Lecture 15: Convolutional Neural Networks

Mar 13, 2023

CIS 4190/5190

Spring 2023

Course Progress

Till now: foundational algorithms applicable to large classes of machine learning problems.

Going forward: applications to specific types of data and specific types of problems.

- New Types of Data: Grids (e.g. Images), Sequences (e.g. Language)
- New Types of Problems: Making Sequences of Decisions (e.g. Robotics), Recommendation Systems
- Ethics

Other Types of Data

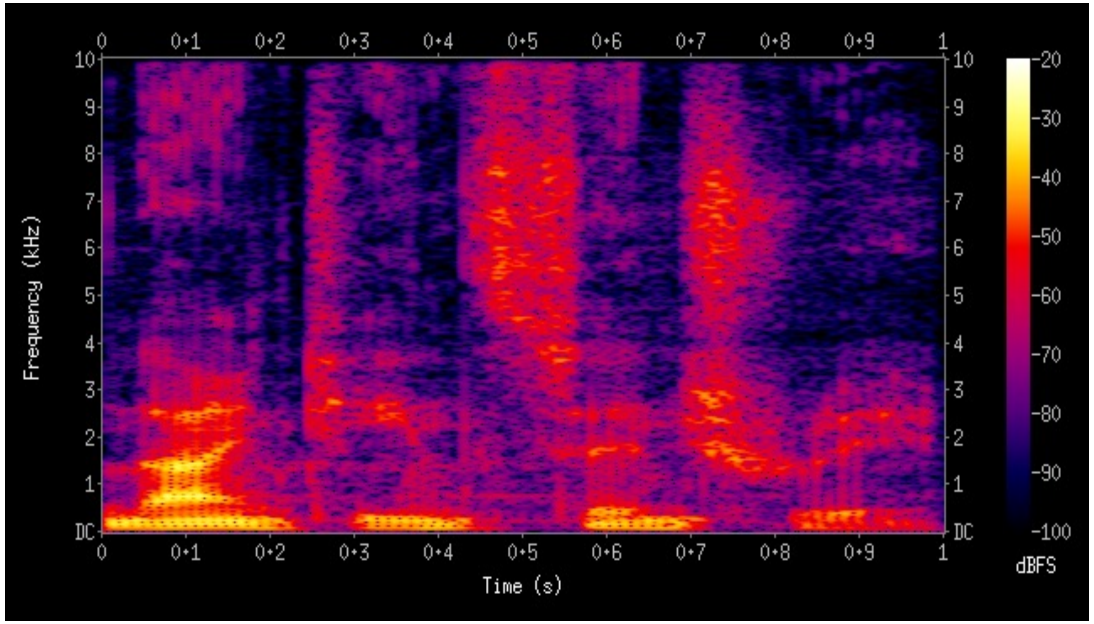
- Until now, the i^{th} sample in our dataset was either naturally a **vector** x_i or we converted it into one.

- What if our data samples were more naturally expressed in a different structure?
  - $\boldsymbol{x}_i$ is a "grid": e.g. images
  - $\boldsymbol{x}_i$ is a "sequence": e.g. text
  - $\boldsymbol{x}_i$ is a "graph": e.g. protein structure

Neural Networks Specialized to Grid Data

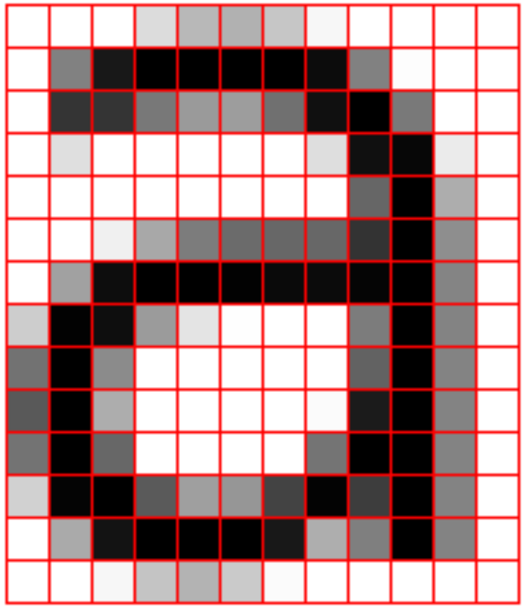
- We will study a class of neural networks called convolutions that specialize to properties often present in *grid* data, particularly images.

https://en.wikipedia.org/wiki/Spectrogram



Spectrogram encoding of audio

https://towardsdatascience.com/convolutional-neural-networks-explained-9cc5188c4939



1.0	1.0	1.0	0.9	0.6	0.6	0.6	1.0	1.0	1.0	1.0	1.0
1.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.5	1.0	1.0	1.0
1.0	0.2	0.2	0.5	0.6	0.6	0.5	0.0	0.0	0.5	1.0	1.0
1.0	0.9	1.0	1.0	1.0	1.0	1.0	0.9	0.0	0.0	0.9	1.0
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.5	0.0	0.5	1.0
1.0	1.0	1.0	0.5	0.5	0.5	0.5	0.5	0.4	0.0	0.5	1.0
1.0	0.4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	1.0
0.9	0.0	0.0	0.6	1.0	1.0	1.0	1.0	0.5	0.0	0.5	1.0
0.5	0.0	0.6	1.0	1.0	1.0	1.0	1.0	0.5	0.0	0.5	1.0
0.5	0.0	0.7	1.0	1.0	1.0	1.0	1.0	0.0	0.0	0.5	1.0
0.6	0.0	0.6	1.0	1.0	1.0	1.0	0.5	0.0	0.0	0.5	1.0
0.9	0.1	0.0	0.6	0.7	0.7	0.5	0.0	0.5	0.0	0.5	1.0
1.0	0.7	0.1	0.0	0.0	0.0	0.1	0.9	0.8	0.0	0.5	1.0
1.0	1.0	1.0	0.8	0.8	0.9	1.0	1.0	1.0	1.0	1.0	1.0

Digital image

Images as 2D Arrays

What we see

0	3	2	5	4	7	6	9	8
3	0	1	2	3	4	5	6	7
2	1	0	3	2	5	4	7	6
5	2	3	0	1	2	3	4	5
4	3	2	1	0	3	2	5	4
7	4	5	2	3	0	1	2	3
6	5	4	3	2	1	0	3	2
9	6	7	4	5	2	3	0	1
8	7	6	5	4	3	2	1	0

What a computer sees

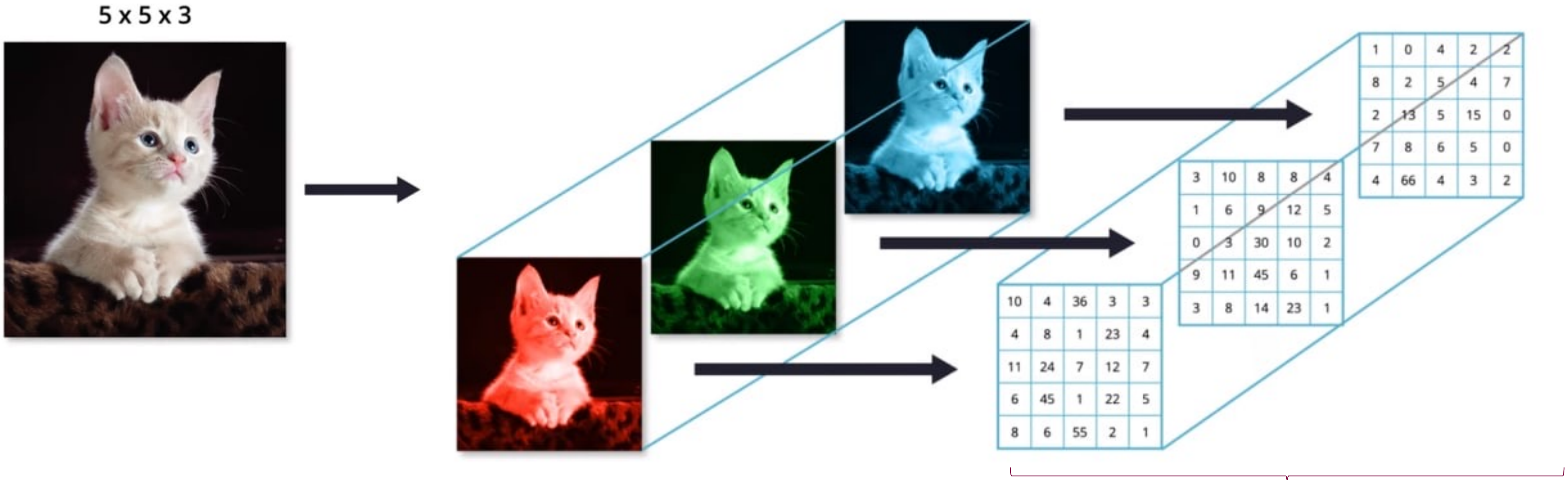
Computer vision:

How to extract meaning out of these 2D arrays?

Note: for color images, a stack of (typically 3) 2D arrays, each called a “channel”.

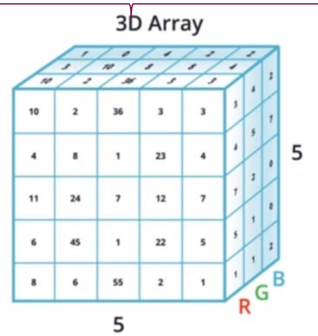
Source: S. Narasimhan, S. Lazebnik

Color Images Are 3D Arrays with 2 Spatial Dimensions



RGB encoding

(Red, Green, Blue “channels”)



We will see: convenient to deal with the ***spatial*** *dimensions* separately, and there are still only two of those.

