

CIS 500 — Software Foundations

Homework Assignment 5

Simple types

Due: Monday, October 9, 2006, by noon

Submit your solutions as `hw5`, for example using the command:

```
~cis500/bin/cis500submit hw5 hw5.pdf
```

1 Exercise Give a typing derivation showing that:

$$\text{if true then 0 else succ (pred 0)} \quad : \quad \text{Nat}$$

2 Exercise Exercise 8.3.4 in TAPL.

3 Exercise Exercise 8.3.6 in TAPL.

4 Exercise Each part of this exercise suggests a different way of changing the language of typed arithmetic and boolean expressions from Chapter 8. (Note that these changes are not cumulative: each part starts from the ordinary language of Chapter 8) For each part, say which of the following properties are true in the enriched system.

- Uniqueness of types
- Preservation
- Progress

For each the properties that fail, give a counter-example. For those that hold, briefly explain why. (You don't need to give a full proof — just an intuition.)

1. Suppose we remove the typing rule E-PREDZERO.
2. Suppose we add the following typing axiom:

$$0 : \text{Bool}$$

3. Suppose we add a new evaluation axiom:

$$\text{if } t_1 \text{ then } t_2 \text{ else } t_3 \longrightarrow t_2$$

4. Suppose we add the following typing rule:

$$\frac{t_1 : \text{Nat} \quad t_2 : T \quad t_3 : T}{\text{if } t_1 \text{ then } t_2 \text{ else } t_3 : T}$$

5. Suppose we add two new evaluation rules

$$\text{pred true} \longrightarrow \text{false}$$
$$\text{pred false} \longrightarrow \text{true}$$

and the following typing rule:

$$\frac{t_1 : \text{Bool}}{\text{pred } t_1 : \text{Bool}}$$

6. Suppose we add a new type `Foo` and two new typing rules:

$$\frac{t_1 : \text{Nat}}{\text{pred } t_1 : \text{Foo}}$$
$$\frac{t_1 : \text{Foo}}{\text{succ } t_1 : \text{Nat}}$$

5 Exercise What rules could we add to the evaluation relation to recover the properties that failed in part 4 of exercise 4?

6 Exercise State the inversion lemma (i.e., the appropriate analog of Lemma 8.2.2) for the enriched typing relation from part 6 of exercise 4.

7 Exercise Read through the explanation and proof of the “Preservation of Types Under Substitution” Lemma (9.3.8). Then *without looking at the book*, write out the proof again in your own words.

8 Exercise Exercise 9.3.9 in TAPL.

9 Exercise Required for groups containing at least one PhD student. Optional for others. Exercise 9.3.10 in TAPL.

10 Debriefing

1. Approximately how many hours (per person, on average) did you spend on this assignment?
2. Would you rate it as easy, moderate, or difficult?
3. How deeply do you feel you understand the material it covers (0%–100%)?
4. Any other comments?