Lecture 22

# CIS 4521/5521: COMPILERS

# Announcements

- HW6: Analysis & Optimizations
  - Alias analysis, constant propagation, dead code elimination, register allocation
  - Available soon
  - Due: **Wednesday,** April 30th

A high-level tour of a variety of optimizations.

# BASIC OPTIMIZATIONS

# Inlining

- Replace a call to a function with the body of the function itself with arguments rewritten to be local variables:
- Example in OAT: inline `pow` into `g`

```
int g(int x) { return x + pow(x); }
int pow(int a) {
    var b = 1; var x = 0;
    while (x < a) {b = 2 * b; x = x + 1}
    return b;
}
```

note: renaming

➔

```
int g(int x) {
    int a = x;
    int b = 1; int x2 = 0;
    while (x2 < a) {b = 2 * b; x2 = x2 + 1};
    tmp = b;
    return x + tmp;
}
```

- May need to rename variables to avoid *capture*
  - See lecture about ***capture avoiding substitution*** for lambda calculus
- Best done at the AST or relatively high-level IR.
- When is it profitable?
  - Eliminates the stack manipulation, jump, etc.
  - Can increase code size.
  - Enables further optimizations

# Code Specialization

- Idea: create specialized versions of a function that is called from different places with different arguments.
- Example: specialize function `f` in:

```
class A implements I { int m() {…} }
class B implements I { int m() {…} }
int f(I x) { x.m(); }        // don't know which m
A a = new A(); f(a);         // know it's A.m
B b = new B(); f(b);         // know it's B.m
```

- `f_A` would have code specialized to dispatch to `A.m`
- `f_B` would have code specialized to dispatch to `B.m`
- You can also inline methods when the run-time type is known statically
  - Often just one class implements a method.

# Common Subexpression Elimination

- ***fold redundant computations together***
  - in some sense, it's the opposite of inlining
- Example:

```
a[i] = a[i] + 1
```

compiles to:

```
MEM[a + i*8] := MEM[a + i*8] + 1
```

Common subexpression elimination removes the redundant add and multiply:

```
t = a + i*8; MEM[t] := MEM[t] + 1
```

- For safety, you must be sure that the shared expression always has the same value in both places!

# Unsafe Common Subexpression Elimination

- Example: consider this OAT function:

```
unit f(int[] a, int[] b, int[] c) {
  var j = …; var i = …; var k = …;
  b[j] = a[i] + 1;
  c[k] = a[i];
  return;
}
```

- The optimization that shares the expression `a[i]` is unsafe… why?

```
unit f(int[] a, int[] b, int[] c) {
  var j = …; var i = …; var k = …;
  t = a[i];
  b[j] = t + 1;
  c[k] = t;
  return;
}
```

# LOOP OPTIMIZATIONS

# Loop Optimizations

- Program hot spots often occur in loops.
  - Especially inner loops
  - Not always: consider operating systems code or compilers vs. a computer game or word processor

- Most program execution time occurs in loops.
  - The 90/10 rule of thumb holds here too.
    (90% of the execution time is spent in 10% of the code)

- Loop optimizations are very important, effective, and numerous
  - Also, concentrating effort to improve loop body code is usually a win

# Loop Invariant Code Motion (revisited)

- Another form of redundancy elimination.
- If the result of a statement or expression does not change during the loop *and* it's pure, it can be hoisted outside the loop body.
- Often useful for array element addressing code
  - Invariant code not visible at the source level

```
for (i = 0; i < a.length; i++) {
    /* a not modified in the body */
}
```

```
t = a.length;
for (i =0; i < t; i++) {
 /* same body as above */
}
```

Hoisted loop-invariant expression

# Strength Reduction (revisited)

- Strength reduction can work for loops too
- Idea: replace expensive operations (multiplies, divides) by cheap ones (adds and subtracts)
- For loops, create a *dependent induction variable*:

- Example:

```
for (int i = 0; i<n; i++) { a[i*3] = 1; }  // stride by 3
```

⬇

```
int j = 0;
for (int i = 0; i<n; i++) {
  a[j] = 1;
  j = j + 3;     // replace multiply by add
}
```

# Loop Unrolling (revisited)

- Branches can be expensive, unroll loops to avoid them.

