Latent Dirichlet Allocation (LDA)


Following slides borrowed (with heavily modification) from:
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“Bag of Words” Models

- Assume that all the words within a document are exchangeable.
  - The order of the words doesn’t matter, just the count.

```
<table>
<thead>
<tr>
<th>Word</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>about</td>
<td>2</td>
</tr>
<tr>
<td>all</td>
<td>2</td>
</tr>
<tr>
<td>Africa</td>
<td>1</td>
</tr>
<tr>
<td>apple</td>
<td>0</td>
</tr>
<tr>
<td>anxious</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>gas</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>oil</td>
<td>1</td>
</tr>
<tr>
<td>zebra</td>
<td>0</td>
</tr>
</tbody>
</table>
```
Mixture of Unigrams = Naïve Bayes (NB)

Model: For each document:
- Choose a topic $z_d$ with $p(topic_i) = \Theta$
- Choose $N$ words $w_n$ by drawing each one independently from a multinomial conditioned on $z_d$ with $p(w_n=word_j|topic_i=z) = \beta_z$
  - *Multinomial*: take a (non-uniform prior) dice with a word on each side; roll the dice $N$ times and count how often each word comes up.

In NB, we have exactly one topic per document.
LDA: Each doc is a mixture of topics

- LDA: each document is a (different) mixture of topics
  - Naïve Bayes assumes each document is on a single topic
  - LDA lets each word be on a different topic
  - For each document, d:
    - Choose a multinomial distribution $\theta_d$ over topics for that document
    - For each of the $N$ words $w_n$ in the document
      - Choose a topic $z_n$ with $p(topic) = \theta_d$
      - Choose a word $w_n$ from a multinomial conditioned on $z_n$ with $p(w=w_n|topic=z_n)$
      - Note: each topic has a different probability of generating each word
In the LDA model, we want the *topic mixture proportions* for each document to be drawn from some *distribution*.

- *distribution* = “probability distribution”, so it sums to one

So, we want to put a prior distribution on multinomials. That is, k-tuples of non-negative numbers that sum to one.

- We want probabilities of probabilities
- These multinomials lie in a (k-1)-*simplex*
  - *Simplex* = generalization of a triangle to (k-1) dimensions.

Our prior:

- Defined for a (k-1)-simplex.
- Conjugate to the multinomial
3 Dirichlet Examples (over 3 topics)

Corners: only one topic
Center: uniform mixture of topics
Colors indicate probability of seeing the topic distribution
Dirichlet Distribution

\[ p(\theta | \alpha) = \frac{\Gamma\left(\sum_{i=1}^{k} \alpha_i\right)}{\prod_{i=1}^{k} \Gamma(\alpha_i)} \prod_{i=1}^{k} \theta_i^{\alpha_i - 1} \]

- **Dirichlet distribution**
  - is defined over a (k-1)-simplex. I.e., it takes k non-negative arguments which sum to one.
  - is the conjugate prior to the multinomial distribution.
    - I.e. if our likelihood is multinomial with a Dirichlet prior, then the posterior is also Dirichlet
  - The Dirichlet parameter \( \alpha_i \) can be thought of as the prior count of the \( i^{th} \) class.

- For LDA, we often use a “symmetric Dirichlet” where all the \( \alpha \) are equal
  - \( \alpha \) is then a “concentration parameter”
Effect of $\alpha$

- When $\alpha < 1.0$, the majority of the probability mass is in the "corners" of the simplex, generating mostly documents that have a small number of topics.
- When $\alpha > 1.0$, most documents contain words from most of the topics.
The LDA Model

For each document,
- Choose the topic distribution $\theta \sim \text{Dirichlet}(\alpha)$
- For each of the N words $w_n$:
  - Choose a topic $z \sim \text{Multinomial}(\theta)$
  - Then choose a word $w_n \sim \text{Multinomial}(\beta_z)$
    - Where each topic has a different parameter vector $\beta$ for the words
The LDA Model: “Plate representation”

- For each of M documents,
  - Choose the topic distribution $\theta \sim \text{Dirichlet}(\alpha)$
  - For each of the N words $w_n$:
    - Choose a topic $z \sim \text{Multinomial}(\theta)$
    - Choose a word $w_n \sim \text{Multinomial}(\beta_z)$
Parameter Estimation

- Given a corpus of documents, find the parameters $\alpha$ and $\beta$ which maximize the likelihood of the observed data (words in documents), marginalizing over the hidden variables $\theta$, $z$.

