Exam policy: This exam allows one one-page, two-sided cheat sheet; No other materials.

Time: 80 minutes. Be sure to write your name and Penn student ID (the 8 bigger digits on your ID card) on the answer form and fill in the associated bubbles in pencil.

If you think a question is ambiguous, mark what you think is the best answer. As always, we will consider written regrade requests if your interpretation of a question differed from what we intended. We will only grade the answer forms.

For the “TRUE or FALSE” questions, note that “TRUE” is (a) and “FALSE” is (b). For the multiple choice questions, select exactly one answer.

Questions follow the course convention that $n$ or $N$ represents the number of observations and $p$ the number of features in the model.

There are 40 questions and 61 points (19 one point questions, 21 two point questions).
1. [2 points] Compared to MLE, MAP solutions tend to have
   (a) lower bias and lower variance
   (b) higher bias and lower variance
   (c) lower bias and higher variance
   (d) higher bias and higher variance

   ★ SOLUTION:  B

2. [2 points] We trained a three-way logistic regression and obtained weights
   \[ w_a = (1, 1, 0), \ w_b = (-1, 1, 1), \ w_c = (2, 1, 2) \]
   What label would be given to the point \( x = (0, 1, 1) \)?
   (a) A  
   (b) B  
   (c) C

   ★ SOLUTION:  C

3. [2 points] As part of building a decision tree, we want to measure the information gain from asking a question about a binary split on feature \( X_1 \) of the following 4 samples:

   \[
   \begin{array}{|c|c|}
   \hline
   X_1 & y \\
   \hline
   T & 0 \\
   F & 1 \\
   F & 1 \\
   F & 0 \\
   \hline
   \end{array}
   \]

   Which of the following measures the information gain of splitting the sample on the value of \( X_1 \)?
   (a) \( -\left(\frac{2}{3}\right) \log_2\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right) \log_2\left(\frac{1}{3}\right) \)
   (b) \( -\log_2\left(\frac{1}{2}\right) + \frac{1}{3} \left(\frac{2}{3} \log_2\left(\frac{2}{3}\right) + \frac{1}{3} \log_2\left(\frac{1}{3}\right)\right) + \frac{2}{3} \left(\frac{2}{3} + \log_2\left(\frac{2}{3}\right) + \frac{1}{3} \log_2\left(\frac{1}{3}\right)\right) \)
   (c) \( -\log_2\left(\frac{1}{2}\right) + \frac{3}{4} \left(\frac{2}{3} \log_2\left(\frac{2}{3}\right) + \frac{1}{3} \log_2\left(\frac{1}{3}\right)\right) \)
   (d) \( -\log_2\left(\frac{1}{2}\right) + \frac{1}{4} \left(\frac{2}{3} \log_2\left(\frac{2}{3}\right) + \frac{1}{3} \log_2\left(\frac{1}{3}\right)\right) \)
   (e) none of the above
★ SOLUTION: C
4. [2 points] The depth of a decision tree is most likely to scale as

(a) the number of training examples  
(b) the square root of the number of training examples  
(c) the log of the number of the training examples

★ SOLUTION:  C

5. [2 points] You are using a dataset for housing with one feature (area) in squared meters. Your friend Margaret is also using the same data set but with the mentioned feature in squared feet. (A meter is roughly 3 feet.) You both are using nearest neighbors with L2 distance. What do you expect to see?

(a) The two models will both give the exact same predictions.
(b) Area is more important when measured in feet than in meters.
(c) Area is less important when measured in feet than in meters.
(d) The two models will give different predictions, but we can’t know the relative effect of area.

★ SOLUTION:  B

6. [2 points] Similar to the question above, you are using a dataset for housing with one feature (area) in squared meters. Your friend Margaret is also using the same data set but with the mentioned feature in squared feet. However, this time, you are both are experimenting with decision trees. What do you expect to see?

(a) The two models will both give the exact same predictions.
(b) Area is more important when measured in feet than in meters.
(c) Area is less important when measured in feet than in meters.
(d) The two models will give different predictions, but we can’t know the relative effect of area.

★ SOLUTION:  A
7. [2 points] Traditionally, when we have a real-valued input attribute during decision-tree learning, we consider 9 binary splits according to whether the attribute is in the lowest 10%, lowest 20%, etc. Your friend Pat suggests that instead we should just have a 10-way split with one branch for each of the 10 “bins” of the attribute (lowest 10%, next 10%, etc). The single biggest problem with Pat’s suggestion is:

(a) It is too computationally expensive.
(b) It would probably result in a decision tree that scores badly on the training set and a test set.
(c) It would probably result in a decision tree that scores well on the training set but badly on a test set.
(d) It would probably result in a decision tree that scores well on a test set but badly on a training set.