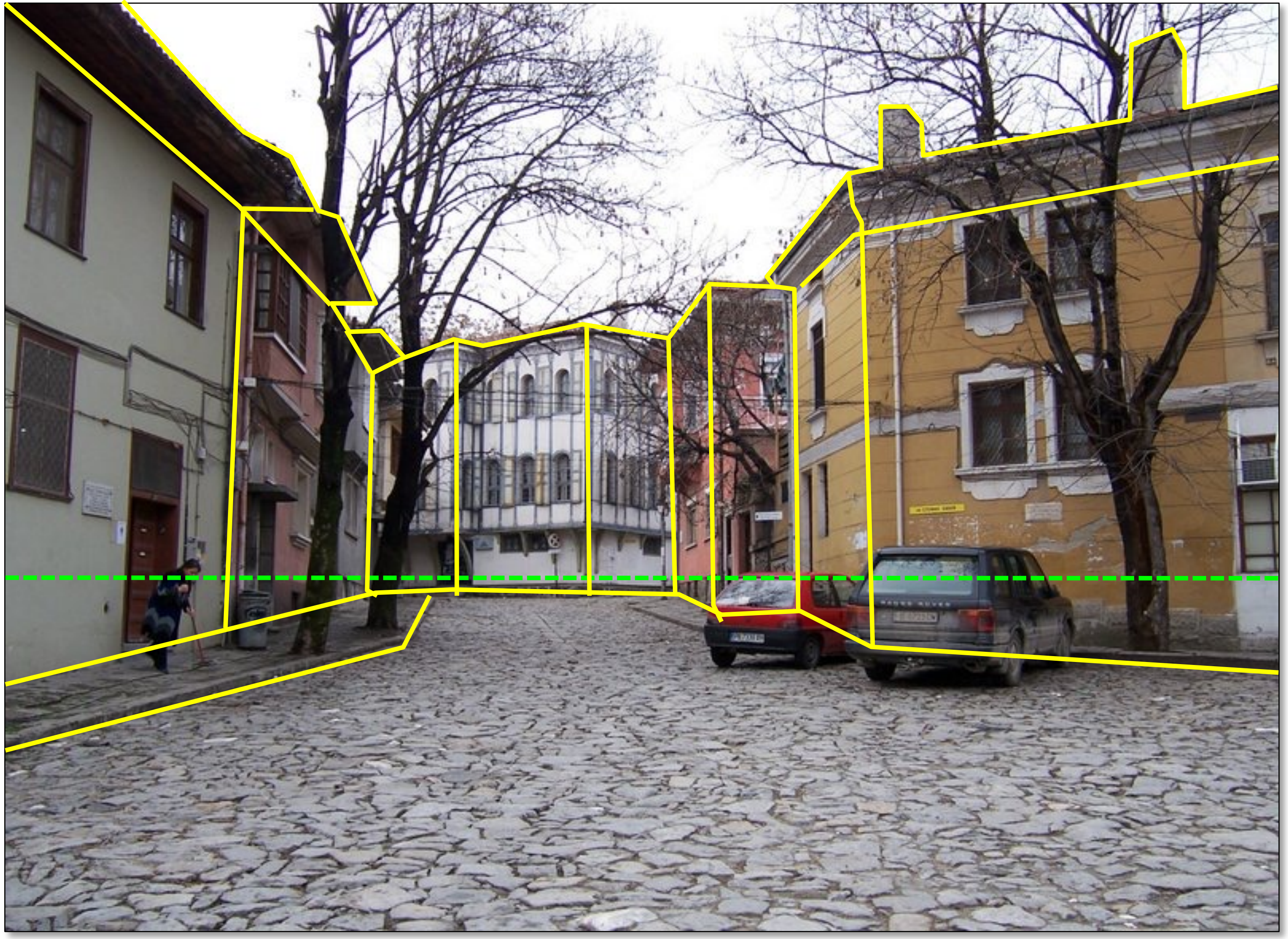
https://dev.to/sandeepbalachandran/machine-learning-going-furthur-with-cnn-part-2-41km

What Info can be Extracted from Images?

Source: S. Lazebnik

What Info can be Extracted from Images?

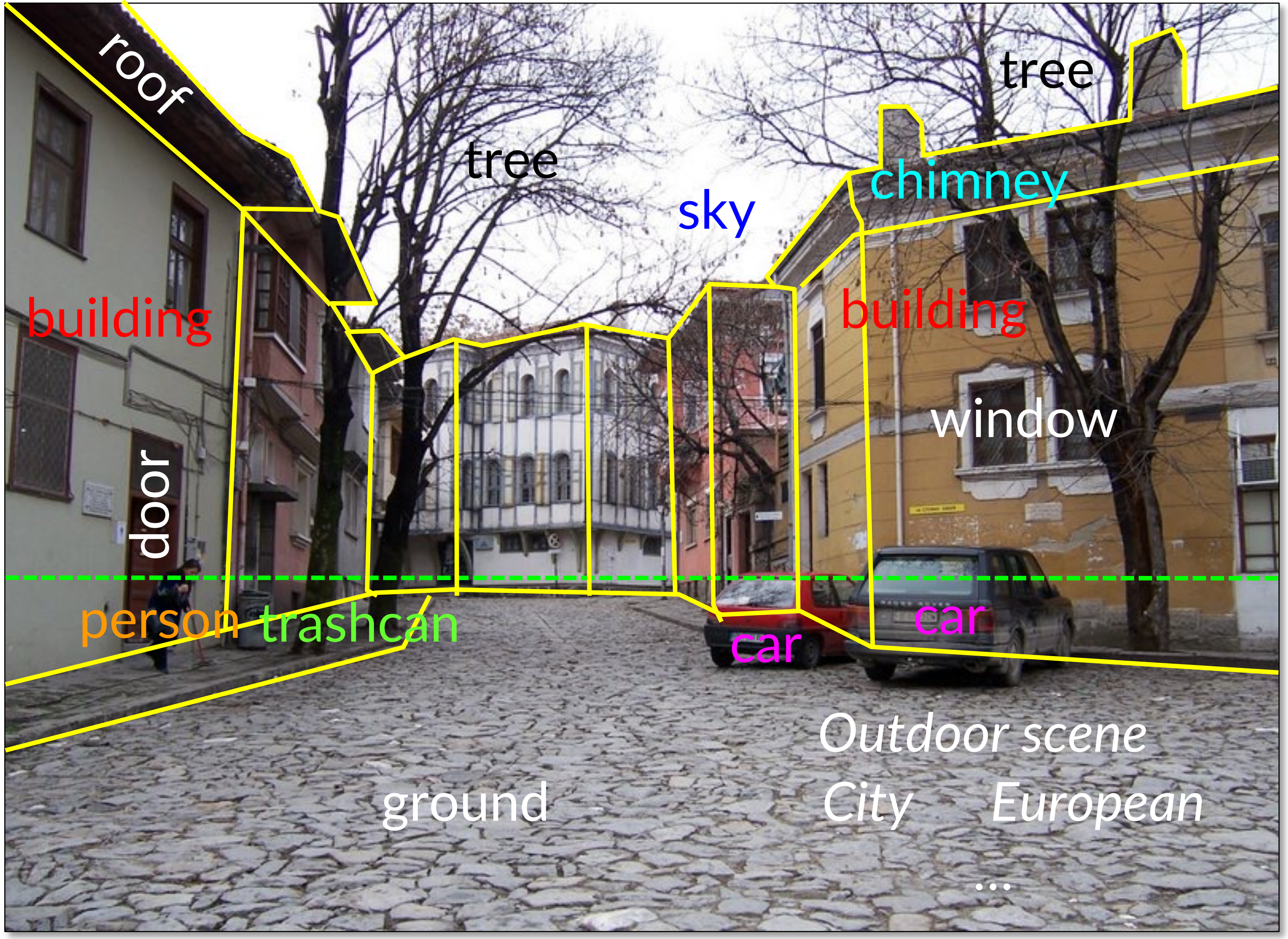
geometric
information



Source: S. Lazebnik

What Info can be Extracted from Images?

geometric information



semantic information

Source: S. Lazebnik

Vision is Deceptively Hard!

In the 1960s, Marvin Minsky assigned a couple of undergrads to spend the summer programming a computer to use a camera to identify objects in a scene. He figured they'd have the problem solved by the end of the summer.

Half a century later, we're still working on it.

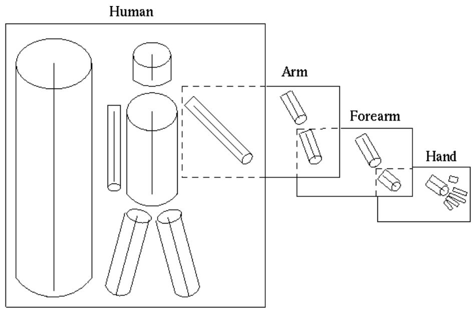
Source: XKCD

The Treachery of Images – Rene Magritte

"This is not a pipe"

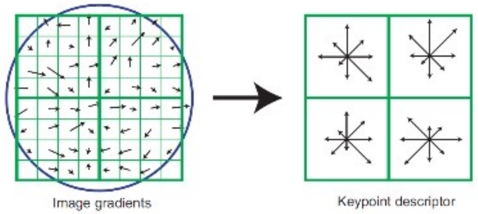
Vision often involves making educated guesses.

ML in Computer Vision



The *very* old: 1960's - Mid 1990's

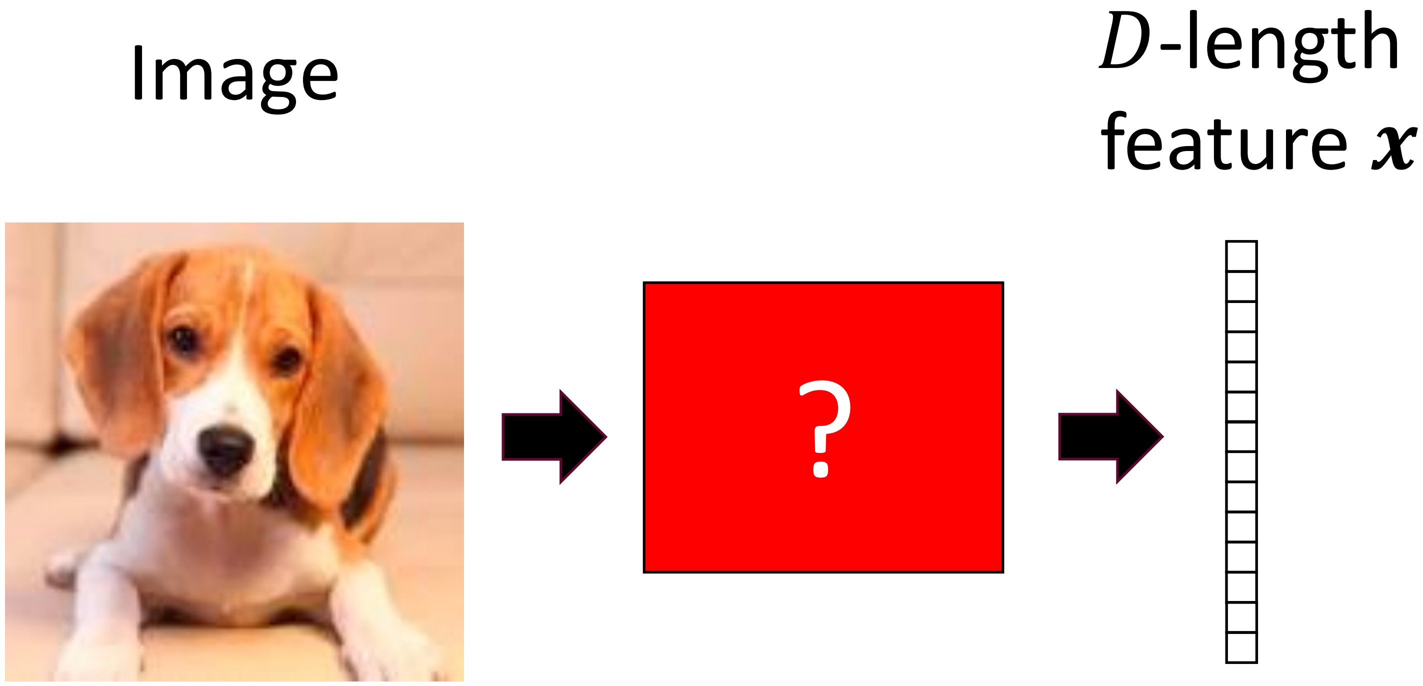
Image → hand-def. features → hand-def. classifier



The old: Mid 1990's – 2012

Image → hand-def. features → learned classifier

What Should Good Visual Representations Do?