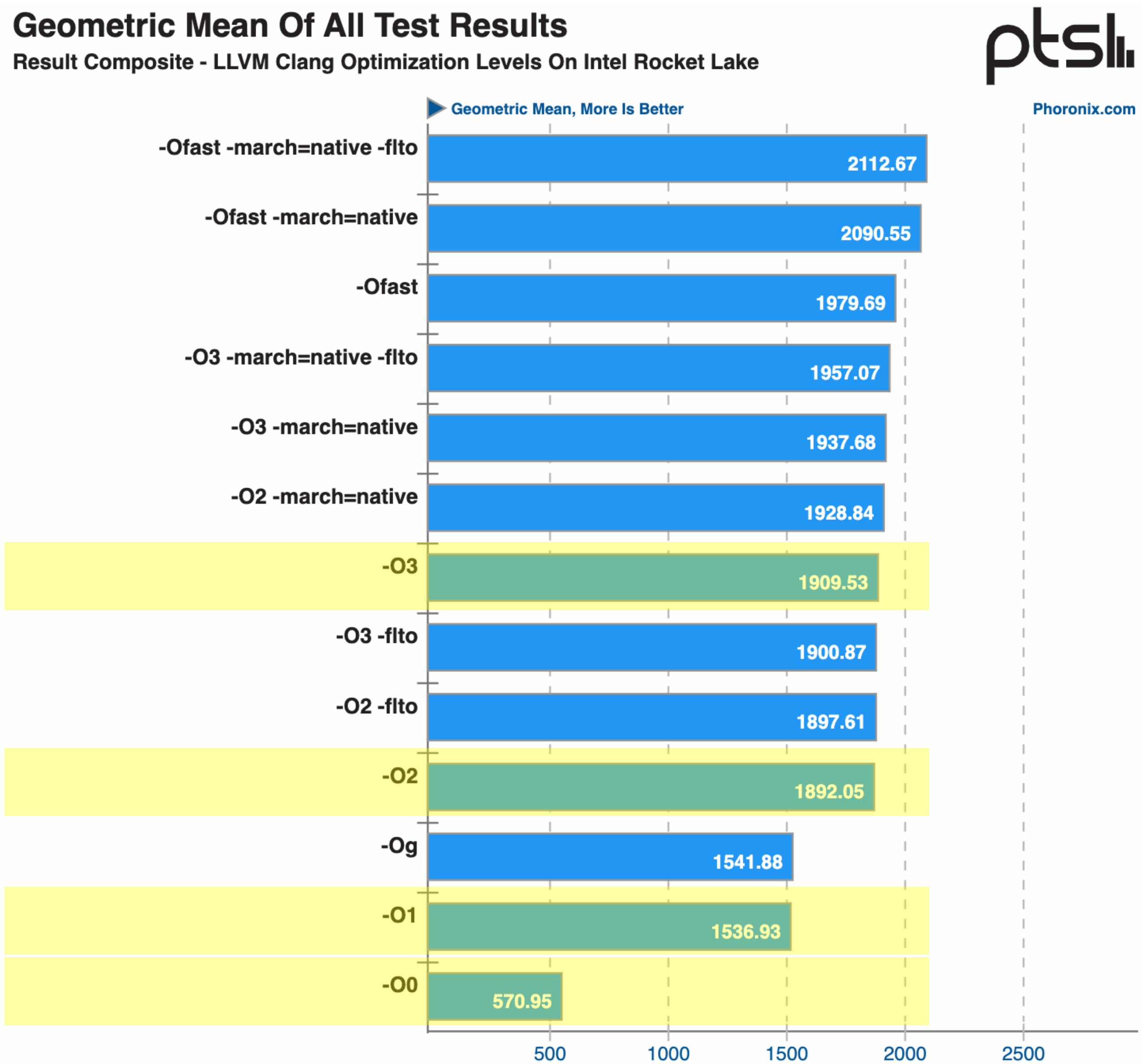
```
for (int i=0; i<n; i++) { S }
```

```
for (int i=0; i<n-3; i+=4) {S;S;S;S};
for (         ; i<n; i++) { S } // left over iterations
```

- With k unrollings, eliminates (k-1)/k conditional branches
  - So for the above program, it eliminates ¾ of the branches
- Space-time tradeoff:
  - Not a good idea for large S or small n
- Interacts with instruction caching, branch prediction