★ SOLUTION: C

8. [2 points] Suppose we compute the MAP estimate of the mean $\mu$ of a Gaussian with a fixed variance, $n$ samples and the following prior on $\mu$:

$$
    p(\mu|\mu_0, \sigma_0) = \frac{1}{\sigma_0\sqrt{2\pi}} e^{-\frac{(\mu-\mu_0)^2}{2\sigma_0^2}}
$$

Under what condition(s) does the MAP estimate of $\mu$ converge to the MLE estimate of $\mu$?

(a) $\mu_0 \to 0$
(b) $n \to \infty$
(c) $\sigma_0 \to \infty$
(d) two of the above
(e) all of the above

★ SOLUTION: D
9. [2 points] As $k$ in k-NN increases:

(a) the bias increases and the variance increases
(b) the bias increases and the variance decreases
(c) the bias decreases and the variance increases
(d) the bias decreases and the variance decreases

★ SOLUTION: B

10. [1 points] True or False? The larger the regularization penalty $\lambda$ in penalized regression, the more the model will tend to overfit the training data.

★ SOLUTION: False

11. [1 points] Which norm most heavily shrinks large weights (weights that are much bigger than 1)?

(a) $L_0$
(b) $L_1$
(c) $L_2$
(d) not enough information to tell

★ SOLUTION: C

12. [2 points] Suppose you have a friend who is an esteemed doctor. Your friend has a dataset containing gene expression data for patients. For each patient, there is a vector containing the expression levels of 10,000 genes, along with a label indicating whether the patient developed a certain disease. Using this, your friend’s goal is to learn a model where, given a new patient (also represented by a similar 10,000-dimensional vector), the model can output the probability of this patient developing the disease. Your friend also believes a reasonable model will depend on at least 9,000 of the genes. Which of the following methods would be most suitable for this problem?
(a) Linear least squares regression with $L_2$ penalty  
(b) Linear least squares regression with $L_1$ penalty  
(c) Logistic regression with $L_2$ penalty  
(d) Logistic regression with $L_1$ penalty  
(e) Decision Tree  

★ SOLUTION: C
Models A-D below are fit by linear regression on a simple three-example data set \((1,1), (0,1), (2,2)\) with one independent variable \(x\) and one dependent variable \(y\), using OLS or \(L_1\) or \(L_2\)-penalized regression.

Use the plots above to answer the next three questions:

13. [2 points] Which model was most likely to have been generated using OLS?

(a) Model A  
(b) Model B  
(c) Model C  
(d) Model D  
(e) Not enough information
★ SOLUTION: (C)
14. [2 points] Which model was most likely to have been generated using \( L_1 \) regularization?

(a) Model A  
(b) Model B  
(c) Model C  
(d) Model D  
(e) Not enough information

★ SOLUTION: (B)

15. [2 points] Which model was most likely to have been generated using \( L_2 \) regularization?

(a) Model A  
(b) Model B  
(c) Model C  
(d) Model D  
(e) Not enough information

★ SOLUTION: (D)

16. [2 points] In order to check whether a function \( k(x_1, x_2) \) is a kernel function, we used a matrix \( X \) to generate a potential kernel matrix \( K \). If the matrix \( K \) has eigenvalues 1.0, 2.3, 3.7, and 4.2 then we can conclude that

(a) \( k(x_1, x_2) \) is definitely a kernel function  
(b) \( k(x_1, x_2) \) cannot be a kernel function  
(c) \( k(x_1, x_2) \) might be a kernel function
17. [2 points] In order to check whether a function $k(x_1, x_2)$ is a kernel function, we used a matrix $X$ to generate a potential kernel matrix $K$. If the matrix $K$ has eigenvalues -1.0, 2.3, 3.7, and 4.2 then we can conclude that

(a) $k(x_1, x_2)$ is definitely a kernel function
(b) $k(x_1, x_2)$ cannot be a kernel function
(c) $k(x_1, x_2)$ might be a kernel function

★ SOLUTION:  C

18. [1 points] True or False? Stepwise linear regression will always find at least as accurate a model as streamwise regression, assuming the same regularization penalty is used in both cases.

★ SOLUTION: False

19. [1 points] True or False? The result of streamwise regression depends on the order in which features are added.

★ SOLUTION: True

20. [1 points] True or False? In stepwise regression, the model which includes all the features will always be considered.

