Lecture 11

CIS 4521/5521: COMPILERS

Announcements

- HW3: LLVM Backend
  - Available on the course web pages.
  - Due: Weds., February 26th at 10:00PM
  - Note: test cases should be submitted 24 hours earlier (so by Tues., Feb. 25th at 10pm)

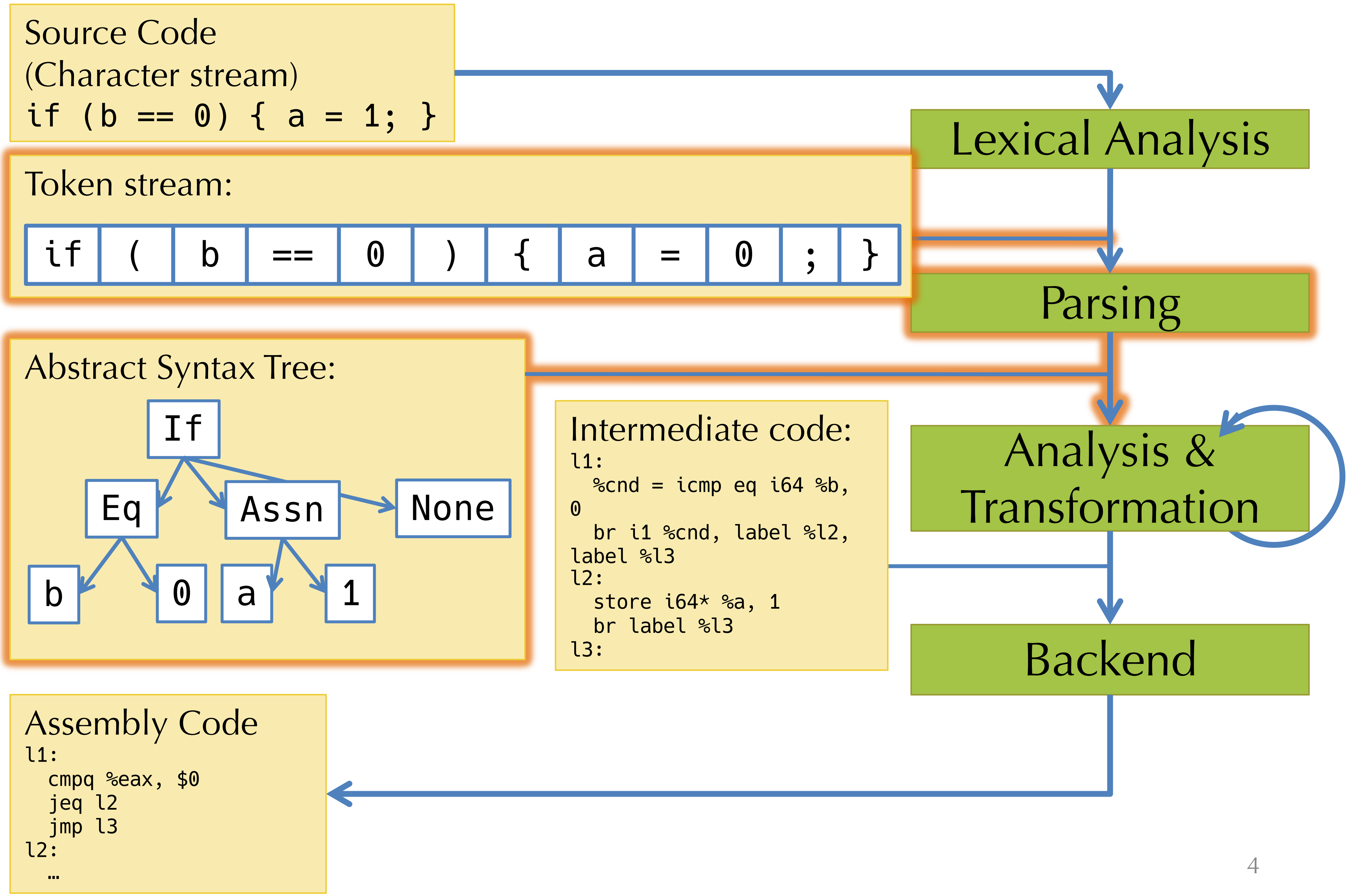
you should have ALREADY STARTED

- Midterm: March 6th
  - In class
  - One-page, letter-sized, double-sided “cheat sheet” of notes permitted
  - See Ed post (soon) for previous exams

Creating an abstract representation of program syntax.