# EFFECTIVENESS?

# Optimization Effectiveness?

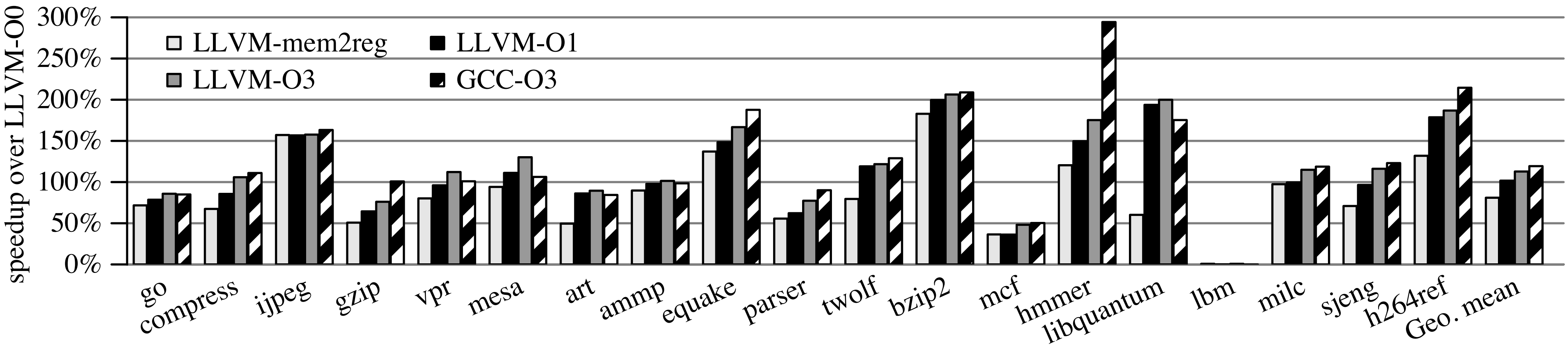



Geom. mean over 44 benchmark programs at various –O levels.

Clang 12

https://www.phoronix.com/review/clang-12-opt
LLVM Clang 12 Benchmarks At Varying Optimization Levels, LTO
25 June 2021

# Optimization Effectiveness?




$$\%\text{speedup} = \left[\frac{\text{base time}}{\text{optimized time}} - 1\right] \times 100\%$$

Example:

base time = 2s

optimized time = 1s ⇒ 100% speedup

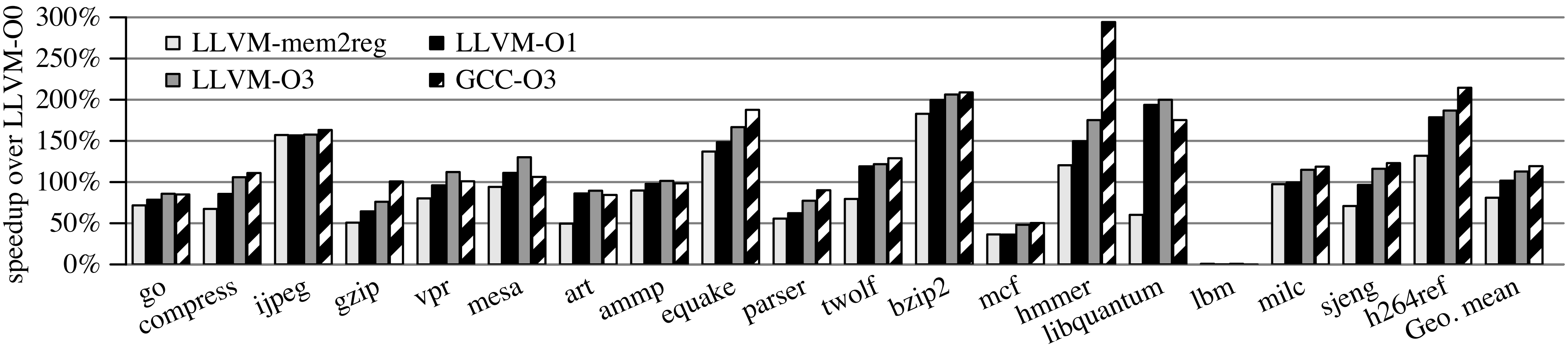
Example:

base time = 1.2s

optimized time = 0.87s ⇒ 38% speedup

Graph taken from:
Jianzhou Zhao, Santosh Nagarakatte, Milo M. K. Martin, and Steve Zdancewic.
Formal Verification of SSA-Based Optimizations for LLVM.
In Proc. 2013 ACM SIGPLAN Conference on Programming Languages Design and Implementation (PLDI), 2013

# Optimization Effectiveness?




- mem2reg: promotes alloca'ed stack slots to temporaries to enable register allocation
- Analysis:
  - mem2reg alone (+ back-end optimizations like register allocation) yields ~78% speedup on average
  - -O1 yields ~100% speedup
    (so all the rest of the optimizations combined account for ~22%)
  - -O3 yields ~120% speedup
- Hypothetical program that takes 10 sec. (base time):
  - Mem2reg alone: expect ~5.6 sec
  - -O1: expect ~5 sec
  - -O3: expect ~4.5 sec

# CODE ANALYSIS

# Motivating Code Analyses

- There are lots of things that might influence the safety/applicability of an optimization
  - What algorithms and data structures can help?