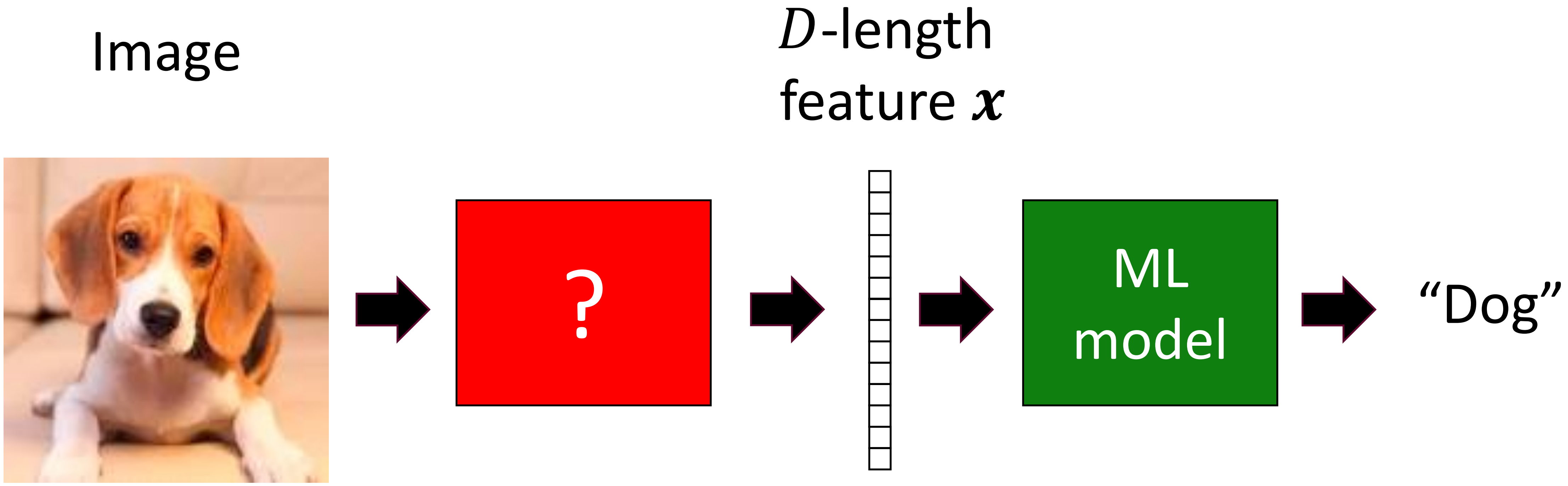


What Should Good Visual Representations Do?

What is a "good" feature space?

Good features make useful tasks easy to perform.

What Should Good Visual Representations Do?

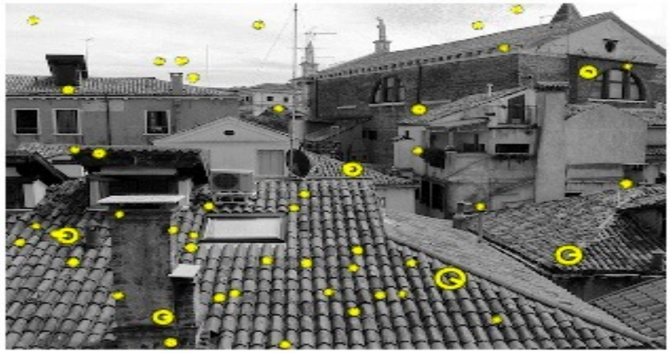


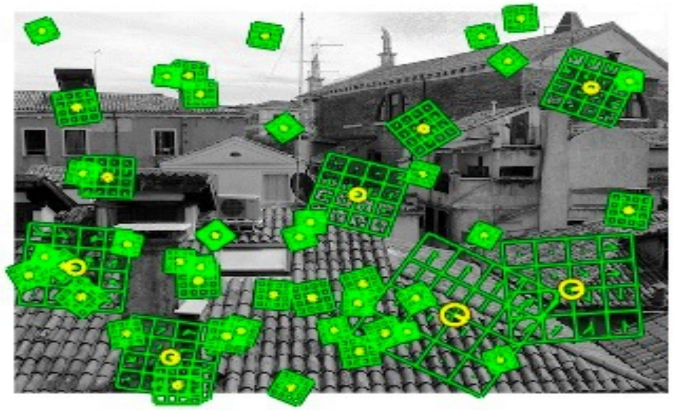
How should we produce such good features?

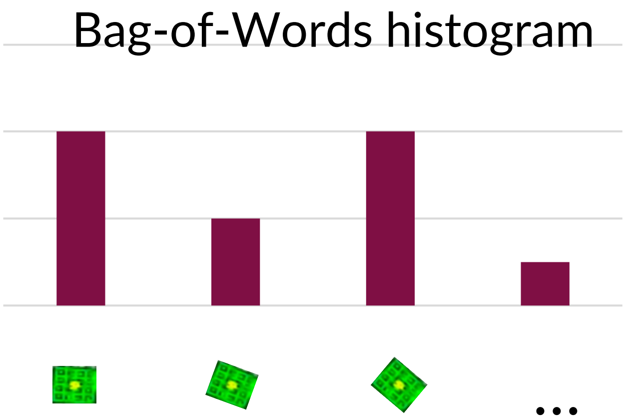
Visual Features Before Deep Learning

Most Feature Extraction Frameworks Pre-2012

- Step 1: Focus on “interest points” rather than all pixels
  - E.g. corner points, “difference of gaussians”, or even a uniform grid
- Step 2: Compute features at interest points.
  - E.g. “SIFT”, “HOG”, “SURF”, “GIST”, etc.
- Step 3: Convert to fixed-dimensional feature vector by measuring statistics of the features such as histograms
  - E.g. “Bag of Words”, “Spatial Pyramids”, etc.



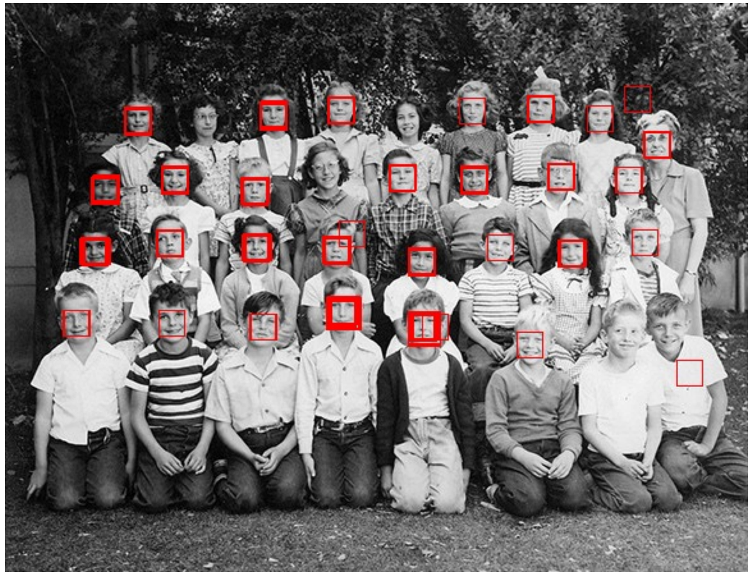




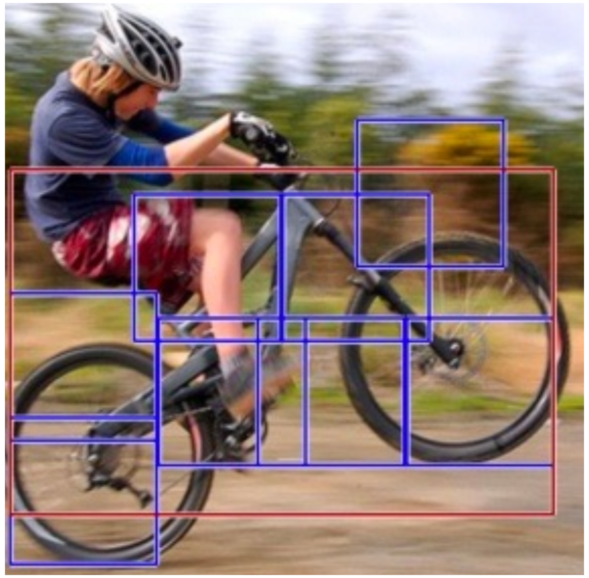
See libraries like VLFeat and OpenCV

Use your favorite ML model now!

Successes of ML for Vision Pre-2012



https://github.com/alexdemartos/ViolaAndJones

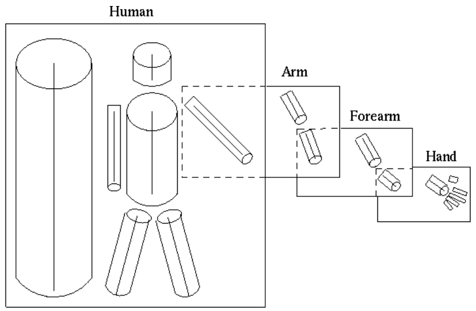


Viola-Jones face detector
(with AdaBoost!)
~2000

Deformable Parts Model
object detection
(with SVMs!)
~2010

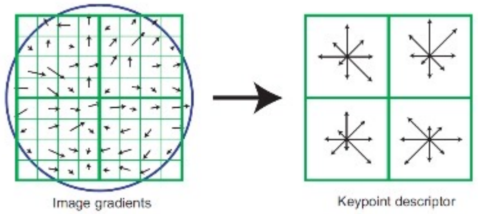
GIST
Scene retrieval
(with nearest neighbors!)
~2006

ML in Computer Vision



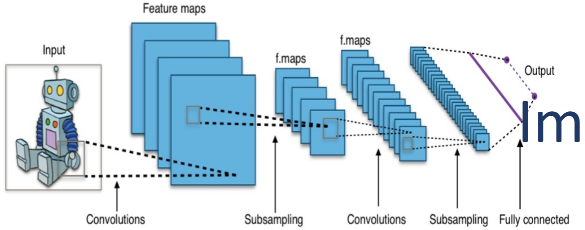
The *very* old: 1960's - Mid 1990's

Image → hand-def. features → hand-def. classifier



The old: Mid 1990's – 2012

Image → hand-def. features → learned classifier



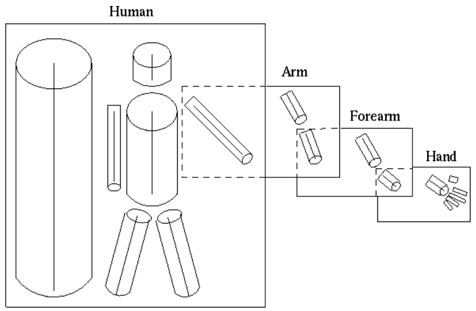
The new: 2012 – ?