PARSING

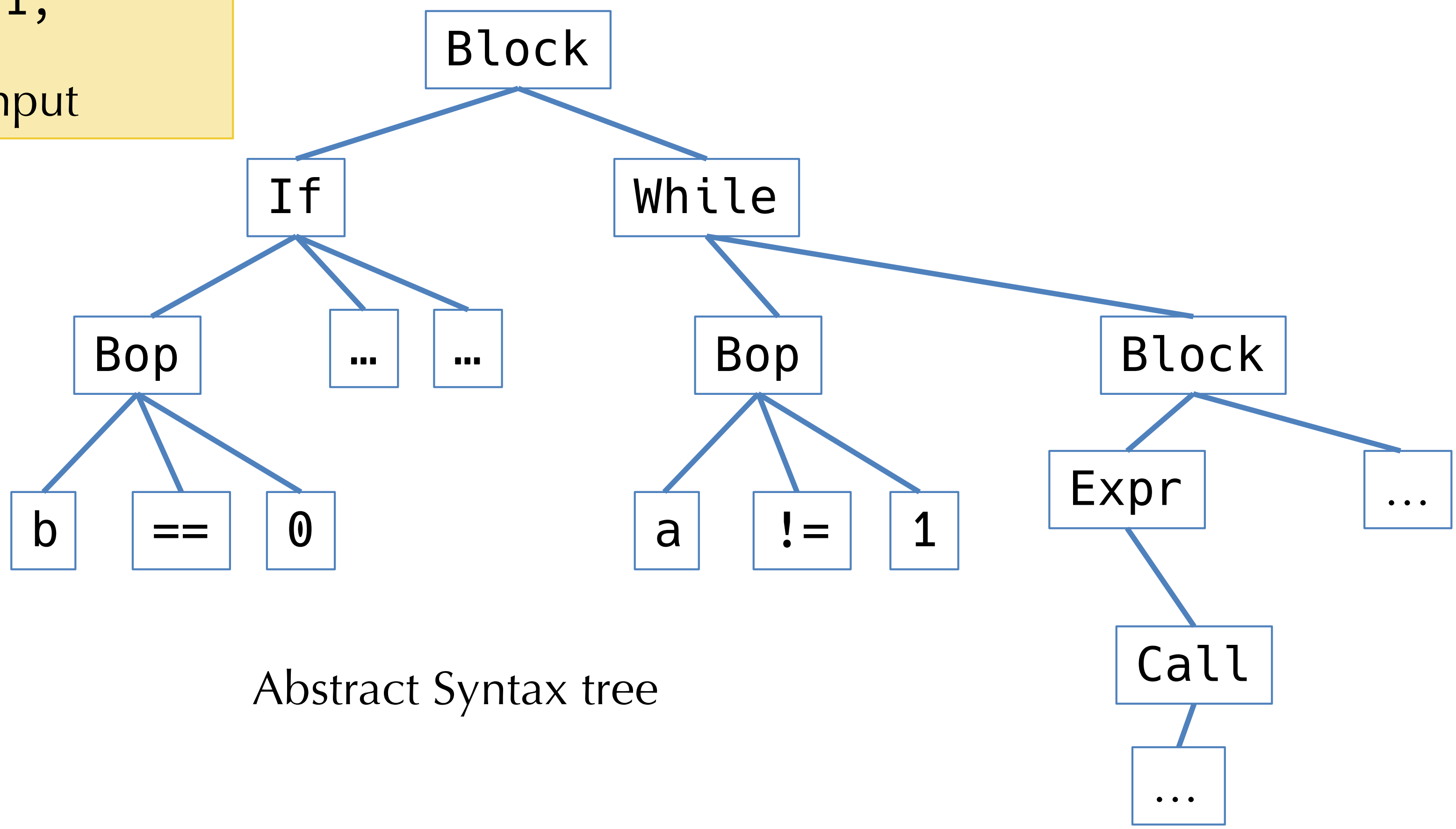
Parsing


Source Code
(Character stream)
if (b == 0) { a = 1; }
Lexical Analysis
Token stream:
if (b == 0) { a = 0 ; }
Parsing
Abstract Syntax Tree:
If
Eq
Assn
None
b
0
a
1
Intermediate code:
l1:
%cnd = icmp eq i64 %b, 0
br i1 %cnd, label %l2, label %l3
l2:
store i64* %a, 1
br label %l3
l3:
Analysis & Transformation
Backend
Assembly Code
l1:
cmpq %eax, $0
jeq l2
jmp l3
l2:
…

Parsing: Finding Syntactic Structure

```
{
  if (b == 0) a = b;
  while (a != 1) {
    print_int(a);
    a = a - 1;
  }
}
```

Source input



Abstract Syntax tree

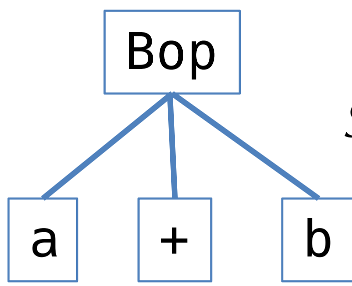
Syntactic Analysis (Parsing): Overview

- Input: stream of tokens (generated by lexer)
- Output: abstract syntax tree

- Strategy:
  - Parse the token stream to traverse the "concrete" syntax
  - During traversal, build a tree representing the "abstract" syntax

- Why abstract? Consider these three *different* concrete inputs:

```
a + b
(a + ((b)))
((a) + (b))
```



Same abstract syntax tree

- Note: parsing doesn't check many things:
  - Variable scoping, type agreement, initialization, …

Specifying Language Syntax

- First question: how to describe language syntax precisely and conveniently?
- Previously we described tokens using regular expressions
  - Easy to implement, efficient DFA representation
  - Why not use regular expressions on tokens to specify programming language syntax?

- Limits of regular expressions:
  - DFA's have only finite # of states
  - So… DFA's can't "count"
  - For example, consider the language of all strings that contain balanced parentheses – easier than most programming languages, but not regular.

- So: we need more expressive power than DFA's

CONTEXT FREE GRAMMARS

Context-free Grammars

- Here is a specification of the language of balanced parens:

$$S \mapsto (S) S$$
$$S \mapsto \varepsilon$$

Note: Once again we have to take care to distinguish meta-language elements (e.g. "S" and "⟼") from object-language elements (e.g. "(").*

- The definition is *recursive* – S mentions itself.

- Idea: "derive" a string in the language by starting with S and rewriting according to the rules:
  - Example:
    $S \mapsto (S) S \mapsto ((S) S) S \mapsto ((\varepsilon) S) S \mapsto ((\varepsilon) S) \varepsilon \mapsto ((\varepsilon) \varepsilon) \varepsilon = (( ))$

- You can replace the *nonterminal* S by one of its definitions anywhere
- A context-free grammar accepts a string iff there is a derivation from the start symbol

* And, since we're writing this description in English, we are careful distinguish the meta-meta-language (e.g. words) from the meta-language and object-language (e.g. symbols) by using quotes.