★ SOLUTION: False

21. [2 points] You want to fit a linear regression model with “the best” 10 features out of 200 candidate features. Which of the following will work best?

(a) Perform $L_2$ regularization
(b) Perform $L_1$ regularization
(c) Use the hinge loss instead of square error loss.
22. [1 points] True or False? For logistic regression, gradient descent can converge to a local minimum and fail to find a global minimum. More advanced optimization methods are thus often used.

★ SOLUTION: False

23. [1 points] True or False? A larger stepsize in gradient descent can lead to faster convergence, but lower accuracy.

★ SOLUTION: True

24. [1 points] True or False? The key idea of AdaGrad is to learn slowly from frequent features but pay attention to rare but informative features.

★ SOLUTION: True

25. [1 points] True or False? The decision boundary for logistic regression in $p$ dimensions is a $p-1$-dimensional hyperplane.

★ SOLUTION: True
26. [2 points] The radial basis functions used in RBFs transform a feature vector of dimension \( p \) to a new space of dimension \( k \) such that

(a) \( k < p \)
(b) \( k = p \)
(c) \( k > p \)
(d) there is no constraint on how \( k \) and \( p \) are related

★ SOLUTION: D

27. [1 points] True or False? SVMs, unlike neural nets, have a global optimum.

★ SOLUTION: True

28. [1 points] True or False? The error surface followed by the gradient descent backpropagation algorithm changes if we change the training data.

★ SOLUTION: True

29. [2 points] Which of the following statement about KL-divergence is not true?

(a) KL-divergence is a measure of how different two distributions are
(b) KL-divergence is 0 if and only if two distributions \( P \) and \( Q \) are equal
(c) KL-divergence can sensibly be measured between any two vectors of equal length
(d) KL-divergence is always non-negative.

★ SOLUTION: (C)

30. [1 points] True or False? A max pooling layer in a ConvNet has an equal number of weights to learn as a filter of the same size.
31. [1 points] True or False? (Standard) GANS are neural networks composed of two subcomponent neural nets. One neural net (the generator) takes as input an image and generates a different, fake image. The other neural net (the discriminator) takes as input either a real image or a fake image and tries to determine whether it was real or fake.

★ SOLUTION: False
32. [2 points] Assume a convolutional neural net where the input is a $4 \times 4$ RGB image (i.e., $4 \times 4 \times 3$), with 2 filters, each of size $2 \times 2 \times 3$, a stride of 2, and we zero pad the image with zeros on all four sides (giving a $6 \times 6 \times 3$ “input”).

How many nodes are there in the first hidden layer?

(a) $6 \times 6 \times 3 \times 2$
(b) $5 \times 5 \times 2$
(c) $4 \times 4 \times 2$
(d) $3 \times 3 \times 2$
(e) none of the above

★ SOLUTION: D

33. [1 points] True or False? When fitting a model using Adaboost, training should stop before the training error reaches zero.

★ SOLUTION: False

34. [2 points] Suppose we are fitting a model using gradient boosting, where we call $\eta$ the learning rate of the model update:

$$F_m(x) = F_{m-1}(x) + \eta \cdot \gamma_{m-1} h_{m-1}(x), \ 0 < \eta \leq 1$$

Additionally, each base learner $h_{m-1}(x)$ is fit with a randomly sub-sampled fraction $f$, $0 < f \leq 1$ of the training data.

Which of the following would most reduce overfitting?

(a) increasing $\eta$, increasing $f$
(b) decreasing $\eta$, increasing $f$
(c) increasing $\eta$, decreasing $f$
(d) decreasing $\eta$, decreasing $f$
35. [1 points] True or False? When fitting a model using gradient boosting and a squared loss, we fit the base learner to the (standard) residuals at each iteration.

★ SOLUTION: True

36. [1 points] True or False? In AdaBoost, in each iteration the weights of the misclassified examples on any given iteration all go up by the same multiplicative factor.

★ SOLUTION: True

37. [1 points] True or False? AdaBoost will eventually give zero training error regardless of the type of weak classifier it uses, provided enough iterations are performed.

★ SOLUTION: False

38. [2 points] Which of the following least penalizes misclassifications as a function of how far they are from the decision boundary?

(a) hinge loss
(b) logistic loss
(c) exponential loss

★ SOLUTION: A

39. [1 points] True or False? Hinge loss functions only penalize points for which the predicted classification label is wrong.

★ SOLUTION: False

40. [1 points] True or False? The number of support vectors found by an SVM is independent of the magnitude of the penalty on the slack variables.
★ SOLUTION: False