Image → jointly learned features + classifier with "deep" multi-layer neural networks

Representation Learning for Images

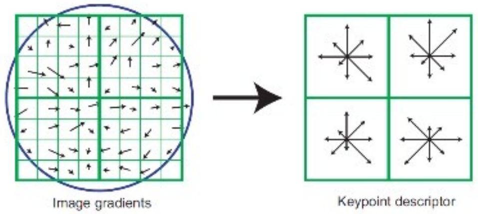
Convolutional Neural Networks

What is Different Now?



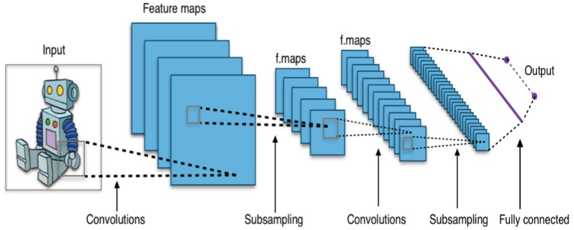
The *very* old: 60’s - Mid 90’s

Image → hand-def. features → hand-def. classifier



The old: Mid 90’s – 2012

Image → hand-def. features → learned classifier



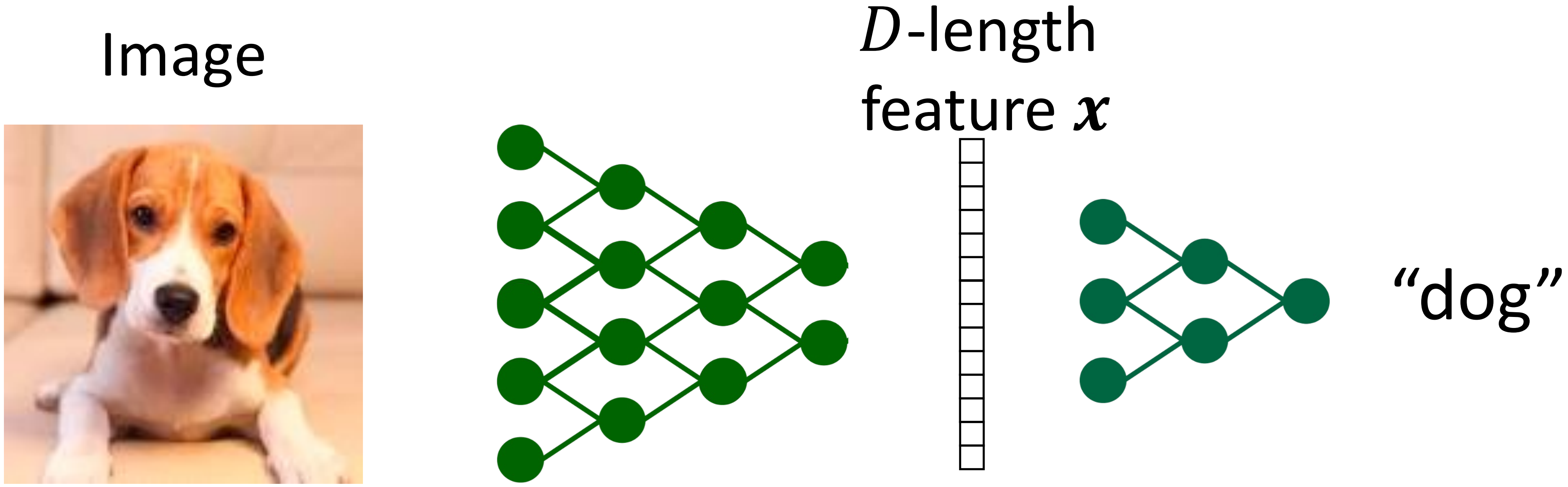
The new: 2012 – ?

Image → jointly learned features + classifier

Answer: Representation learning

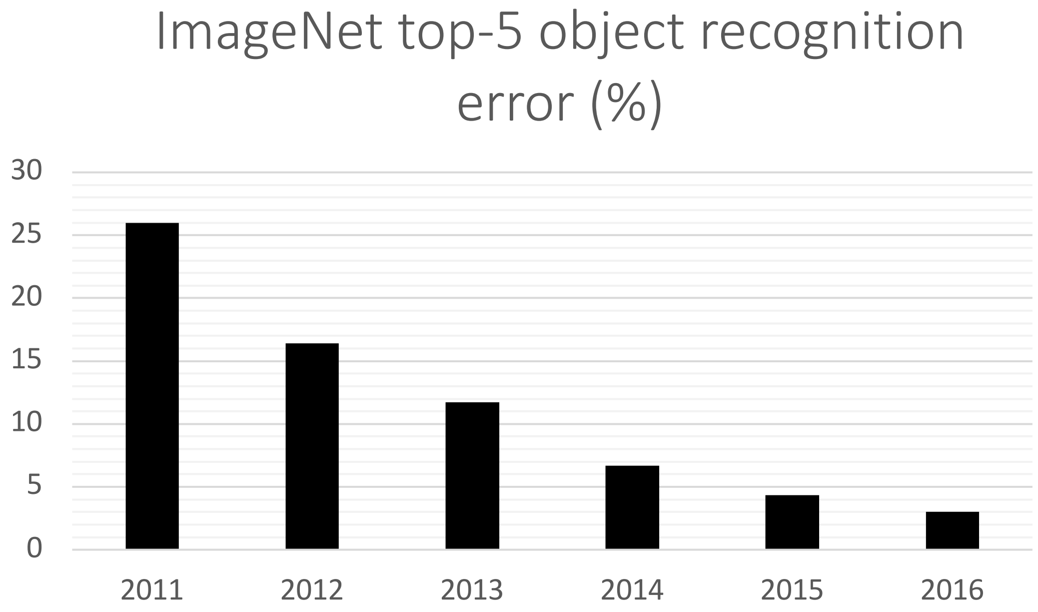
“Deep” Learning

- “Deep” multi-layer neural networks are **representation learners**.
- Every layer improves upon its preceding layer, tailoring the representation to the task.



Impact of Deep Learning in Computer Vision

ImageNet 1000-object category recognition challenge



But the neural networks you have seen so far won't work well on images!

What’s special about images?

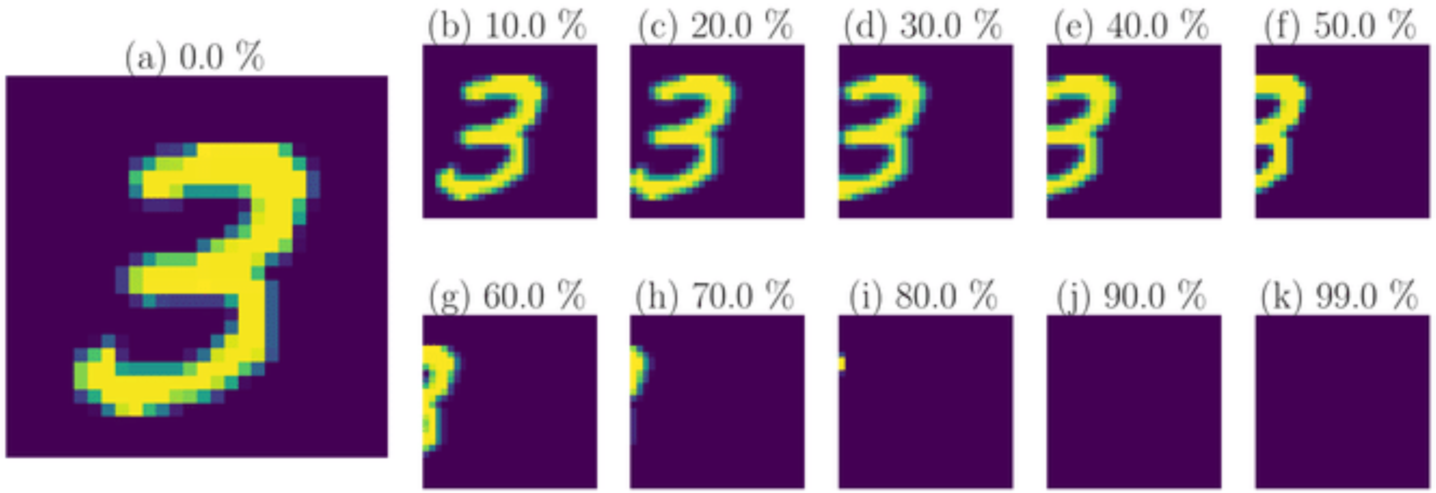
- Images are special. Why?
- Bad news: They are very high-dimensional, which makes all ML harder.
- Good news: We don’t have to treat images as just vectors of pixels. We know more about them, and can exploit that knowledge.

Structure in Images

- **2D image structure**
  - Location associations and spatial neighborhoods are meaningful
  - So far, we can shuffle the features without changing the problem (e.g., $\beta^\top x$)
  - Not true for images!

Structure in Images

- **Translation invariance**
  - Consider image classification (e.g., labels are cat, dog, etc.)
  - **Invariance:** If we translate an image, it does not change the category label



Source: Ott et al., Learning in the machine: To share or not to share?

Structure in Images

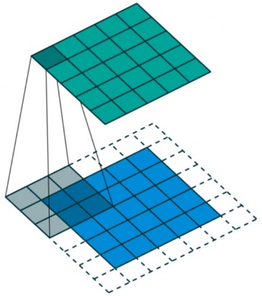
- **Translation equivariance**
  - Consider object detection (e.g., find the position of the cat in an image)
  - **Equivariance:** If we translate an image, the the object is translated similarly

We will exploit this through image-specific operations in neural networks.

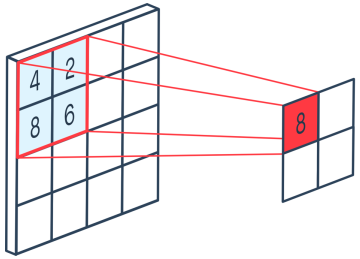
“Image”-Specific Operators/Layers

- We want to retain useful **location associations,** and exploit **translation invariance** and **equivariance**.

- **Two key operations in neural networks for images:**



Convolution layers
(capture equivariance)



Pooling layers
(capture invariance)

https://towardsdatascience.com/types-of-convolutions-in-deep-learning-717013397f4d
https://peltarion.com/static/2d_max_pooling_pa1.png

Convolutions Beyond Photographs

- Recall: convolutions try to gather useful **location associations,** and exploit **translation invariance** and **equivariance**.

- These properties are useful beyond just photographic images. Need not even be 2D grids.
  - E.g. detecting spikes in a time series of stock prices, or an audio stream. (1-D)
    - Also important to retain location associations
    - Local operations, invariance, equivariance.