CFGs Mathematically

- A Context-free Grammar (CFG) consists of
  - A set of *terminals* (e.g., a lexical token or ε)
  - A set of *nonterminals* (e.g., S and other syntactic variables)
  - A designated nonterminal called the *start symbol*
  - A set of productions: LHS $\longmapsto$ RHS
    - LHS is a nonterminal
    - RHS is a *string* of terminals and nonterminals

- Example: The balanced parentheses language:

$$S \longmapsto (\,S\,)\,S$$

$$S \longmapsto \varepsilon$$

- How many terminals? How many nonterminals? Productions?

Another Example: Sum Grammar

- A grammar that accepts parenthesized sums of numbers:

$$
\begin{array}{lll}
S \mapsto E + S & | & E \\
E \mapsto \text{number} & | & (\; S \;)
\end{array}
$$

e.g.: `(1 + 2 + (3 + 4)) + 5`

- Note the vertical bar '|' is shorthand for multiple productions:

$S \mapsto E + S$
$S \mapsto E$
$E \mapsto \text{number}$
$E \mapsto (S)$

4 productions
2 nonterminals: S, E
4 terminals: (,), +, number
Start symbol: S

Derivations in CFGs

- Example: derive `(1 + 2 + (3 + 4)) + 5`
- $\underline{S} \mapsto \underline{E} + S$

  $\mapsto (\underline{S}) + S$

  $\mapsto (\underline{E} + S) + S$

  $\mapsto (1 + \underline{S}) + S$

  $\mapsto (1 + \underline{E} + S) + S$

  $\mapsto (1 + 2 + \underline{S}) + S$

  $\mapsto (1 + 2 + \underline{E}) + S$

  $\mapsto (1 + 2 + (\underline{S})) + S$

  $\mapsto (1 + 2 + (\underline{E} + S)) + S$

  $\mapsto (1 + 2 + (3 + \underline{S})) + S$

  $\mapsto (1 + 2 + (3 + \underline{E})) + S$

  $\mapsto (1 + 2 + (3 + 4)) + \underline{S}$

  $\mapsto (1 + 2 + (3 + 4)) + \underline{E}$

  $\mapsto (1 + 2 + (3 + 4)) + 5$

$S \mapsto E + S \mid E$
$E \mapsto \text{number} \mid (S)$

For arbitrary strings α, β, γ and production rule $A \mapsto \beta$
a single step of the derivation is:

$$\alpha A \gamma \mapsto \alpha \beta \gamma$$

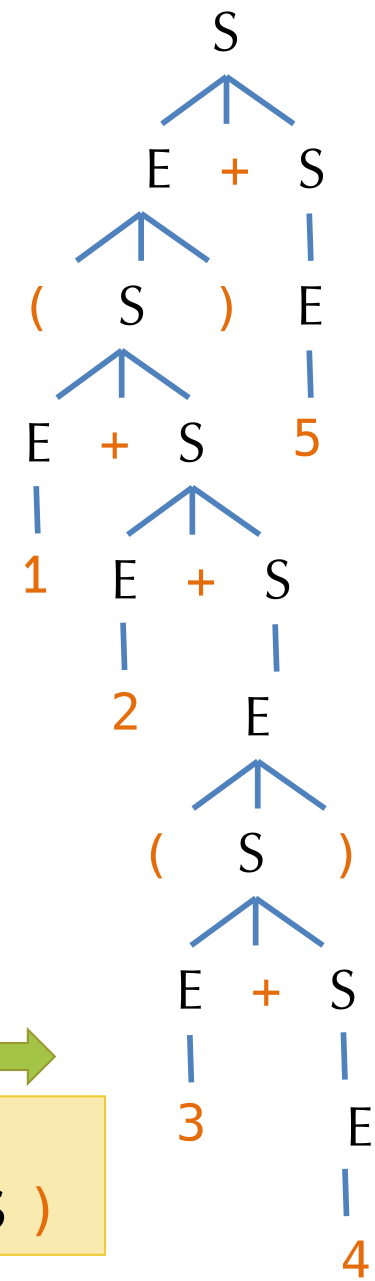
(*substitute* β for an occurrence of A)

In general, there are many possible derivations for a given string.

Note: Underline indicates symbol being expanded.

From Derivations to Parse Trees

- Tree representation of the derivation
- Leaves of the tree are terminals
  - In-order traversal yields the input sequence of tokens
- Internal nodes: nonterminals
- No information about the *order* of the derivation steps



Parse Tree

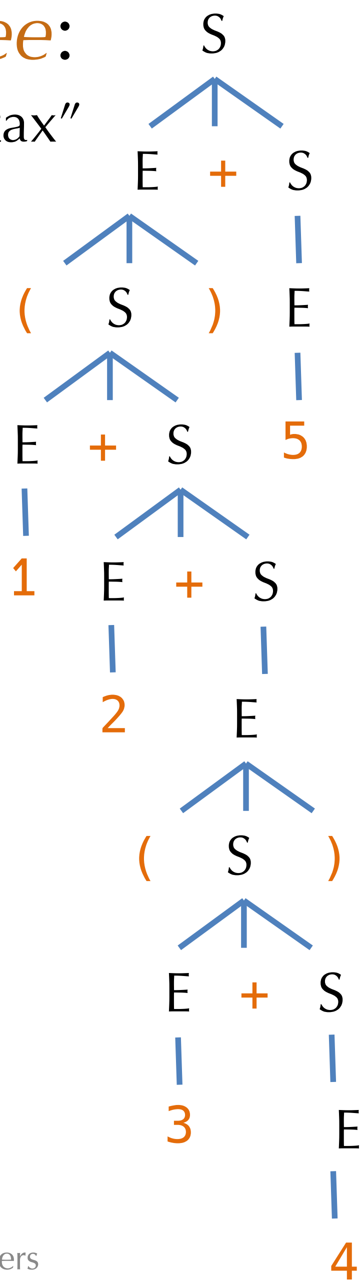
`(1 + 2 + (3 + 4)) + 5`

S ⟼ E + S | E
E ⟼ number | (S)

