Do you have Poll Everywhere?

A) Yes
B) No
Norms
Norms

For all $a \in \mathbb{R}$ and all $u, v \in V$,

- $L_p(a v) = |a| L_p(v)$
- $L_p(u + v) \leq L_p(u) + L_p(v)$  
  - triangle inequality or subadditivity
- If $L_p(v) = 0$ then $v$ is the zero vector
  - implies $|v| = 0$ iff $v$ is the zero vector

$L_p$ norm: $(\sum_j |x_j|^p)^{1/p}$
What is $||(1,2,3)||_1$?

A) 1  
B) 3  
C) $\sqrt{14}$  
D) $\sqrt{\frac{14}{3}}$  
E) none of the above
What is $\|(1,2,3)\|_2$?

A) 1
B) 3
C) $\sqrt{14}$
D) $\sqrt{14/3}$
E) none of the above
What is \( |(1,2,3)|_{1/2} \) ?

A) 1
B) 3
C) \( \sqrt{14} \)
D) \( \sqrt{14/3} \)
E) none of the above
What is $|(1,2,3)|_0$?
A) 1
B) 3
C) $\sqrt{14}$
D) $\sqrt{14/3}$
E) none of the above
$L_0$ pseudo-norm

$|x|_0 = \text{number of } x_j \neq 0$

How is this not a real norm?
Norms

Is $|x|^{1/2}$ convex?
Distance

◆ How do norms relate to distance?
Distance

How do norms relate to distance?

\[ d_p(x, y) = |x - y|_p \]
A symmetric function $K: \mathbf{X} \times \mathbf{X} \rightarrow \mathbb{R}$ is a positive semi-definite (psd) kernel on $\mathbf{X}$ if

$$\sum_{i,j} c_i c_j K(x_i, x_j) \geq 0$$
If $x$ is a vector containing only non-negative numbers, does the following satisfy the 3 properties of a norm?

$$\sum_j x_j$$ - the sum of the elements of $x$

Technically, that doesn’t make it a norm, since the criteria should hold over a full vector space.
If $x$ is a vector containing only non-negative numbers, does the following satisfy the 3 properties of a norm?

$$\sum_j j x_j$$ - the sum of the elements of $x$, each weighted by its index, $j$
If $x$ is a vector containing only non-negative numbers, does the following satisfy the 3 properties of a norm?

$$\sum_j x_j^2$$
If $x$ is a vector containing only non-negative numbers, does the following satisfy the 3 properties of a norm?

$\text{length}(x) - \text{number of the elements of } x$
If $x$ is a vector containing only non-negative numbers, does the following satisfy the 3 properties of a norm?

$d(x,y)$ - the Euclidean distance between $x$ and some other (arbitrary, but fixed, vector $y$, also non-negative)
True or False: The only important thing you need to pick which doing k-nearest neighbors is k