- How do you know what code participates in a loop?
- How do you know an expression is invariant?
- How do you know if an expression has no side effects?
- How do you keep track of where a variable is defined?
- How do you know where a variable is used?
- How do you know if two reference values may be aliases of one another?

# Moving Towards Register Allocation

- The OAT compiler currently generates as *many* temporary variables as it needs
  - These are the `%uids` you should be very familiar with by now.

- Current compilation strategy:
  - Each %`uid` maps to a stack location.
  - This yields programs with many loads/stores to memory.
  - Very inefficient.

- Ideally, we'd like to map as many `%uid`'s as possible into registers.
  - Eliminate the use of the `alloca` instruction?
  - Only 16 max registers available on 64-bit X86
  - %rsp and %rbp are reserved and some have special semantics, so really only 10 or 12 available
  - This means that a register must hold more than one slot
- When is this safe?

# Scope vs. Liveness

- We can already get some coarse liveness information from variable scoping.
- Consider the following OAT program:

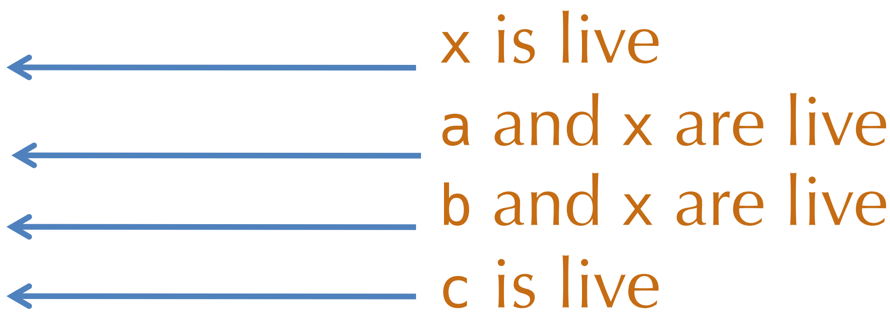
```
int f(int x) {
  var a = 0;
  if (x > 0) {
    var b = x * x;
    a = b + b;
  }
  var c = a * x;
  return c;
}
```

- Note that due to OAT's scoping rules, variables `b` and `c` can never be live at the same time.
  - `c`'s scope is disjoint from `b`'s scope
- So, we could assign `b` and `c` to the same alloca'ed slot and potentially to the same register at the x86 level.

# But Scope is too Coarse

- Consider this program:

```
int f(int x) {
  int a = x + 2;
  int b = a * a;
  int c = b + x;
  return c;
}
```




- The scopes of a,b,c,x all overlap – they're all in scope at the end of the block.
- But, a, b, c are never live at the same time.
  - So they can share the same stack slot / register

# Live Variable Analysis

- A variable v is ***live*** at a program point if v is defined before the program point and used after it.
- Liveness is defined in terms of where variables are ***defined*** and where variables are ***used***

- ***Liveness analysis***: Compute the live variables between each statement.
  - May be *conservative* (i.e. it may claim a variable is live when it isn't) so because that's a safe approximation
  - To be useful, it should be more *precise* than simple scoping rules.

- Liveness analysis is one example of ***dataflow analysis***
  - Other examples: Available Expressions, Reaching Definitions, Constant-Propagation Analysis, …

# Liveness information

- Consider this program:

```
int f(int x) {
  int a = x + 2;
  int b = a * a;
  int c = b + x;
  return c;
}
```

← x is live

← a and x are live

← b and x are live

← c is live

- The scopes of a,b,c,x all overlap – they're all in scope at the end of the block.
- But, a, b, c are never live at the same time.
  - So they can share the same stack slot / register

# Liveness

- Observation: `%uid1` and `%uid2` can be assigned to the same register if their values will not be needed at the same time.
  - What does it mean for an `%uid` to be "needed"?
  - Ans: its contents will be used as a source operand in a later instruction.
- Such a variable is called "***live***"

> A variable is ***live*** if its value *might* be used by some future part of the execution path when the program is executed.
>
> Notes:
> - the use of the variable might depend on user input or other data not available until the program is run
> - even if not, in general, such a property is undecidable
>
> ⇒ liveness is a static approximation of the dynamic behavior

- Observe: two variables can share the same register if they are *not* live at the same time.