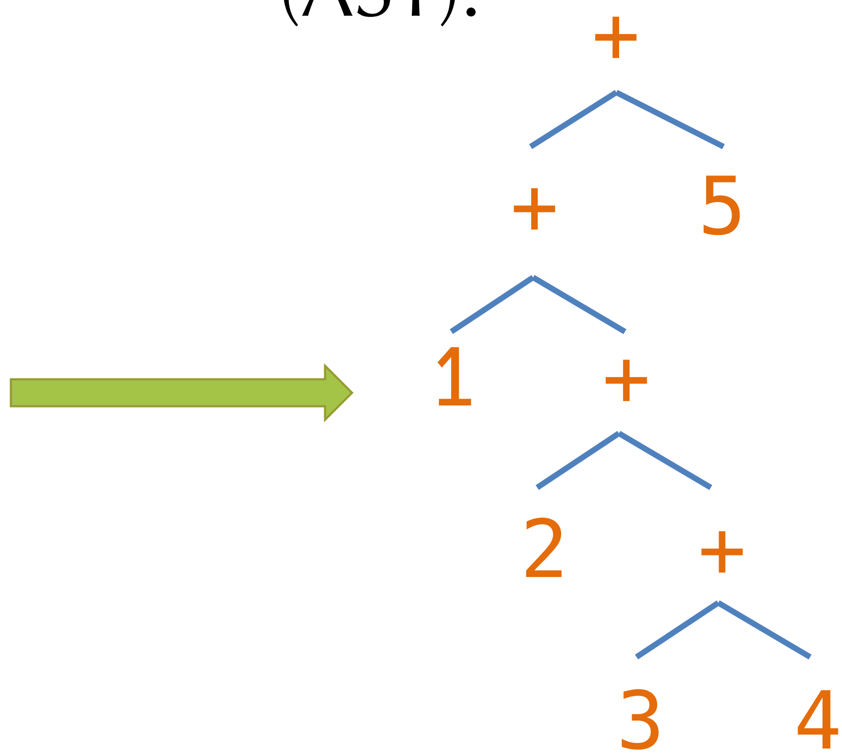
From Parse Trees to Abstract Syntax

- *Parse tree*:

"concrete syntax"



- *Abstract syntax tree* (AST):



- Hides, or *abstracts*, unneeded information.

Derivation Orders

- Productions of the grammar can be applied in any order.
- There are two standard orders:
  - *Leftmost derivation*: Find the left-most nonterminal and apply a production to it.
  - *Rightmost derivation*: Find the right-most nonterminal and apply a production there.

- Note that both strategies (and any other) yield the same parse tree!
  - Parse tree doesn't contain the information about what order the productions were applied.

Example: Left- and rightmost derivations

- Leftmost Derivation
- **S** ⟼ **E** + S
  ⟼ (**S**) + S
  ⟼ (**E** + S) + S
  ⟼ (1 + **S**) + S
  ⟼ (1 + **E** + S) + S
  ⟼ (1 + 2 + **S**) + S
  ⟼ (1 + 2 + **E**) + S
  ⟼ (1 + 2 + (**S**)) + S
  ⟼ (1 + 2 + (**E** + S)) + S
  ⟼ (1 + 2 + (3 + **S**)) + S
  ⟼ (1 + 2 + (3 + **E**)) + S
  ⟼ (1 + 2 + (3 + 4)) + **S**
  ⟼ (1 + 2 + (3 + 4)) + **E**
  ⟼ (1 + 2 + (3 + 4)) + 5

- Rightmost derivation:
- **S** ⟼ E + **S**
  ⟼ E + **E**
  ⟼ **E** + 5
  ⟼ (**S**) + 5
  ⟼ (E + **S**) + 5
  ⟼ (E + E + **S**) + 5
  ⟼ (E + E + **E**) + 5
  ⟼ (E + E + (**S**)) + 5
  ⟼ (E + E + (E + **S**)) + 5
  ⟼ (E + E + (E + **E**)) + 5
  ⟼ (E + E + (**E** + 4)) + 5
  ⟼ (E + **E** + (3 + 4)) + 5
  ⟼ (**E** + 2 + (3 + 4)) + 5
  ⟼ (1 + 2 + (3 + 4)) + 5

Loops and Termination

- Some care is needed when defining CFGs
- Consider:

  $S \longmapsto E$
  $E \longmapsto S$

  – This grammar has nonterminal definitions that are "nonproductive". (i.e. they don't mention any terminal symbols)
  – There is no finite derivation starting from S, so the language is empty.

- Consider: $S \longmapsto (S)$

  – This grammar is productive, but again there is no finite derivation starting from S, so the language is empty

- It is easy to generalize these examples to a "chain" of many nonterminals, which can be harder to find in a large grammar

- Upshot: be aware of "vacuously empty" CFG grammars.
  – Every nonterminal should eventually rewrite to an alternative that contains only terminal symbols.

Associativity, ambiguity, and precedence.

