CIS 500
Software Foundations
Fall 2006

November 6

Some Hints

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- There *will* be a question that is also a one-star exercise from the book.
- There *will* be a question similar to problem 6 from midterm 1 ("Which properties remain true if we change one fo the type systems we've studied in the following way...?")
- There *will* be (at least) one question based on one of the proofs in chapter 15.
- For PhD students, there *will* be a question involving subtyping and references.

Review

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  ```
- ```
  try
    (if (λx:Bool. x) error
       then (error false)
       else error)
  with
    λy:Bool→Bool. y
  ```

For reference: Typing rules for exceptions

$$\Gamma \vdash \texttt{error} : \texttt{T} \qquad \text{(T-Error)}$$

$$\frac{\Gamma \vdash \texttt{t}_1 : \texttt{T} \qquad \Gamma \vdash \texttt{t}_2 : \texttt{T}}{\Gamma \vdash \texttt{try } \texttt{t}_1 \texttt{ with } \texttt{t}_2 : \texttt{T}} \qquad \text{(T-Try)}$$

Give the result of evaluation and the final store after each of these expressions is evaluated to a normal form starting in the empty store.

- ```
  let x = ref 0 in
  let y = ref 1 in
  y:=3
  !x
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  ```

For reference: Evaluation rules for references

$$\frac{l \notin dom(\mu)}{\texttt{ref } \texttt{v}_1 \mid \mu \longrightarrow l \mid (\mu, l \mapsto \texttt{v}_1)} \qquad \text{(E-RefV)}$$

$$\frac{\mu(l) = \texttt{v}}{\texttt{!}l \mid \mu \longrightarrow \texttt{v} \mid \mu} \qquad \text{(E-DerefLoc)}$$

$$l \texttt{:=} \texttt{v}_2 \mid \mu \longrightarrow \texttt{unit} \mid [l \mapsto \texttt{v}_2]\mu \qquad \text{(E-Assign)}$$

(Plus several congruence rules.)

Which of the following functions *could* evaluate to 42 when applied to a single argument and evaluated with a store of the appropriate type?

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- λx:Ref Nat. x
- λx:Ref Nat. (x:=3; l_1:=42; !l_1)
- λf:Unit$\to$Unit. (l_1:=3; f unit; !l_1)

Preservation and progress for chapter 13

- The preservation and progress proofs for $\lambda_{\rightarrow}$ with references are just sketched in TAPL.
- Working out the details for yourself is an excellent exercise
- A question based on this proof may appear on the final exam, but will *not* appear on the coming midterm

Subtyping

For each of the following pairs of terms, say whether the one on the left is a subtype of the one on the right, a supertype, equivalent, or incomparable.

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- `{a:Top, b:{c:Top,d:Top}}` and `{b:{d:Top,c:Top},a:Top}`

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- `<l:Top,m:{n:Top}>` and `<m:{n:Top,o:Top}>`
- `<>→Top` and `{}→Top`

Subtyping

Draw a subtyping derivation for the following statement:

$$\texttt{(Top}\rightarrow\texttt{\{x:Nat\})}\rightarrow\texttt{\{x:Nat,y:Nat\}} \quad <: \quad \texttt{(\{\}}\rightarrow\texttt{\{\})}\rightarrow\texttt{\{y:Nat\}}$$

For reference: Subtyping rules

$$\mathtt{S} <: \mathtt{S} \qquad \text{(S-Refl)}$$

$$\frac{\mathtt{S} <: \mathtt{U} \qquad \mathtt{U} <: \mathtt{T}}{\mathtt{S} <: \mathtt{T}} \qquad \text{(S-Trans)}$$

$$\{\mathtt{l}_i\mathtt{:T}_i \;^{i\in 1..n+k}\} <: \{\mathtt{l}_i\mathtt{:T}_i \;^{i\in 1..n}\} \qquad \text{(S-RcdWidth)}$$

$$\frac{\text{for each } i \qquad \mathtt{S}_i <: \mathtt{T}_i}{\{\mathtt{l}_i\mathtt{:S}_i \;^{i\in 1..n}\} <: \{\mathtt{l}_i\mathtt{:T}_i \;^{i\in 1..n}\}} \qquad \text{(S-RcdDepth)}$$

$$\frac{\{\mathtt{k}_j\mathtt{:S}_j \;^{j\in 1..n}\} \text{ is a permutation of } \{\mathtt{l}_i\mathtt{:T}_i \;^{i\in 1..n}\}}{\{\mathtt{k}_j\mathtt{:S}_j \;^{j\in 1..n}\} <: \{\mathtt{l}_i\mathtt{:T}_i \;^{i\in 1..n}\}} \qquad \text{(S-RcdPerm)}$$

$$\frac{\mathtt{T}_1 <: \mathtt{S}_1 \qquad \mathtt{S}_2 <: \mathtt{T}_2}{\mathtt{S}_1 \to \mathtt{S}_2 <: \mathtt{T}_1 \to \mathtt{T}_2} \qquad \text{(S-Arrow)}$$

$$\mathtt{S} <: \mathtt{Top} \qquad \text{(S-Top)}$$

Ascription as a derived form

- Someone asked to work exercise 11.4.1 part 2 today.
- But the solution is somewhat technical and would take too much time to discuss in detail.
- This exercise is not needed for the exam.

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