# Control-flow Graphs Revisited

- For the purposes of dataflow analysis, we use the *control-flow graph* (CFG) intermediate form.
- Recall that a basic block is a sequence of instructions such that:
  - There is a distinguished, labeled entry point (no jumps into the middle of a basic block)
  - There is a (possibly empty) sequence of non-control-flow instructions
  - The block ends with a single control-flow instruction (jump, conditional branch, return, etc.)

- A *control flow graph*
  - Nodes are blocks
  - There is an edge from B1 to B2 if the control-flow instruction of B1 might jump to the entry label of B2
  - There are no "dangling" edges – there is a block for every jump target.

**Note:** the following slides are intentionally a bit ambiguous about the exact nature of the code in the control flow graphs:

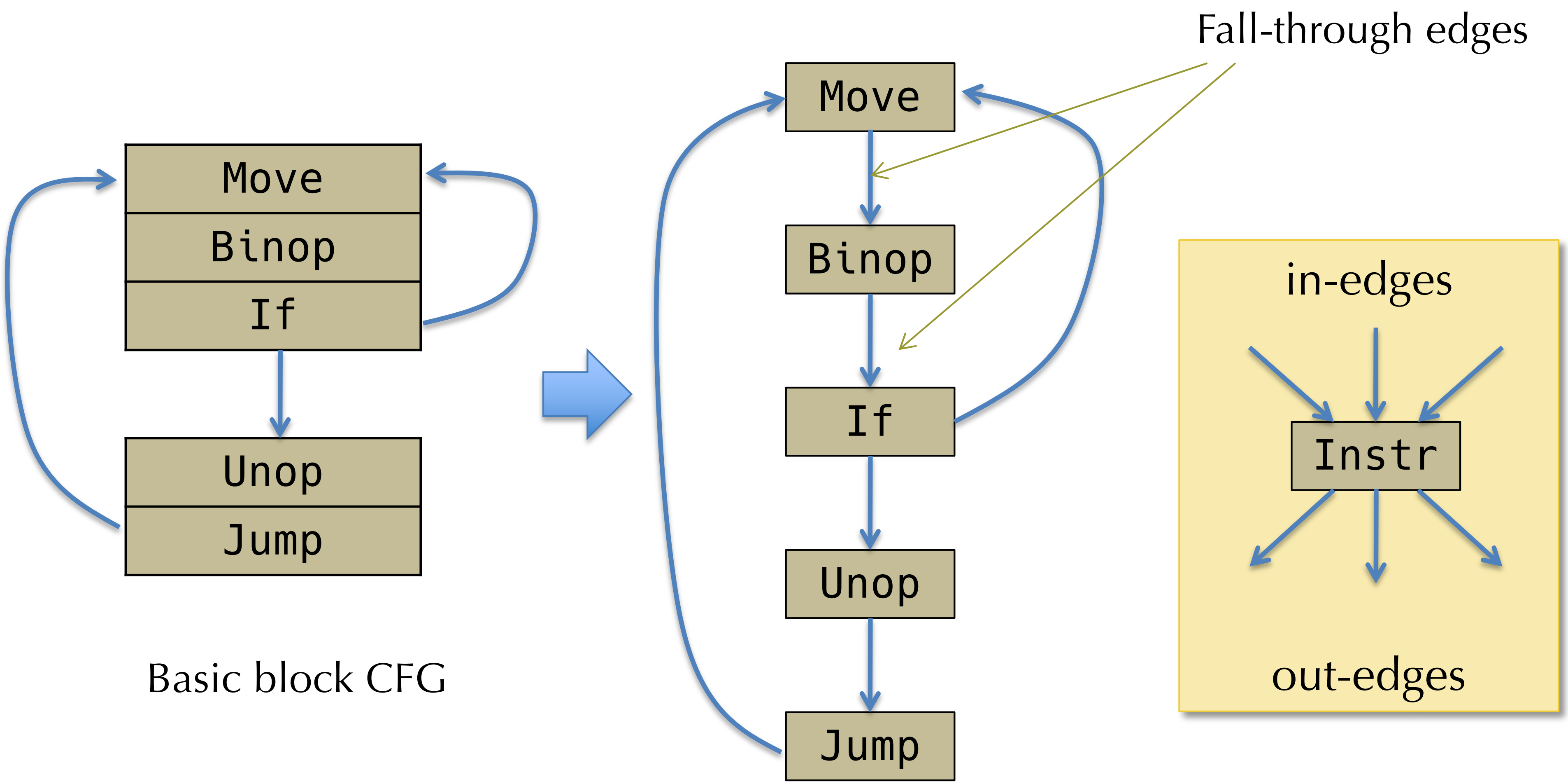
an "imperative" C-like source level
at the x86 assembly level
the LLVM IR level

Each setting applies the same general idea, but the exact details will differ.