GRAMMARS FOR PROGRAMMING LANGUAGES

Associativity

S ⟼ E + S | E
E ⟼ number | (S)

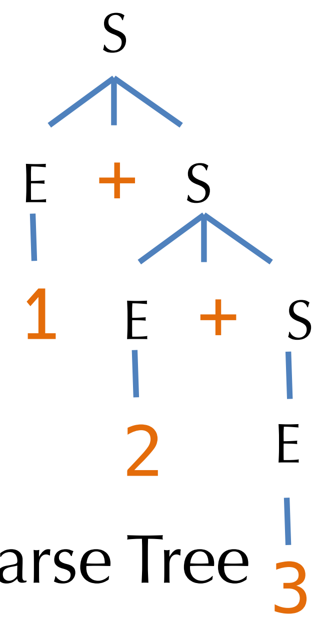
Consider the input: 1 + 2 + 3

Leftmost derivation:

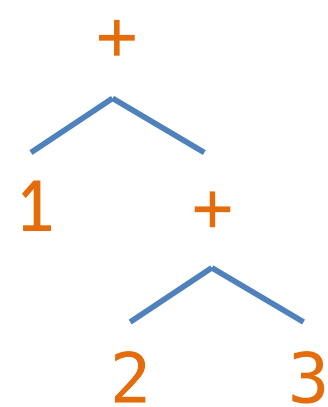
S ⟼ **E** + S
⟼ 1 + **S**
⟼ 1 + **E** + S
⟼ 1 + 2 + **S**
⟼ 1 + 2 + **E**
⟼ 1 + 2 + 3

Rightmost derivation:

S ⟼ E + **S**
⟼ E + E + **S**
⟼ E + E + **E**
⟼ E + **E** + 3
⟼ **E** + 2 + 3
⟼ 1 + 2 + 3



Parse Tree



AST

Associativity

- This grammar makes '+' *right associative*…
  - i.e., the abstract syntax tree is the same for both `1 + 2 + 3` and `1 + (2 + 3)`
- Note that the grammar is *right recursive*…

S refers to itself on the right of +

$$S \mapsto E + S \mid E$$
$$E \mapsto \text{number} \mid (\ S\ )$$

- How would you make '+' left associative?
- What are the trees for "`1 + 2 + 3`"?

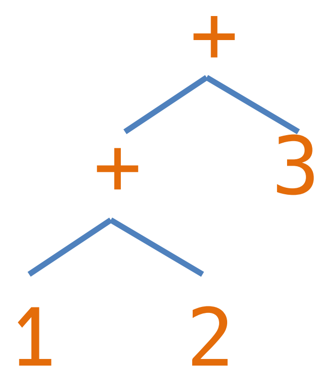
Ambiguity

- Consider this grammar:

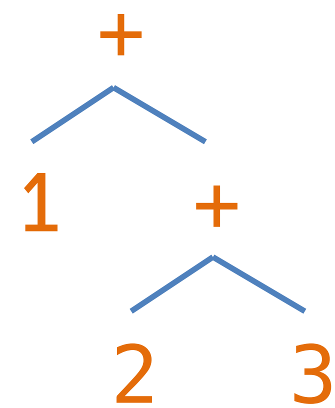
$$S \mapsto S + S \mid (S) \mid \text{number}$$

- Claim: it accepts the *same* set of strings as the previous one.
- What's the difference?
- Consider these *two* leftmost derivations:
  - **S** ↦ **S** + S ↦ 1+ **S** ↦ 1+ **S** + S ↦ 1+2+ **S** ↦ 1+2+3
  - **S** ↦ **S** + S ↦ **S** + S + S ↦ 1+ **S** + S ↦ 1+2+ **S** ↦ 1+2+3

- One derivation gives left associativity, the other gives right associativity to '+'
  - Which is which?



AST 1



AST 2

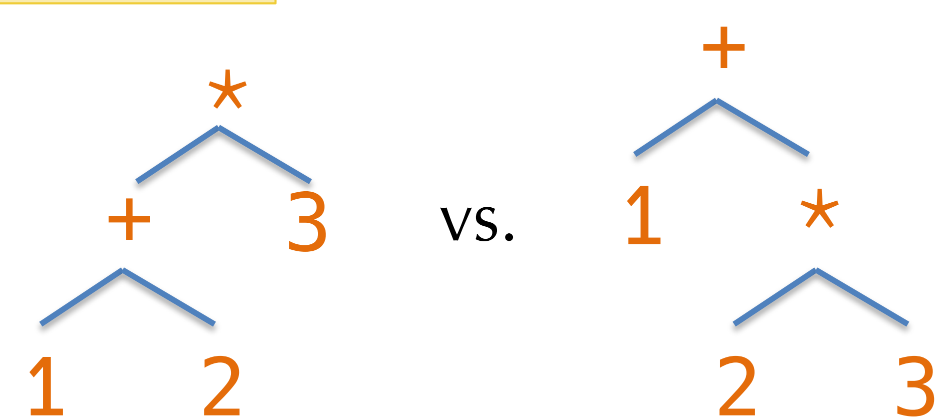
Why do we care about ambiguity?

- The '+' operation is associative, so it doesn't matter which tree we pick. Mathematically, x + (y + z) = (x + y) + z
  - But, some binary operations aren't associative. Examples?
  - Some operations are only left (or right) associative. Examples?

- Moreover, if there are multiple operations, ambiguity in the grammar leads to ambiguity in their *precedence*
- Consider:

S ⟼ S + S | S * S | (S) | number

- Input: `1 + 2 * 3`
  - One parse = `(1 + 2) * 3` = 9
  - The other = `1 + (2 * 3)` = 7



Eliminating Ambiguity

- We can often eliminate ambiguity by adding nonterminals and allowing recursion only on the left (or right) .
- Higher-precedence operators go *farther* from the start symbol.
- Example:

$$S \mapsto S + S \mid S * S \mid (S) \mid \text{number}$$

- To disambiguate:
  - Decide (following math) to make '*' higher precedence than '+'
  - Make '+' left associative
  - Make '*' right associative
- Note:
  - S_2 corresponds to 'atomic' expressions

$$S_0 \mapsto S_0 + S_1 \mid S_1$$
$$S_1 \mapsto S_2 * S_1 \mid S_2$$
$$S_2 \mapsto \text{number} \mid (S_0)$$

Context Free Grammars: Summary

- Context-free grammars allow concise specifications of programming languages.
  - An unambiguous CFG specifies how to parse: convert a token stream to a (parse tree)
  - Ambiguity can (often) be removed by encoding precedence and associativity in the grammar.