- **E-step:**
  - Compute $p(\theta, z|w, \alpha, \beta)$, the posterior of the hidden variables $(\theta, z)$ given each document $w$, and parameters $\alpha$ and $\beta$.

- **M-step**
  - Estimate parameters $\alpha$ and $\beta$ given the current hidden variable distribution estimates.

- **Unfortunately, the E-step cannot be solved in a closed form**
  - So people use a “variational” approximation.

$\theta$: topic distribution for the document, $z$: topic for each word in the document.
Variational Inference

In variational inference, we consider a simplified graphical model with variational parameters $\gamma$, $\phi$ and minimize the KL Divergence between the variational and posterior distributions.

- $q$ approximates $p$

$$(\gamma^*, \phi^*) = \arg\min_{(\gamma, \phi)} KL(q(\theta, z|\gamma, \phi) || p(\theta, z|w, \alpha, \beta))$$
Parameter Estimation: Variational EM

- Given a corpus of documents, find the parameters $\alpha$ and $\beta$ which maximize the likelihood of the observed data.

- **E-step:**
  - Estimate the variational parameters $\gamma$ and $\phi$ in $q(\gamma, \phi; \alpha, \beta)$ by minimizing the KL-divergence to $p$ (with $\alpha$ and $\beta$ fixed)

- **M-step**
  - Maximize (over $\alpha$ and $\beta$) the lower bound on the log likelihood obtained using $q$ in place of $p$ (with $\gamma$ and $\phi$ fixed)

You don’t need to know the details; only what is hidden and what observed; and that EM works here.
<table>
<thead>
<tr>
<th>“Arts”</th>
<th>“Budgets”</th>
<th>“Children”</th>
<th>“Education”</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEW</td>
<td>MILLION</td>
<td>CHILDREN</td>
<td>SCHOOL</td>
</tr>
<tr>
<td>FILM</td>
<td>TAX</td>
<td>WOMEN</td>
<td>STUDENTS</td>
</tr>
<tr>
<td>SHOW</td>
<td>PROGRAM</td>
<td>PEOPLE</td>
<td>SCHOOLS</td>
</tr>
<tr>
<td>MUSIC</td>
<td>BUDGET</td>
<td>CHILD</td>
<td>EDUCATION</td>
</tr>
<tr>
<td>MOVIE</td>
<td>BILLION</td>
<td>YEARS</td>
<td>TEACHERS</td>
</tr>
<tr>
<td>PLAY</td>
<td>FEDERAL</td>
<td>FAMILIES</td>
<td>HIGH</td>
</tr>
<tr>
<td>MUSICAL</td>
<td>YEAR</td>
<td>WORK</td>
<td>PUBLIC</td>
</tr>
<tr>
<td>BEST</td>
<td>SPENDING</td>
<td>PARENTS</td>
<td>TEACHER</td>
</tr>
<tr>
<td>ACTOR</td>
<td>NEW</td>
<td>SAYS</td>
<td>BENNETT</td>
</tr>
<tr>
<td>FIRST</td>
<td>STATE</td>
<td>FAMILY</td>
<td>MANIGAT</td>
</tr>
<tr>
<td>YORK</td>
<td>PLAN</td>
<td>WELFARE</td>
<td>NAMPHY</td>
</tr>
<tr>
<td>OPERA</td>
<td>MONEY</td>
<td>MEN</td>
<td>STATE</td>
</tr>
<tr>
<td>THEATER</td>
<td>PROGRAMS</td>
<td>PERCENT</td>
<td>PRESIDENT</td>
</tr>
<tr>
<td>ACTRESS</td>
<td>GOVERNMENT</td>
<td>CARE</td>
<td>ELEMENTARY</td>
</tr>
<tr>
<td>LOVE</td>
<td>CONGRESS</td>
<td>LIFE</td>
<td>HAITI</td>
</tr>
</tbody>
</table>