- e.g., LLVM IR doesn't have "imperative" update of %uid temporaries.
  (The SSA structure of the LLVM IR by design makes some of these analyses simpler!)

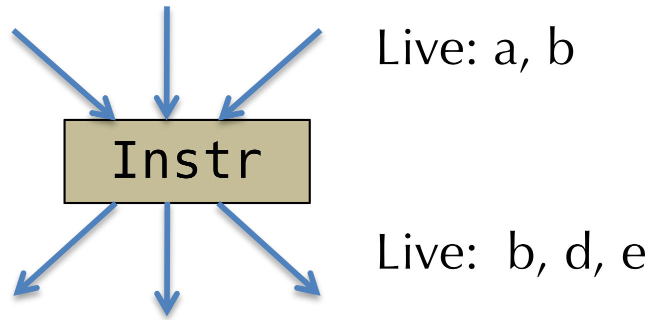
# Dataflow over CFGs

- For precision, it is helpful to think of the "fall through" between sequential instructions as an edge of the control-flow graph too.
  - Different implementation tradeoffs in practice…

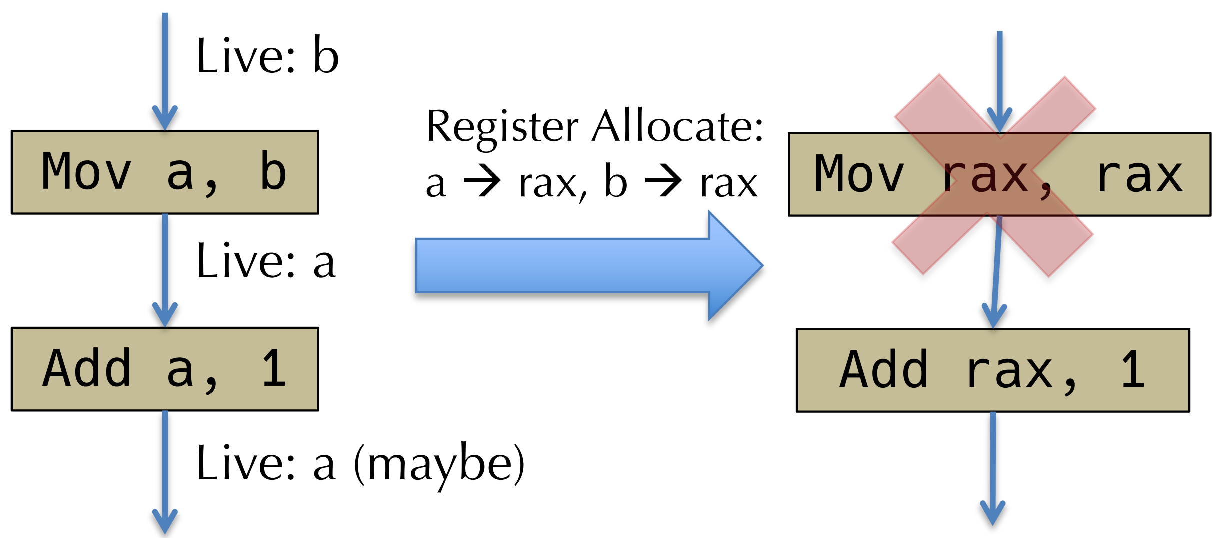



"Exploded" CFG

# Liveness is Associated with *Edges*




- This is useful so that the same register can be used for different temporaries in the same statement.
- Example: `a = b + 1`
- Compiles to:



# Uses and Definitions

- Every instruction/statement ***uses*** some set of variables
  - i.e. reads from them
- Every instruction/statement ***defines*** some set of variables
  - i.e. writes to them

- For a node/statement s define:
  - **use[s]** : set of variables used by s
  - **def[s]** : set of variables defined by s

- General Examples:

| | | |
|---|---|---|
| `s:    a = b + c` | use[s] = {b,c} | def[s] = {a} |
| `s:    a = a + 1` | use[s] = {a} | def[s] = {a} |

# Liveness, Formally

- A variable v is ***live*** on edge e if:
  There is
  - a node n in the CFG such that use[n] contains v, *and*
  - a directed path from e to n such that for every statement s′ on the path, def[s′] does not contain v

- The first clause says that v will be used on some path starting from edge e.
- The second clause says that v won't be redefined on that path before the use.