- Even with an unambiguous CFG, there may be more than one derivation
  - Though all derivations correspond to the same abstract syntax tree.

- Still to come: finding a derivation
  - But first: menhir

Searching for derivations.

LL & LR PARSING

CFGs Mathematically

- A Context-free Grammar (CFG) consists of
  - A set of *terminals* (e.g., a token or ε)
  - A set of *nonterminals* (e.g., S and other syntactic variables)
  - A designated nonterminal called the *start symbol*
  - A set of productions: LHS $\longmapsto$ RHS
    - LHS is a nonterminal
    - RHS is a *string* of terminals and nonterminals

- Example: The balanced parentheses language:

$$S \longmapsto (\,S\,)\,S$$

$$S \longmapsto \varepsilon$$

- How many terminals? How many nonterminals? Productions?

Consider finding left-most derivations

- Look at only one input symbol at a time.

S ⟼ E + S | E
E ⟼ number | (S)

Partly-derived String	Look-ahead	Parsed/Unparsed Input
<u>S</u>	(	(1 + 2 + (3 + 4)) + 5
⟼ **<u>E</u>** + S	(	(1 + 2 + (3 + 4)) + 5
⟼ (**<u>S</u>**) + S	1	(1 + 2 + (3 + 4)) + 5
⟼ (**<u>E</u>** + S) + S	1	(1 + 2 + (3 + 4)) + 5
⟼ (1 + **<u>S</u>**) + S	2	(1 + 2 + (3 + 4)) + 5
⟼ (1 + **<u>E</u>** + S) + S	2	(1 + 2 + (3 + 4)) + 5
⟼ (1 + 2 + **<u>S</u>**) + S	(	(1 + 2 + (3 + 4)) + 5
⟼ (1 + 2 + **<u>E</u>**) + S	(	(1 + 2 + (3 + 4)) + 5
⟼ (1 + 2 + (**<u>S</u>**)) + S	3	(1 + 2 + (3 + 4)) + 5
⟼ (1 + 2 + (**<u>E</u>** + S)) + S	3	(1 + 2 + (3 + 4)) + 5
⟼ …		

There is a problem

$S \longmapsto E + S \mid E$
$E \longmapsto \text{number} \mid (S)$

- We want to decide which production to apply based on the look-ahead symbol.
- But, there is a choice:

(1) $\quad S \longmapsto E \longmapsto (S) \longmapsto (E) \longmapsto (1)$

vs.

(1) + 2 $\quad S \longmapsto E + S \longmapsto (S) + S \longmapsto (E) + S \longmapsto (1) + S \longmapsto (1) + E$
$\longmapsto (1) + 2$

- Given the look-ahead symbol: '(' it isn't clear whether to pick $S \longmapsto E$ or $S \longmapsto E + S$ first.

LL(1) GRAMMARS

Grammar is the problem

- Not all grammars can be parsed "top-down" with only a single lookahead symbol.
- *Top-down*: starting from the start symbol (root of the parse tree) and going down

- LL(1) means
  - **L**eft-to-right scanning
  - **L**eft-most derivation,
  - **1** lookahead symbol

$$S \mapsto E + S \mid E$$
$$E \mapsto \text{number} \mid (S)$$

- This language isn't "LL(1)"
- Is it LL(k) for some k?

- What can we do?

Making a grammar LL(1)

- *Problem:* We can't decide which S production to apply until we see the symbol after the first expression.
- *Solution:* "Left-factor" the grammar. There is a common S prefix for each choice, so add a new non-terminal S′ at the decision point:

$$
\begin{aligned}
S &\longmapsto E + S \mid E \\
E &\longmapsto \text{number} \mid (S)
\end{aligned}
\quad\Longrightarrow\quad
\begin{aligned}
S &\longmapsto E S' \\
S' &\longmapsto \varepsilon \\
S' &\longmapsto + S \\
E &\longmapsto \text{number} \mid (S)
\end{aligned}
$$

- Also need to eliminate left-recursion somehow. Why?
- Consider:

$$
\begin{aligned}
S &\longmapsto S + E \mid E \\
E &\longmapsto \text{number} \mid (S)
\end{aligned}
$$

Infinite regress if we want to find the left-most derivation:

$$\underline{S} \longmapsto \underline{S} + E \longmapsto \underline{S} + E + E \longmapsto \underline{S} + E + E + E \longmapsto \underline{S} + E + E + E + E \ldots$$

(this can't be resolved by left factoring!)

LL(1) Parse of the input string

- Look at only one input symbol at a time.