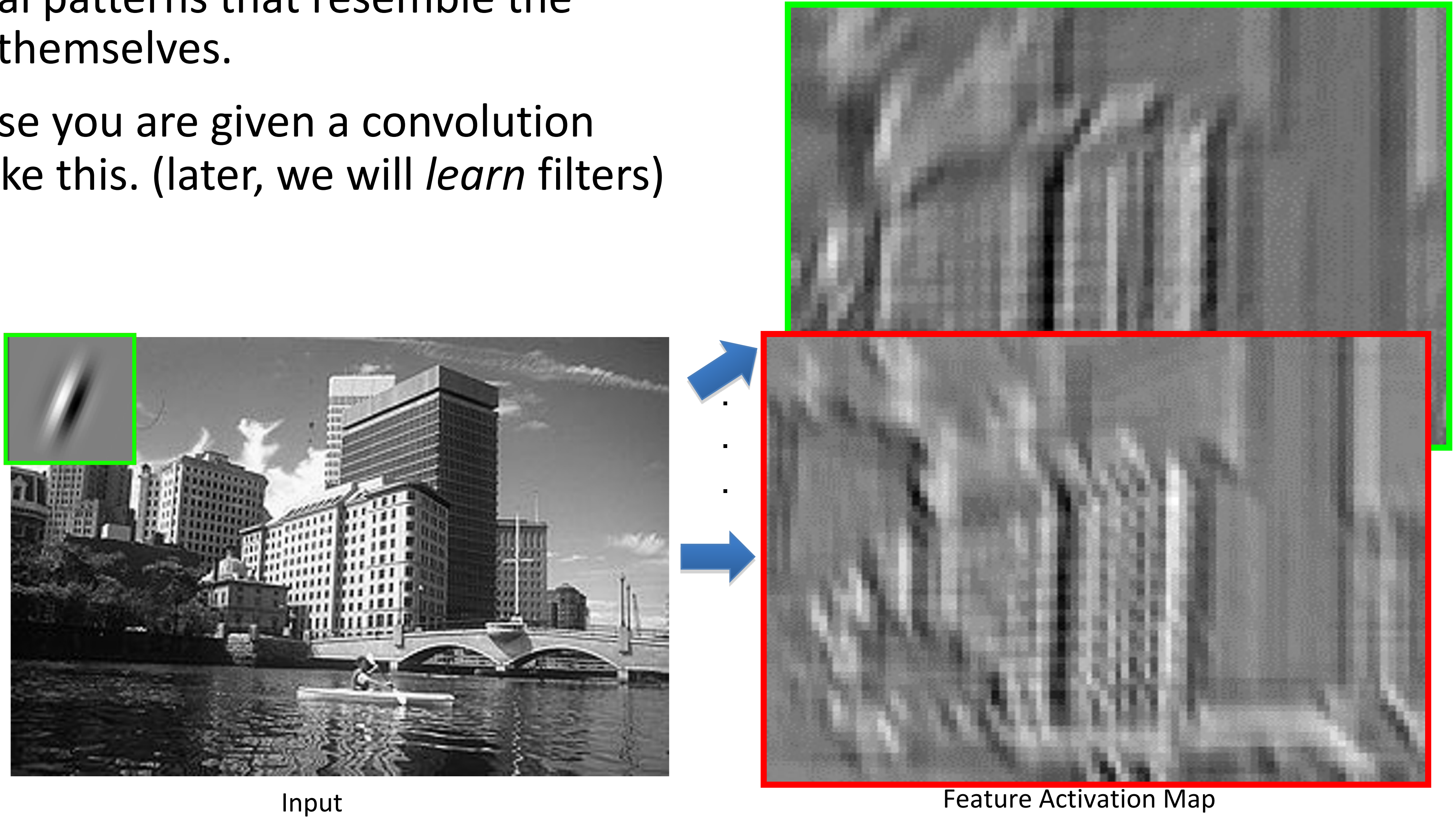
  - Can also apply in higher dimensions. E.g. convolving over a 3D "grid" of voxels to detect objects.

https://towardsdatascience.com/types-of-convolutions-in-deep-learning-717013397f4d
https://peltarion.com/static/2d_max_pooling_pa1.png

Convolution

Convolution Filters: Template Matching over an Image

- Intuitively, convolutional filters search for local patterns that resemble the filters themselves.
- Suppose you are given a convolution filter like this. (later, we will *learn* filters)



graphic credit: S. Lazebnik

Convolutional filtering in 1D

- Suppose your input x is a 1-D sequence, such as a time sequence, e.g. the stock market: $\boldsymbol{x} = [25000, 28000, 30000, 21000, 18000, \ldots]$
- Given a "kernel" sequence, e.g. $\boldsymbol{k} = [-1, 1, -1]$
- Convolution is defined by the following operation:

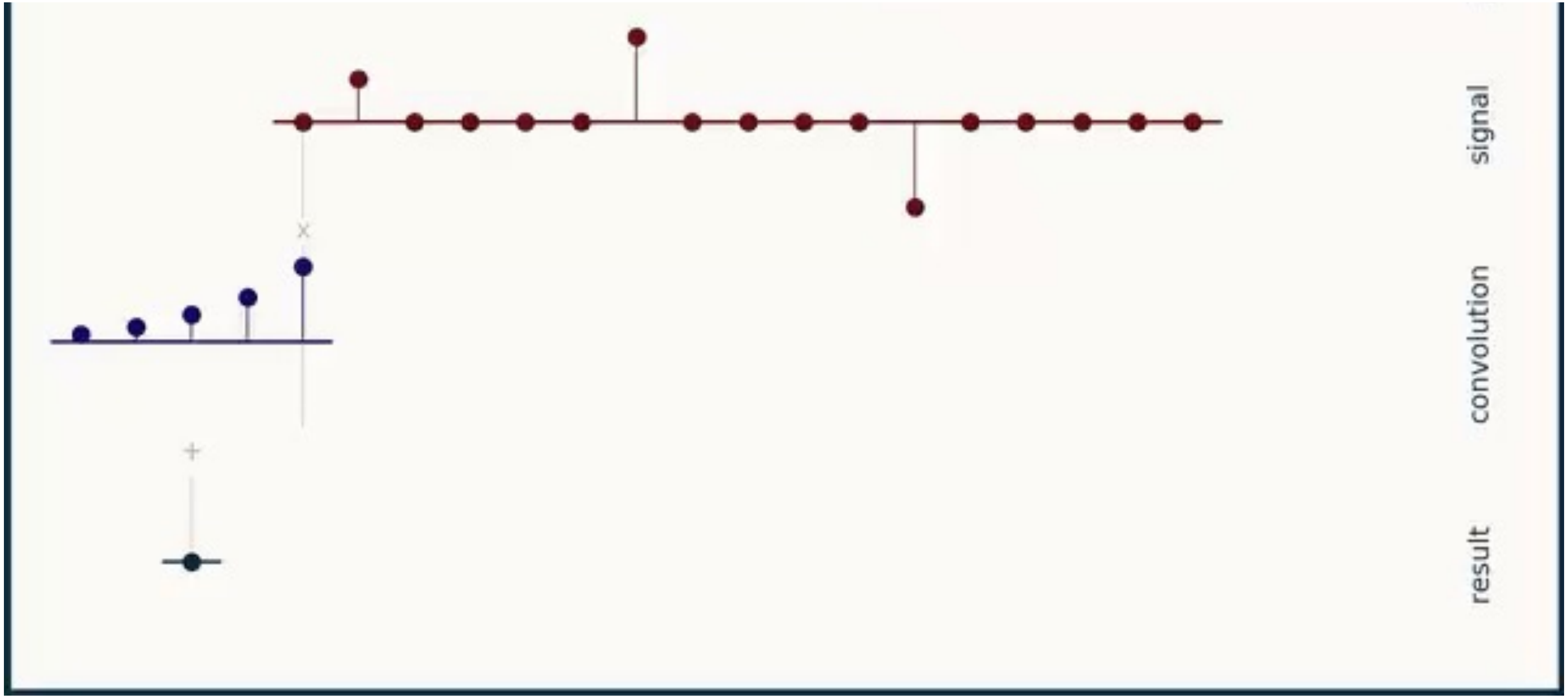
Strictly "cross-correlation", but we'll call it "convolution"

$$y[t] = \sum_{\tau=0}^{|\boldsymbol{k}|-1} k[\tau]x[t+\tau]$$

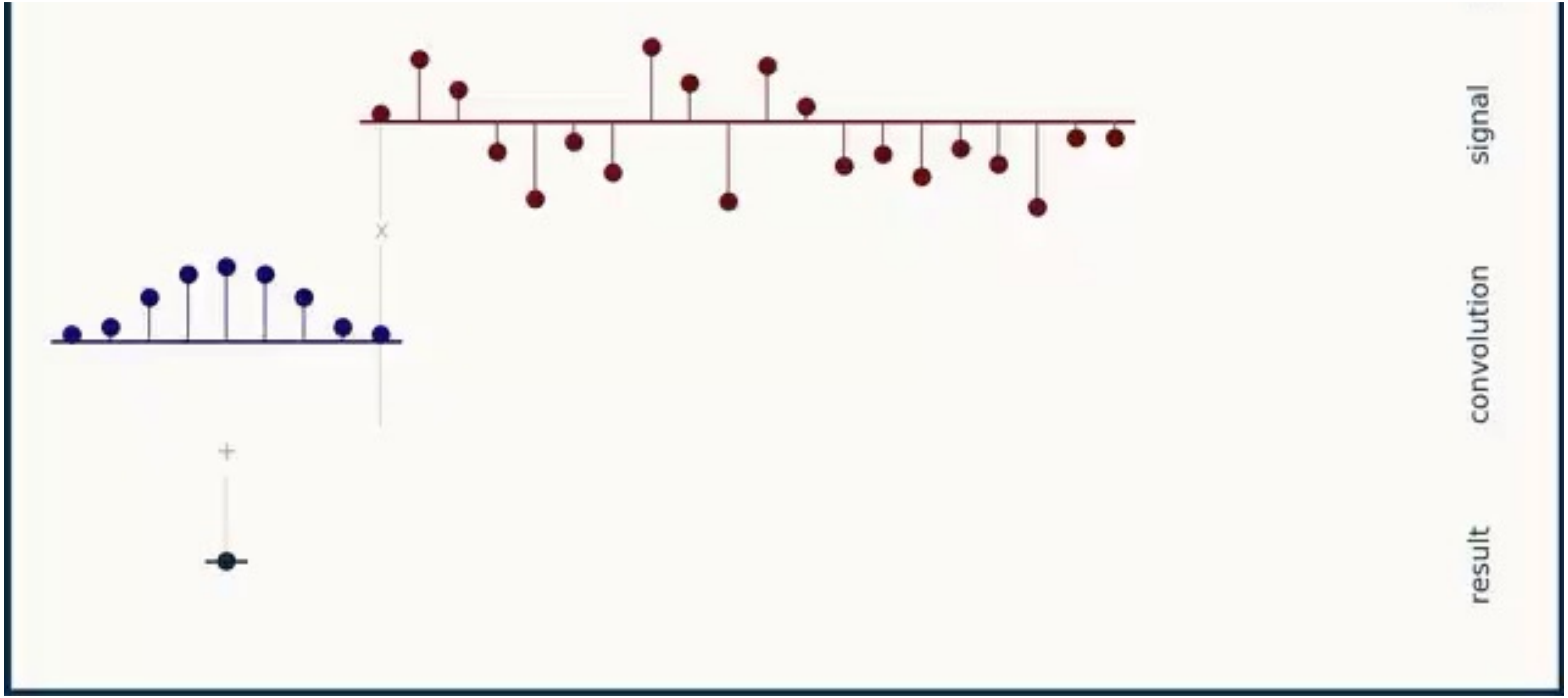
In neural networks, the weights $\boldsymbol{k}$ are learned. (Plus a bias)

$$y[0] = k[0]x[0] + k[1]x[1] + k[2]x[2] = -25000 + 28000 - 30000$$
$$y[1] = k[0]x[1] + k[1]x[2] + k[2]x[3] = -28000 + 30000 - 21000$$
$$y[2] = k[0]x[2] + k[1]x[3] + k[2]x[4] = -30000 + 21000 - 18000$$