- Questions:
  - How to compute this efficiently?
  - How to use this information (e.g. for register allocation)?
  - How does the choice of IR affect this?
    (e.g. LLVM IR uses SSA, so it doesn't allow redefinition ⇒ simplify liveness analysis)

# Simple, inefficient algorithm

- "A variable v is live on an edge e  if there is a node n in the CFG using it  *and* a directed path from e to n pasing through no def of v."

- Backtracking Algorithm:
  - For each variable v…
  - Try all paths from each use of v, tracing backwards through the control-flow graph until either v is defined or a previously visited node has been reached.
  - Mark the variable v live across each edge traversed.

- Inefficient because it explores the same paths many times (for different uses and different variables)

# Dataflow Analysis

- *Idea*: compute liveness information for all variables simultaneously.
  - Keep track of sets of information about each node

- Approach: define *equations* that must be satisfied by any liveness determination.
  - Equations based on "obvious" constraints.

- Solve the equations by iteratively converging on a solution.
  - Start with a "rough" approximation to the answer
  - Refine the answer at each iteration
  - Keep going until no more refinement is possible: a ***fixpoint*** has been reached

- This is an instance of a general framework for computing program properties: dataflow analysis

# Dataflow Value Sets for Liveness

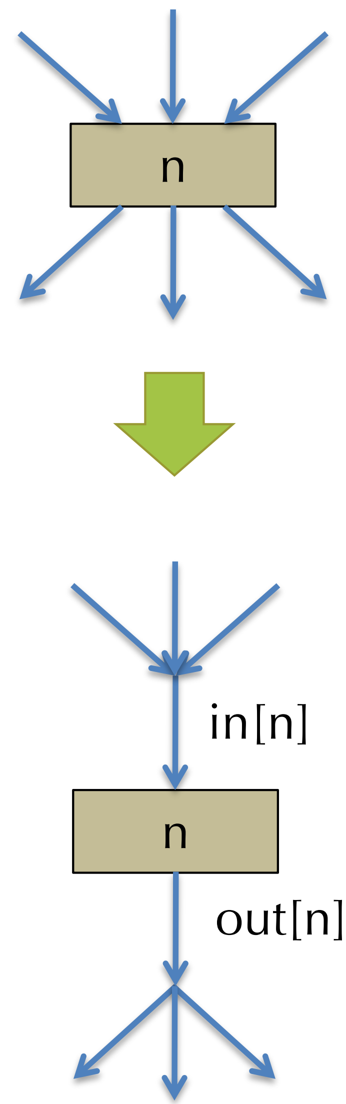
- Nodes are program statements, so:
- **use[n]** : set of variables used by n
- **def[n]** : set of variables defined by n
- **in[n]** : set of variables live on entry to n
- **out[n]** : set of variables live on exit from n

- Associate in[n] and out[n] with the “collected” information about incoming/outgoing edges

- For Liveness: what constraints are there among these sets?
- Clearly:

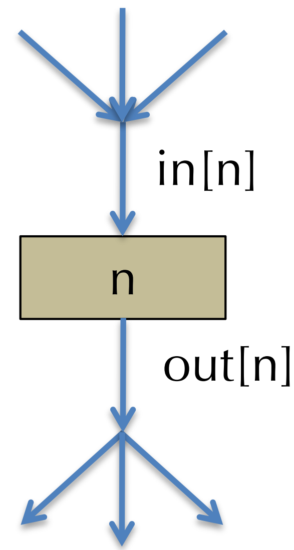
$$in[n] \supseteq use[n]$$

- What other constraints?



# Other Dataflow Constraints

- We have: $in[n] \supseteq use[n]$
  - "A variable must be live on entry to n if it is used by n"
- Also: $in[n] \supseteq out[n] - def[n]$
  - "If a variable is live on exit from n, and n doesn't define it, it is live on entry to n"
  - Note: here '-' means "set difference"
- And: $out[n] \supseteq in[n']$ if $n' \in succ[n]$
  - "If a variable is live on entry to a successor node of n, it must be live on exit from n."



# Iterative Dataflow Analysis

- Find a solution to those constraints by starting from a rough guess.
  - Start with: in[n] = Ø and out[n] = Ø
- The guesses don't satisfy the constraints:
  - in[n] ⊇ use[n]
  - in[n] ⊇ out[n] - def[n]
  - out[n] ⊇ in[n′] if n′ ∈ succ[n]

- Idea: iteratively re-compute in[n] and out[n] where forced to by the constraints.
  - Each iteration will add variables to the sets in[n] and out[n] (i.e. the live variable sets will increase monotonically)
- We stop when in[n] and out[n] satisfy these equations: (which are derived from the constraints above)
  - in[n] = use[n] ∪ (out[n] - def[n])
  - out[n] = $\bigcup_{n' \in succ[n]} in[n']$