$S \longmapsto ES'$
$S' \longmapsto \varepsilon$
$S' \longmapsto + S$
$E \longmapsto \text{number} \mid (S)$

Partly-derived String	Look-ahead	Parsed/Unparsed Input
S	(	(1 + 2 + (3 + 4)) + 5
⟼ **E** S′	(	(1 + 2 + (3 + 4)) + 5
⟼ (**S**) S′	1	(1 + 2 + (3 + 4)) + 5
⟼ (**E** S′) S′	1	(1 + 2 + (3 + 4)) + 5
⟼ (1 **S′**) S′	+	(1 + 2 + (3 + 4)) + 5
⟼ (1 + **S**) S′	2	(1 + 2 + (3 + 4)) + 5
⟼ (1 + **E** S′) S′	2	(1 + 2 + (3 + 4)) + 5
⟼ (1 + 2 **S′**) S′	+	(1 + 2 + (3 + 4)) + 5
⟼ (1 + 2 + **S**) S′	(	(1 + 2 + (3 + 4)) + 5
⟼ (1 + 2 + **E** S′) S′	(	(1 + 2 + (3 + 4)) + 5
⟼ (1 + 2 + (**S**)S′) S′	3	(1 + 2 + (3 + 4)) + 5

Predictive Parsing

- Given an LL(1) grammar:
  - For a given nonterminal, the lookahead symbol uniquely determines the production to apply.
  - Top-down parsing = predictive parsing
  - Driven by a predictive parsing table:
    nonterminal * input token → production

T ⟼ S$
S ⟼ ES′
S′ ⟼ ε
S′ ⟼ + S
E ⟼ number | (S)

	number	+	(	)	$ (EOF)
T	⟼ S$		⟼S$		
S	⟼ E S′		⟼E S′		
S′		⟼ + S		⟼ ε	⟼ ε
E	⟼ num.		⟼ (S)		

- Note: it is convenient to add a special *end-of-file* token $ and a start symbol T (top-level) that requires $.

How do we construct the parse table?

- Consider a given production: A → γ
- Construct the set of all input tokens that may appear *first* in strings that can be derived from γ
  - Add the production → γ to the entry (A,token) for each such token.
- If γ can derive ε (the empty string), then we construct the set of all input tokens that may *follow* the nonterminal A in the grammar.
  - Add the production → γ to the entry (A, token) for each such token.

- Note: if there are two different productions for a given entry, the grammar is not LL(1)

Example

- First(T) = First(S)
- First(S) = First(E)
- First(S′) = { + }
- First(E) = { number, '(' }

- Follow(S′) = Follow(S)
- Follow(S) = { \$, ')' } ∪ Follow(S′)

T ⟼ S\$
S ⟼ ES′
S′ ⟼ ε
S′ ⟼ + S
E ⟼ number | (S)

Note: we want the *least* solution to this system of set equations… a *fixpoint* computation. More on these later in the course.

	number	+	(	)	\$ (EOF)
T	⟼ S\$		⟼S\$		
S	⟼ E S′		⟼E S′		
S′		⟼ + S		⟼ ε	⟼ ε
E	⟼ num.		⟼ (S)		

Converting the table to code

- Define n mutually recursive functions
  - one for each nonterminal A: parse_A
  - The type of parse_A is `unit -> ast` if A is *not* an auxiliary nonterminal
  - Parse functions for auxiliary nonterminals (e.g. S′) take extra ast's as inputs, one for each nonterminal in the "factored" prefix.

- Each function "peeks" at the lookahead token and then follows the production rule in the corresponding entry.
  - Consume terminal tokens from the input stream
  - Call parse_X to create sub-tree for nonterminal X
  - If the rule ends in an auxiliary nonterminal, call it with appropriate ast's. (The auxiliary rule is responsible for creating the ast after looking at more input.)
  - Otherwise, this function builds the ast tree itself and returns it.

	number	+	(	)	$ (EOF)
T	$\longmapsto$ S$		$\longmapsto$S$		
S	$\longmapsto$ E S′		$\longmapsto$E S′		
S′		$\longmapsto$ + S		$\longmapsto$ ε	$\longmapsto$ ε
E	$\longmapsto$ num.		$\longmapsto$ (S)		

Hand-generated LL(1) code for the table above.

DEMO: HANDWRITTEN.ML

LL(1) Summary

- Top-down parsing that finds the leftmost derivation.
- Language Grammar ⇒ LL(1) grammar ⇒ prediction table ⇒ recursive-descent parser

- Problems:
  - Grammar must be LL(1)
  - Can extend to LL(k) (it just makes the table bigger)
  - Grammar cannot be left recursive (parser functions will loop!)

- Is there a better way?

LR GRAMMARS

Bottom-up Parsing (LR Parsers)

- LR(k) parser:
  - Left-to-right scanning
  - Rightmost derivation
  - k lookahead symbols
- LR grammars are more expressive than LL
  - Can handle left-recursive (and right recursive) grammars; virtually all programming languages
  - Easier to express programming language syntax (no left factoring)
- Technique: "Shift-Reduce" parsers
  - Work bottom up instead of top down
  - Construct right-most derivation of a program in the grammar
  - Used by many parser generators (e.g. yacc, CUP, ocamlyacc, merlin, etc.)
  - Better error detection/recovery