Convolutional Filtering in 1D

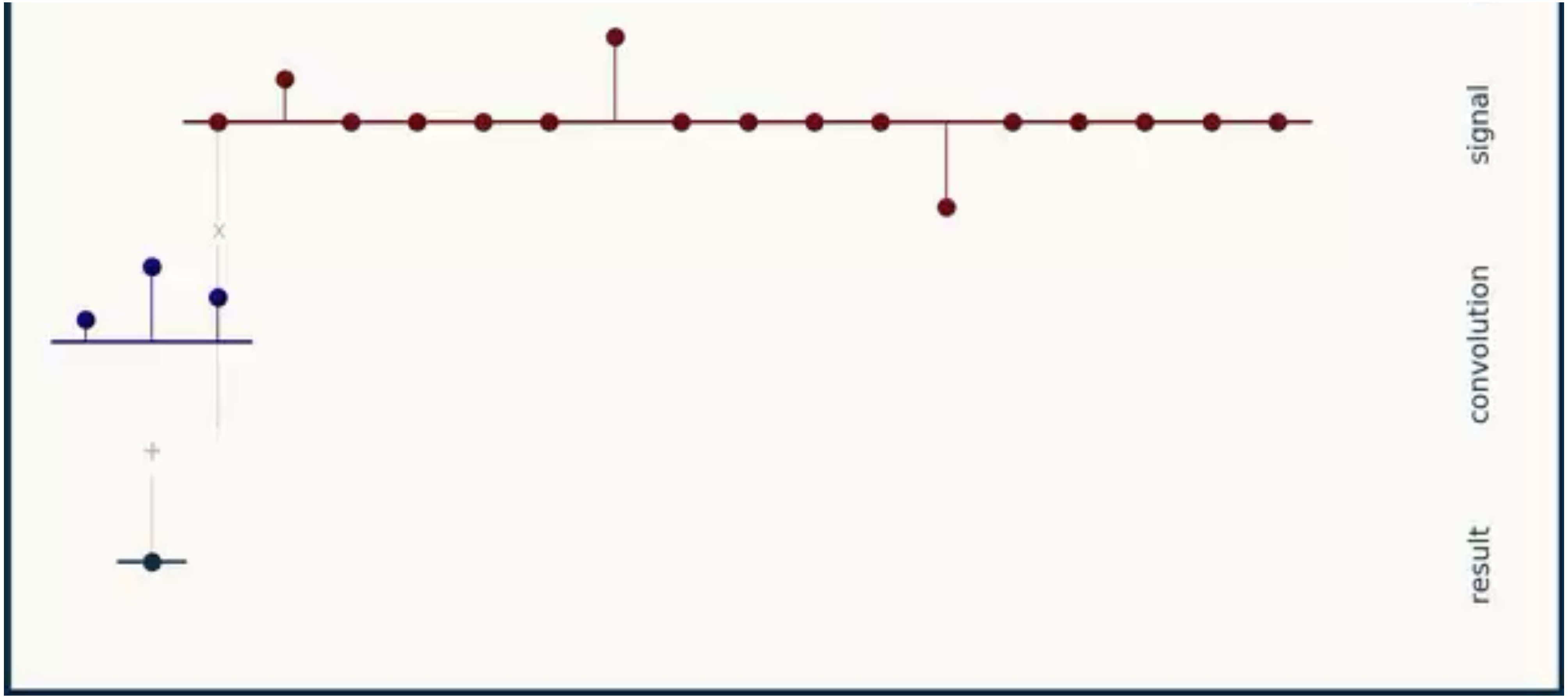


Convolutional Filtering in 1D



No good positive match, but good *negative* match?

Convolutional Filtering in 1D



Convolutional filtering in 2D

- 1-D convolution is defined by the following operation:

$$y[t] = \sum_{\tau=0}^{|k|-1} k[\tau]x[t+\tau]$$

- With a 2-D signal $\boldsymbol{x}$ and 2-D $h \times w$ kernel $\boldsymbol{k}$, 2-D convolution is defined by the following operation:

$$y[s,t] = \sum_{\tau=0}^{h-1} \sum_{\gamma=0}^{w-1} k[\tau,\gamma]x[s+\tau,t+\gamma]$$

Again, in convolutional neural networks, the weights $\boldsymbol{k}$ will be learned.

Convolutional filtering in 2D

$$y[s,t] = \sum_{\tau=0}^{h-1} \sum_{\gamma=0}^{w-1} k[\tau,\gamma]x[s+\tau, t+\gamma]$$

- To compute:
  - Slide kernel over image
  - Take the element-wise multiplication over the window and sum

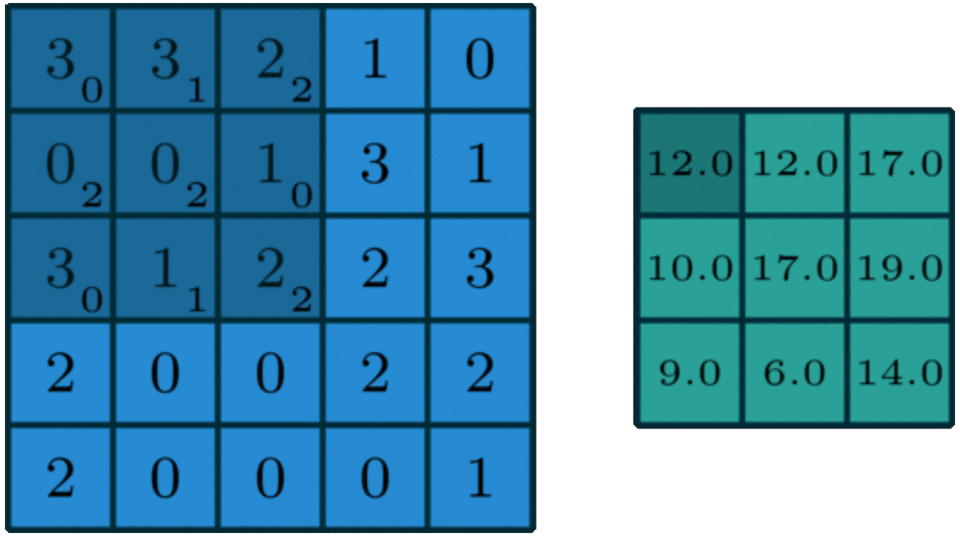


Image: https://towardsdatascience.com/intuitively-understanding-convolutions-for-deep-learning-1f6f42faee1

Example: Edge Detection via Convolution

Example Edge Detection Kernels

-1	-1	-1
2	2	2
-1	-1	-1

Horizontal lines

-1	2	-1
-1	2	-1
-1	2	-1

Vertical lines

-1	-1	2
-1	2	-1
2	-1	-1

45 degree lines

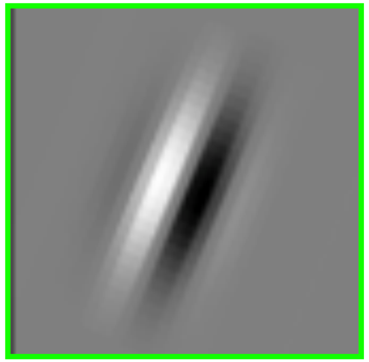
2	-1	-1
-1	2	-1
-1	-1	2

135 degree lines

Result of Convolution with Horizontal Kernel

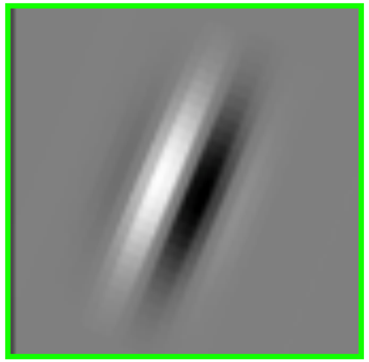
https://aishack.in/tutorials/image-convolution-examples/

Back To Our Example



graphic credit: S. Lazebnik

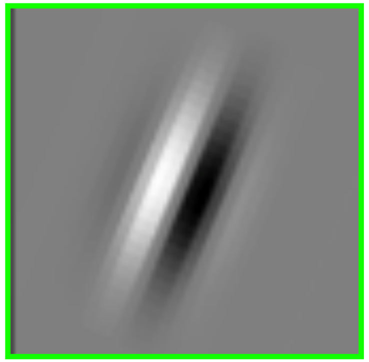
Back To Our Example



$$\text{output}[0,0] = \sum_{\tau=0}^{k-1} \sum_{\gamma=0}^{k-1} \text{filter}[\tau, \gamma] \cdot \text{image}[0+\tau, 0+\gamma]$$

graphic credit: S. Lazebnik

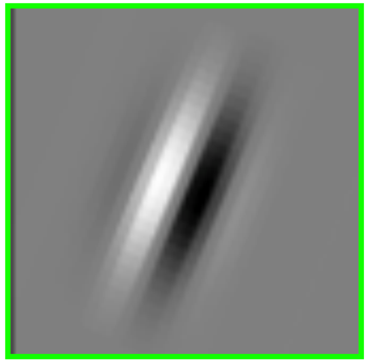
Back To Our Example



$$\text{output}[0,1] = \sum_{\tau=0}^{k-1} \sum_{\gamma=0}^{k-1} \text{filter}[\tau, \gamma] \cdot \text{image}[0+\tau, 1+\gamma]$$

graphic credit: S. Lazebnik

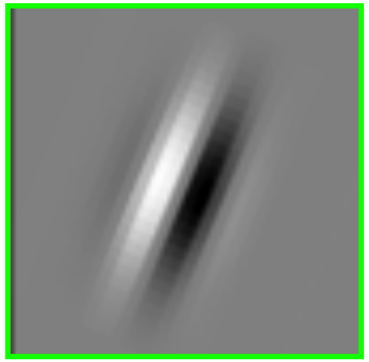
Back To Our Example



$$\text{output}[0,2] = \sum_{\tau=0}^{k-1} \sum_{\gamma=0}^{k-1} \text{filter}[\tau, \gamma] \cdot \text{image}[0+\tau, 2+\gamma]$$

