignment?
2. Would you rate it as easy, moderate, or difficult?
3. How deeply do you feel you understand the material it covers (0%–100%)?
4. Any other comments?