Top-down vs. Bottom up

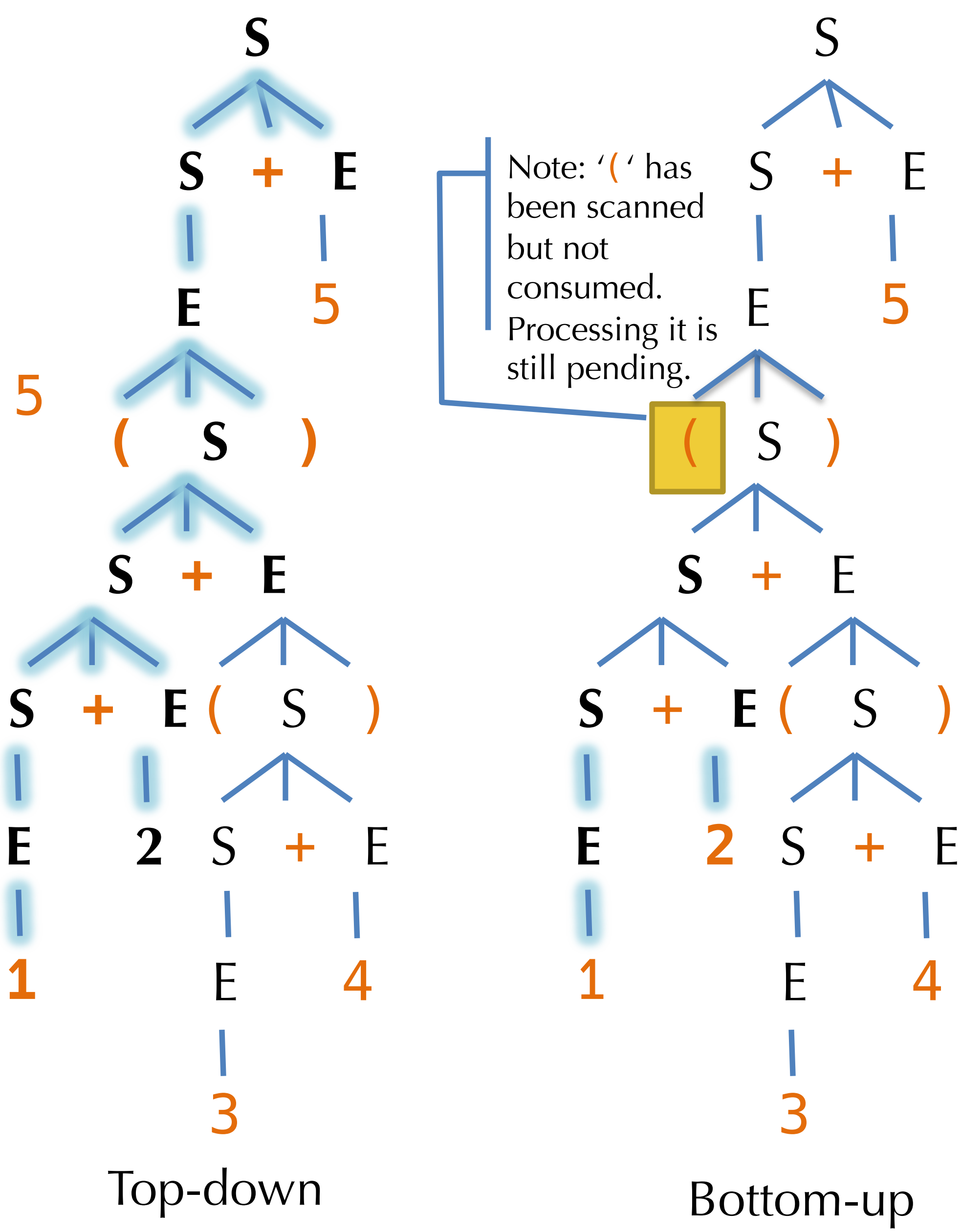
- Consider the left-recursive grammar:

$S \mapsto S + E \mid E$
$E \mapsto \text{number} \mid (S)$

- (1 + 2 + (3 + 4)) + 5

- What part of the tree must we know after scanning just "(1 + 2" ?

- In top-down, must be able to guess which productions to use…



Progress of Bottom-up Parsing

Rightmost derivation ↑

Reductions	Scanned	Input Remaining
(1 + 2 + (3 + 4)) + 5 ⟻		(1 + 2 + (3 + 4)) + 5
(**E** + 2 + (3 + 4)) + 5 ⟻	(	1 + 2 + (3 + 4)) + 5
(**S** + 2 + (3 + 4)) + 5 ⟻	(1	+ 2 + (3 + 4)) + 5
(S + **E** + (3 + 4)) + 5 ⟻	(1 + 2	+ (3 + 4)) + 5
(**S** + (3 + 4)) + 5 ⟻	(1 + 2	+ (3 + 4)) + 5
(S + (**E** + 4)) + 5 ⟻	(1 + 2 + (3	+ 4)) + 5
(S + (**S** + 4)) + 5 ⟻	(1 + 2 + (3	+ 4)) + 5
(S + (S + **E**)) + 5 ⟻	(1 + 2 + (3 + 4	)) + 5
(S + (**S**)) + 5 ⟻	(1 + 2 + (3 + 4	)) + 5
(S + **E**) + 5 ⟻	(1 + 2 + (3 + 4)	) + 5
(**S**) + 5 ⟻	(1 + 2 + (3 + 4)	) + 5
E + 5 ⟻	(1 + 2 + (3 + 4))	+ 5
S + 5 ⟻	(1 + 2 + (3 + 4))	+ 5
S + **E** ⟻	(1 + 2 + (3 + 4)) + 5	
S		

S ⟼ S + E | E
E ⟼ number | (S)

Shift/Reduce Parsing

$S \longmapsto S + E \mid E$
$E \longmapsto \text{number} \mid (S)$

- Parser state:
  - Stack of terminals and nonterminals.
  - Unconsumed input is a string of terminals
  - Current derivation step is stack + input
- Parsing is a sequence of *shift* and *reduce* operations:
- Shift: move look-ahead token to the stack
- Reduce: Replace symbols γ at top of stack with nonterminal X such that X ⟼ γ is a production. (pop γ, push X)

Stack	Input	Action
	`(1 + 2 + (3 + 4)) + 5`	shift (
(	`1 + 2 + (3 + 4)) + 5`	shift 1
(1	`+ 2 + (3 + 4)) + 5`	reduce: E ⟼ number
(E	`+ 2 + (3 + 4)) + 5`	reduce: S ⟼ E
(S	`+ 2 + (3 + 4)) + 5`	shift +
(S +	`2 + (3 + 4)) + 5`	shift 2
(S + 2	`+ (3 + 4)) + 5`	reduce: E ⟼ number

parser.mly, lexer.mll, range.ml, ast.ml, main.ml

DEMO: BOOLEAN LOGIC