graphic credit: S. Lazebnik

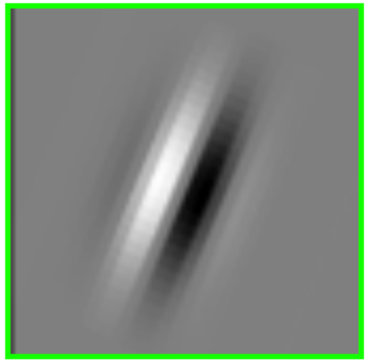
Back To Our Example

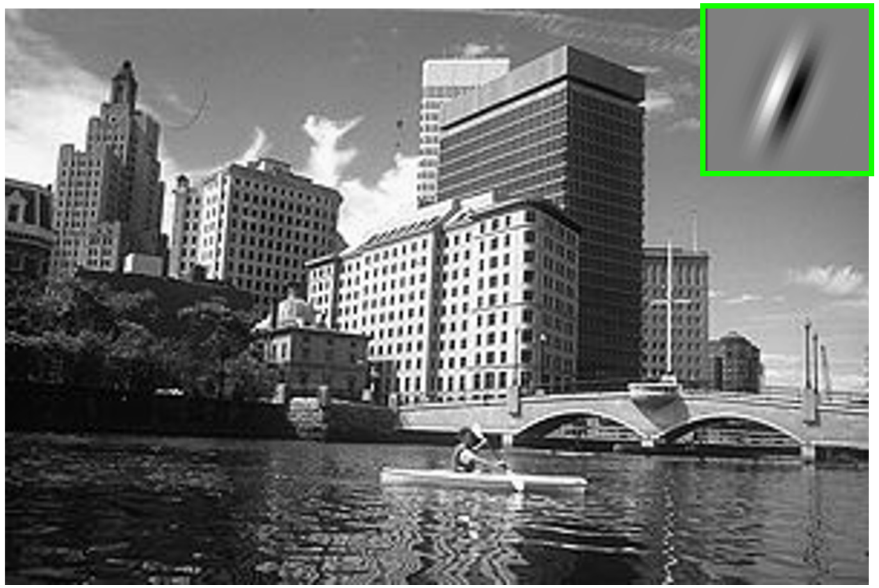


$$\text{output}[i,j] = \sum_{\tau=0}^{k-1} \sum_{\gamma=0}^{k-1} \text{filter}[\tau,\gamma] \cdot \text{image}[i+\tau, j+\gamma]$$

graphic credit: S. Lazebnik

Back To Our Example

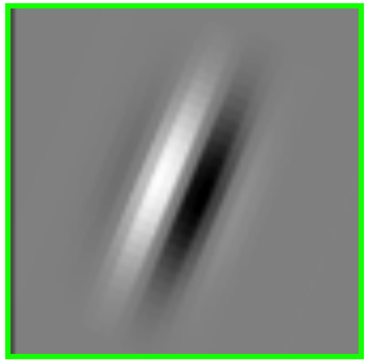


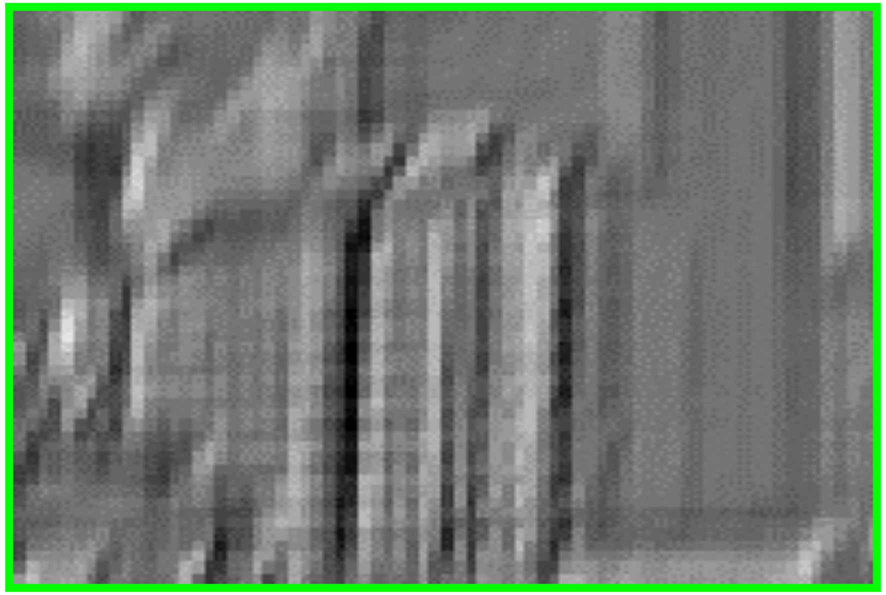


$$\text{output}[i,j] = \sum_{\tau=0}^{k-1} \sum_{\gamma=0}^{k-1} \text{filter}[\tau,\gamma] \cdot \text{image}[i+\tau, j+\gamma]$$

graphic credit: S. Lazebnik

Back To Our Example





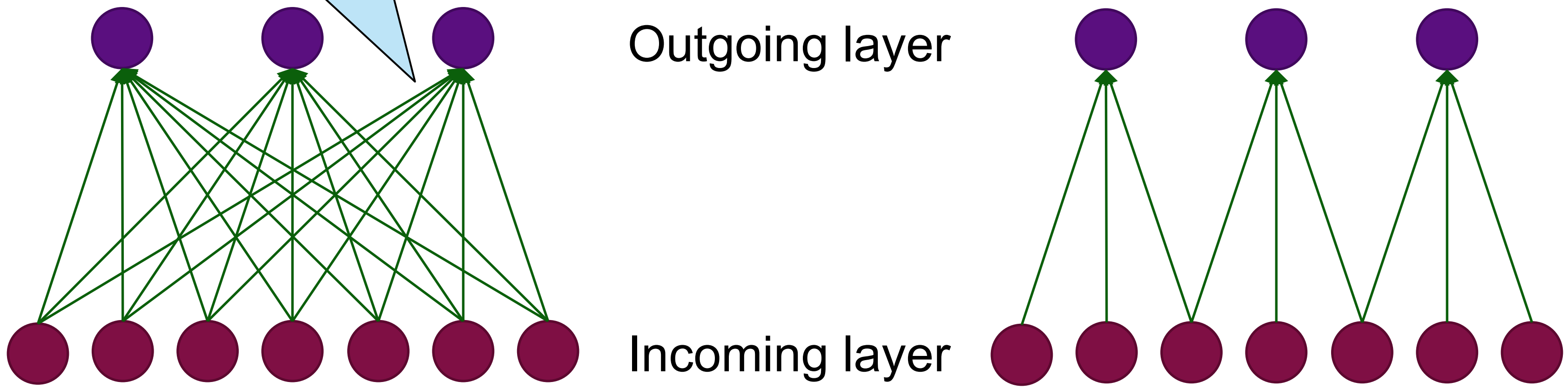
$$\text{output}[i,j] = \sum_{\tau=0}^{k-1}\sum_{\gamma=0}^{k-1} \text{filter}[\tau,\gamma] \cdot \text{image}[i+\tau, j+\gamma]$$

graphic credit: S. Lazebnik

From Convolutions to Convolutional Layers

Convolutional Layer: Local Connectivity

Hence “fully connected” / “fc” layers.



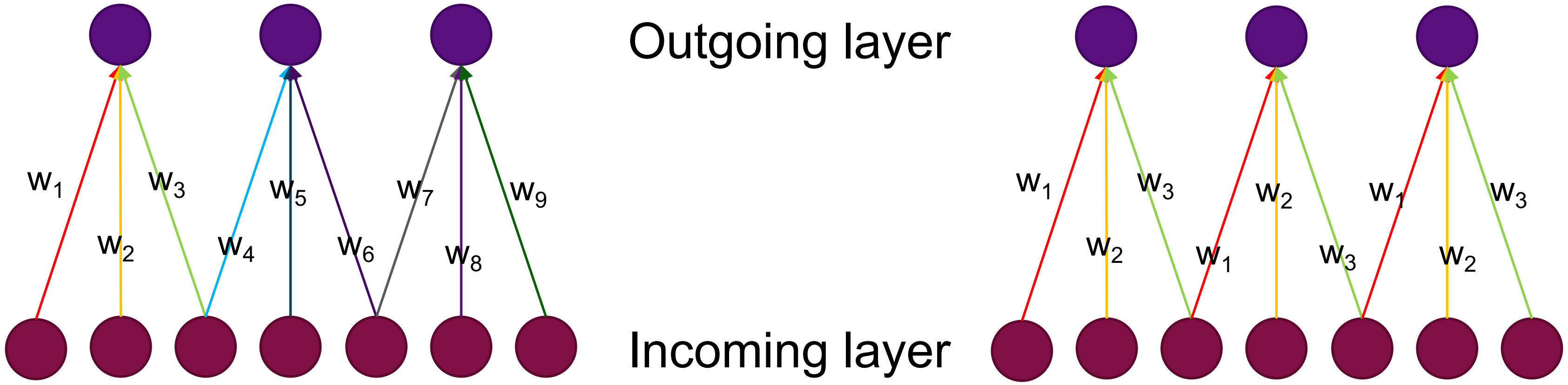
Global connectivity

Local connectivity

- # input units (neurons): 7
- # hidden units: 3
- Number of parameters (ignoring bias)
  - Global connectivity: 3 x 7 = 21
  - Local connectivity: 3 x 3 = 9

Slide credit: Jia-Bin Huang

Convolutional Layer: Weight Sharing



Without weight sharing

With weight sharing

- # input units (neurons): 7
- # hidden units: 3
- Number of parameters (ignoring bias)
  - Without weight sharing: 3 x 3 = 9
  - With weight sharing : 3 x 1 = 3

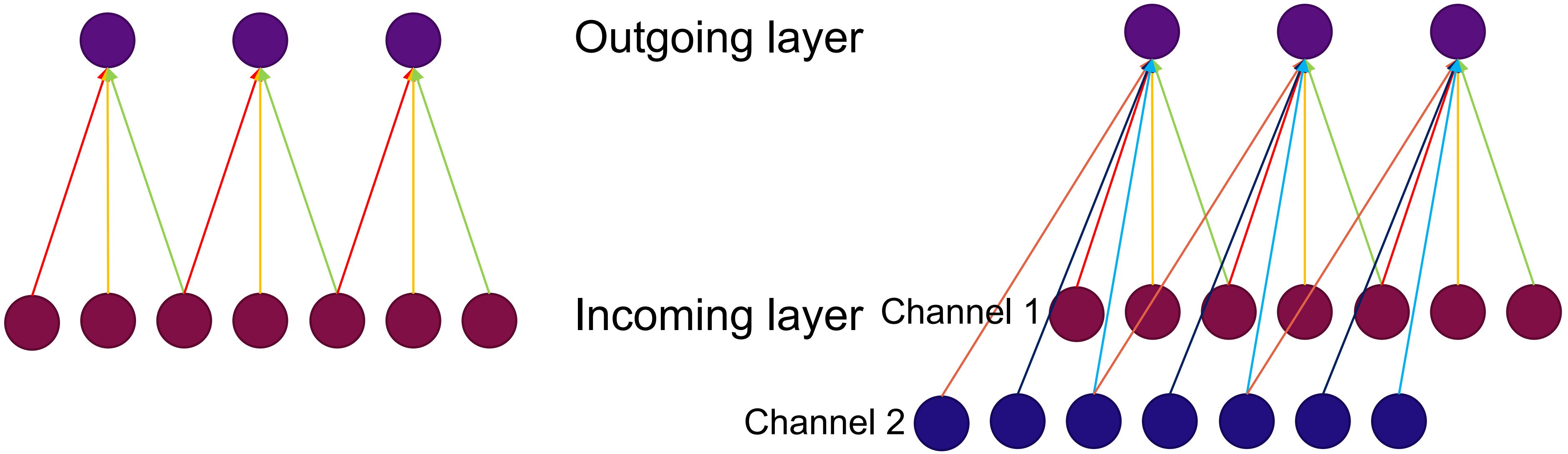
Slide credit: Jia-Bin Huang

Extending convolutions

- We have just discussed the connection between normal “fully connected” layers to convolutions.