The William Randolph Hearst Foundation will give $1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. “Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services,” Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center’s share will be $200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive $400,000 each. The Juilliard School, where music and the performing arts are taught, will get $250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual $100,000 donation, too.
LDA topics can be used for semi-supervised learning
LDA requires fewer topics than NB

perplexity = \(2^{H(p)}\) per word

i.e., \(\log_2(\text{perplexity}) = \text{entropy}\)
There are many LDA extensions

The author-topic model

Topic (LDA)  Author  Author-Topic

\(\alpha, \theta, z\) \(\alpha, \theta, x\) \(\alpha, \theta, a_d, z\)

\(\beta, \phi, w\) \(\beta, \phi, w\) \(\beta, \phi, a_d, w\)

\(z = \text{topic}\) \(x = \text{author}\)
Set the background switching binomial $\lambda$
- Draw an ailment distribution $\eta \sim \text{Dir}(\sigma)$
- Draw word multinomials $\phi \sim \text{Dir}(\beta)$ for the topic, ailment, and background distributions
- For each message $1 \leq m \leq D$:
  - Draw a switching distribution $\pi \sim \text{Beta}(\gamma_0, \gamma_1)$
  - Draw an ailment $a \sim \text{Mult}(\eta)$
  - Draw a topic distribution $\theta \sim \text{Dir}(\alpha_0)$
  - For each word $w_i \in N_m$
    - Draw aspect $y_i \in \{0, 1, 2\}$ (observed)
    - Draw background switcher $\ell \in \{0, 1\} \sim \text{Bi}(\lambda)$
    - If $\ell == 0$:
      - Draw $w_i \sim \text{Mult}(\phi_{B,y})$ (a background)
    - Else:
      - Draw $x_i \in \{0, 1\} \sim \text{Bi}(\pi)$
      - If $x_i == 0$: (draw word from topic $z$)
        - Draw topic $z_i \sim \text{Mult}(\theta)$
        - Draw $w_i \sim \text{Mult}(\phi_z)$
      - Else: (draw word from ailment $a$ aspect $y$)
        - Draw $w_i \sim \text{Mult}(\phi_{a,y})$

Paul & Dredze
What you should know about LDA

- Each document is a mixture over topics
- Each topic looks like a Naïve Bayes model
  - It produces words with some probability
- Estimation of LDA is messy
  - Requires variational EM or Gibbs sampling
- In a plate model, each “plate” represents repeated nodes in a network
  - The plate model shows conditional independence, but not the form of the statistical distribution (e.g. Gaussian, Poisson, Dirichlet, ....)
**LDA generation - example**

- **Topics** = \{sports, politics\}
- **Words** = \{football, baseball, TV, win, president\}
- \( \alpha = (0.8, 0.2) \)
- \( \beta = \)

<table>
<thead>
<tr>
<th>Word</th>
<th>sports</th>
<th>politics</th>
</tr>
</thead>
<tbody>
<tr>
<td>football</td>
<td>0.30</td>
<td>0.01</td>
</tr>
<tr>
<td>baseball</td>
<td>0.25</td>
<td>0.01</td>
</tr>
<tr>
<td>TV</td>
<td>0.10</td>
<td>0.15</td>
</tr>
<tr>
<td>win</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>president</td>
<td>0.01</td>
<td>0.20</td>
</tr>
<tr>
<td>OOV</td>
<td>0.04</td>
<td>0.38</td>
</tr>
</tbody>
</table>
For each document, \( d \)
- Pick a topic distribution, \( \theta_d \) using \( \alpha \)
- For each word in the document
  - pick a topic, \( z \)
  - given that topic, pick a word using \( \beta \)

\[
\begin{array}{c|cc}
\text{topic} & \text{sports} & \text{politics} \\
\hline
\text{football} & 0.30 & 0.01 \\
\text{baseball} & 0.25 & 0.01 \\
\text{TV} & 0.10 & 0.15 \\
\text{win} & 0.30 & 0.25 \\
\text{president} & 0.01 & 0.20 \\
\text{OOV} & 0.04 & 0.38 \\
\end{array}
\]

\( \alpha = (0.8, 0.2) \)
\( \beta = \)