1 Exercise Show the derivations of the following typing judgments:

1. `if false then 0 else succ 0 : Nat`

2. `succ (if iszero 0 then succ 0 else pred 0) : Nat`

1 Solution This solution uses the `prooftree` package.

1.

$$
\cfrac{
\cfrac{}{\texttt{false : Bool}}\text{ T-FALSE}
\qquad
\cfrac{}{\texttt{0 : Nat}}\text{ T-ZERO}
\qquad
\cfrac{\cfrac{}{\texttt{0 : Nat}}\text{ T-ZERO}}{\texttt{succ 0 : Nat}}\text{ T-SUCC}
}{\texttt{if false then 0 else succ 0 : Nat}}\text{ T-IF}
$$

2.

$$
\cfrac{
\cfrac{
\cfrac{\cfrac{}{\texttt{0 : Nat}}\text{ T-ZERO}}{\texttt{iszero 0 : Bool}}\text{ T-ISZERO}
\quad
\cfrac{\cfrac{}{\texttt{0 : Nat}}\text{ T-ZERO}}{\texttt{succ 0 : Nat}}\text{ T-SUCC}
\quad
\cfrac{\cfrac{}{\texttt{0 : Nat}}\text{ T-ZERO}}{\texttt{pred 0 : Nat}}\text{ T-PRED}
}{\texttt{if iszero 0 then succ 0 else pred 0 : Nat}}\text{ T-IF}
}{\texttt{succ (if iszero 0 then succ 0 else pred 0) : Nat}}\text{ T-SUCC}
$$

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2 Exercise Show the derivations of the following typing judgments:

1. `if false then 0 else succ 0 : Nat`

2 Solution This solution uses the `proof` package.

1.

$$
\cfrac{
\cfrac{}{\texttt{false : Bool}}\text{ T-FALSE}
\quad
\cfrac{}{\texttt{0 : Nat}}\text{ T-ZERO}
\quad
\cfrac{\cfrac{}{\texttt{0 : Nat}}\text{ T-ZERO}}{\texttt{succ 0 : Nat}}\text{ T-SUCC}
}{\texttt{if false then 0 else succ 0 : Nat}}\text{ T-IF}
$$