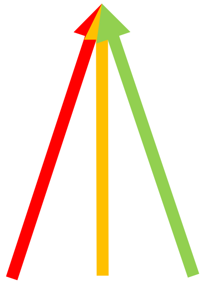
- But convolutional layers in neural networks extend this a bit more (next 3 slides):
  - They can handle multiple input channels (e.g. RGB channels in color image)
  - They can also handle multiple *output* channels
  - They can modify the inputs to maintain desired activation sizes

Convolutional Layer with >1 input “channels” / “maps”

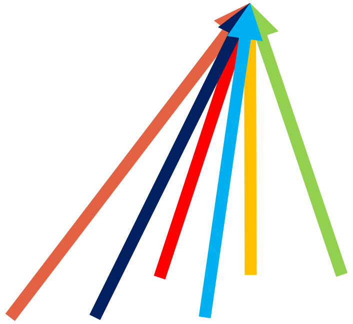


Single input channel

Multiple input channels



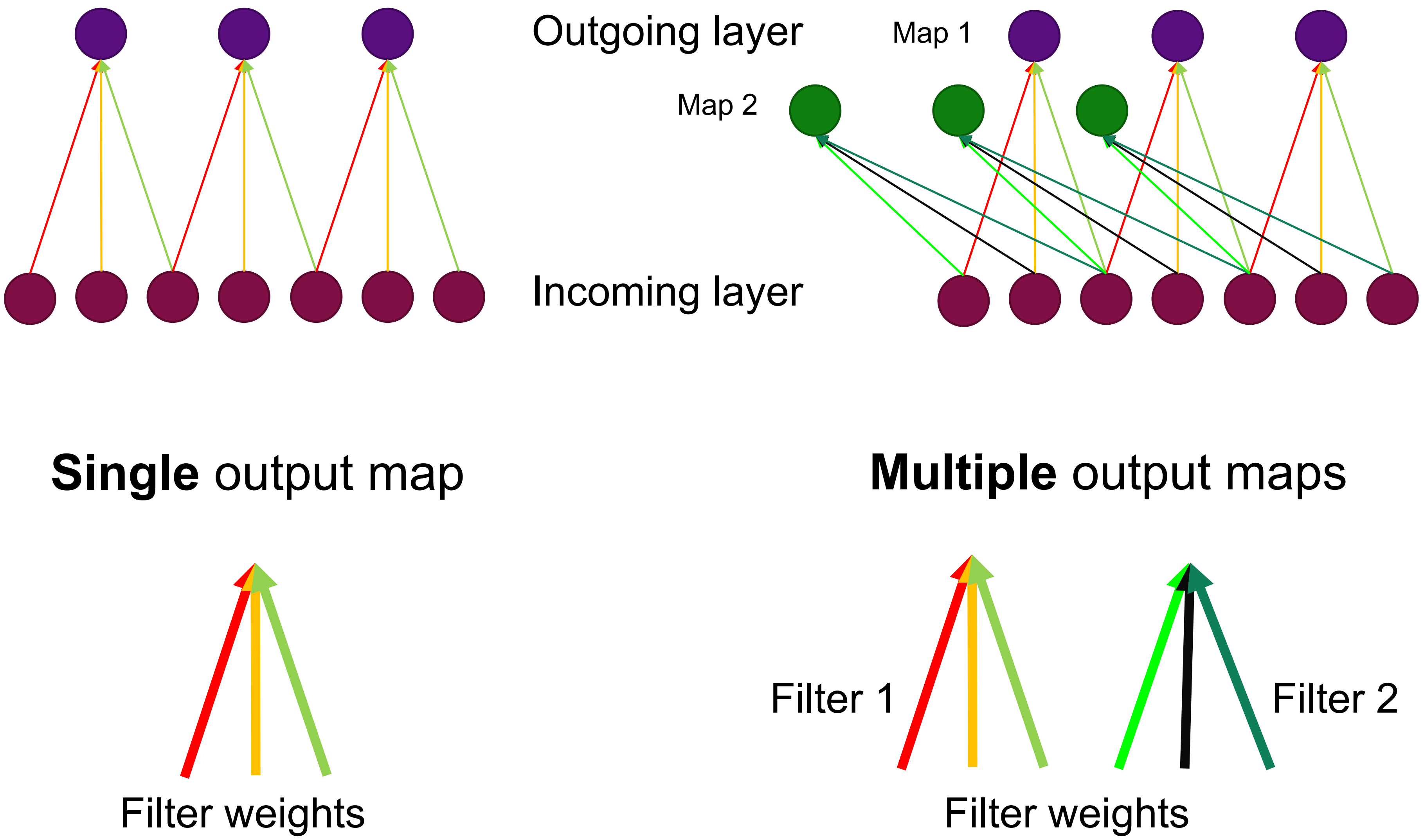
Filter weights



Filter weights

Slide credit: Jia-Bin Huang

Convolutional Layer with >1 output “channels” / “maps”



Slide credit: Jia-Bin Huang

Convolutional Layer Summary

- Local connectivity
- Weight sharing
- Handling multiple input/output channels
- Retains location associations

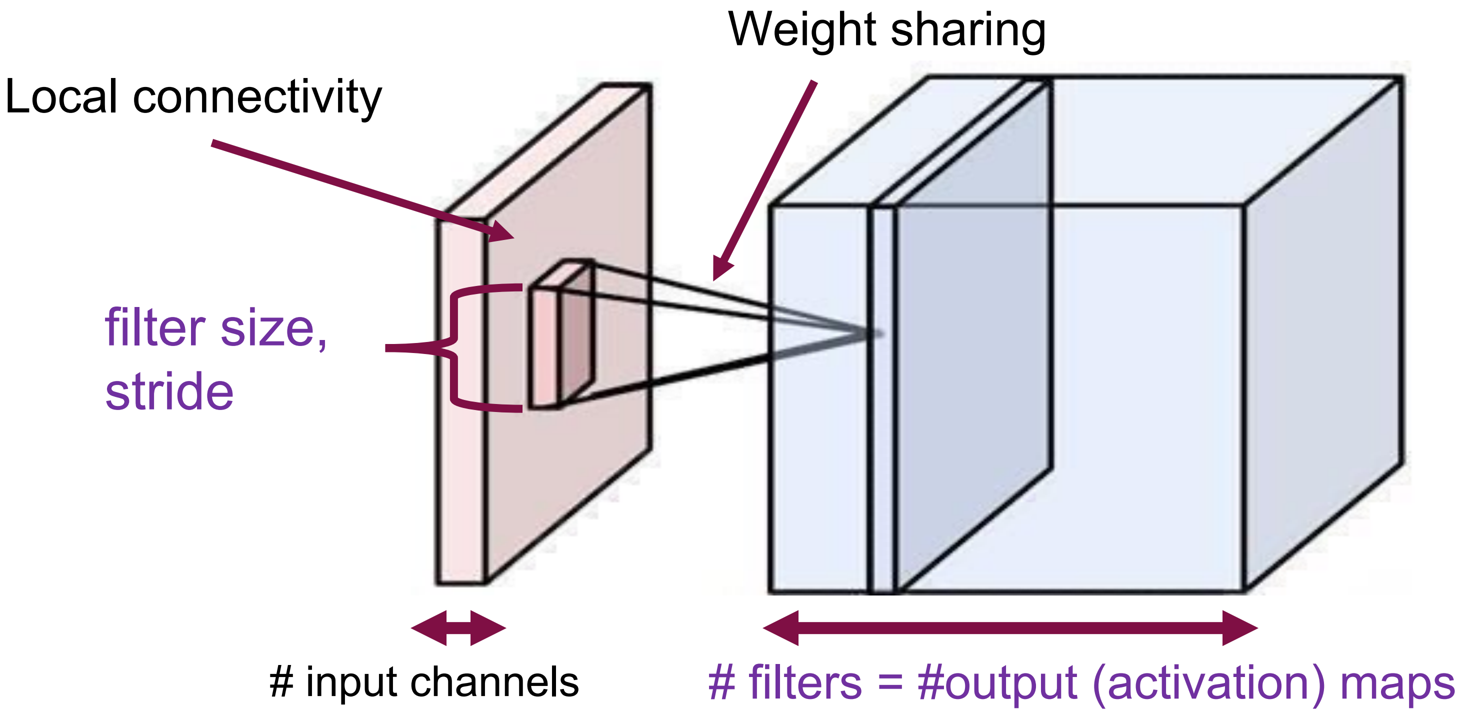
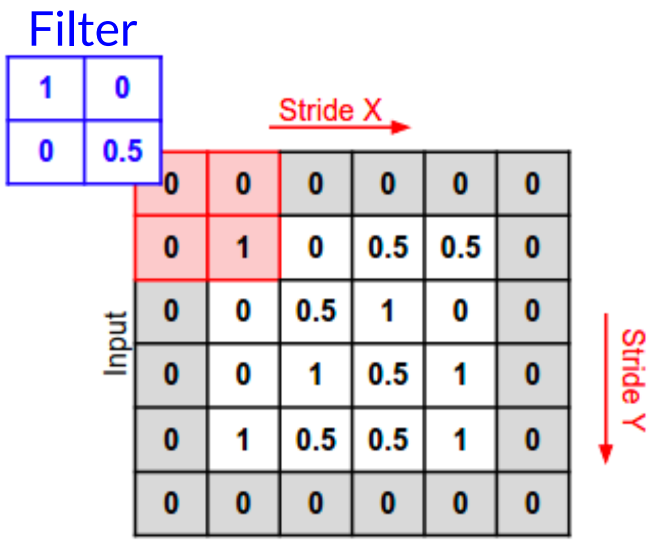


Image credit: A. Karpathy

Slide credit: Jia-Bin Huang

Stride



Output with stride 1

0.5	0	0.25	0.25	0
0	1.25	0.5	0.5	0.5
0	0.5	0.75	1.5	0
0.5	0.25	1.25	1	1
0.	1	0.5	0.5	1

Output with stride 2

0.50	0.25	0.00
0.00	0.75	0.00
0.00	0.50	1.00

https://medium.com/@ayeshmanthaperera/what-is-padding-in-cnns-71b21fb0dd7