# Complete Liveness Analysis Algorithm

for all n, in[n] := Ø, out[n] := Ø

repeat until no change in 'in' and 'out'

    for all n

        $out[n] := \bigcup_{n' \in succ[n]} in[n']$

        $in[n] := use[n] \cup (out[n] - def[n])$

    end

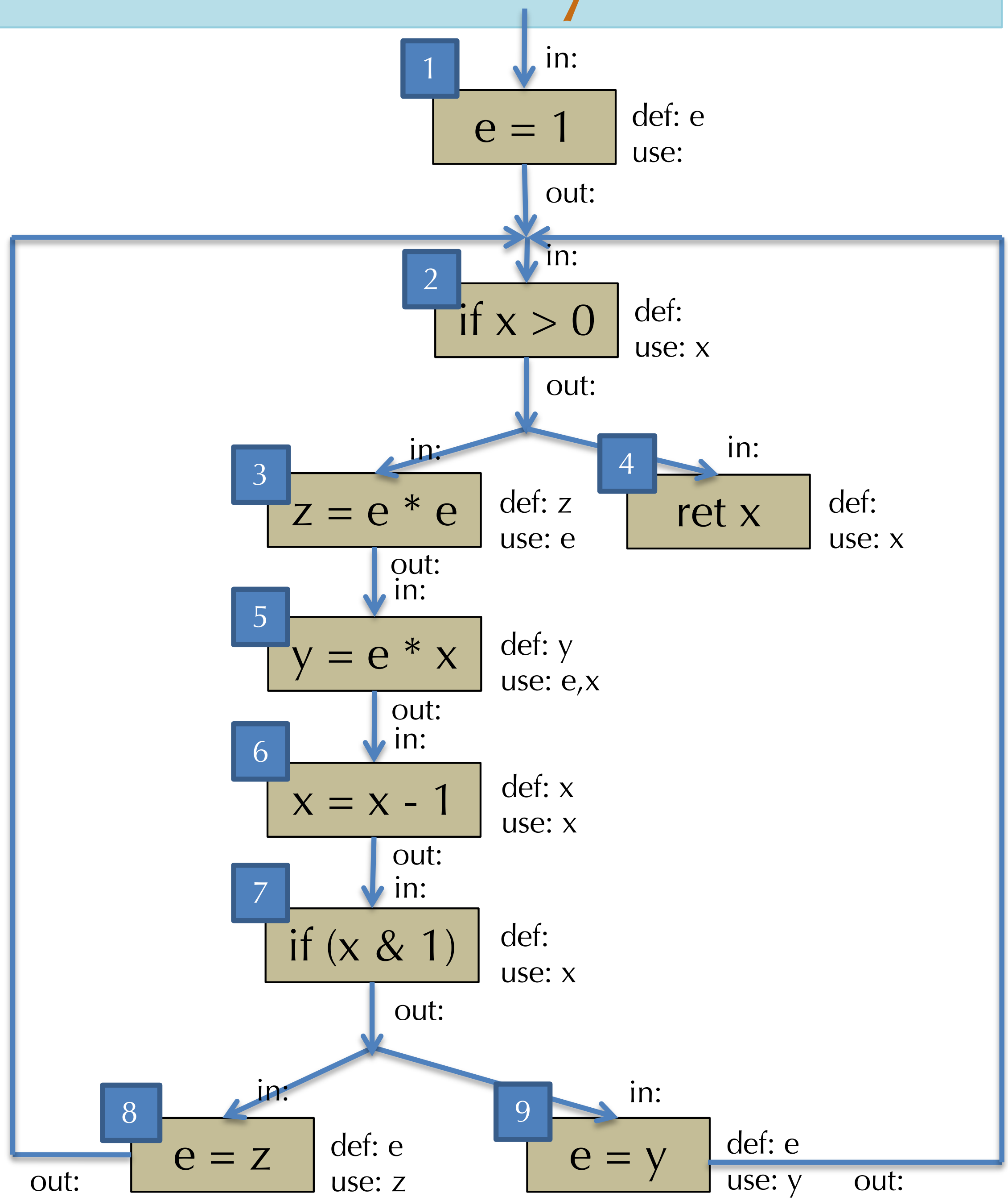
end

- Finds a *fixpoint* of the in and out equations.
  - The algorithm is guaranteed to terminate… Why?
- Why do we start with Ø?

# Example Liveness Analysis

- Example flow graph:

```
e = 1;
while(x>0) {
  z = e * e;
  y = e * x;
  x = x - 1;
  if (x & 1) {
    e = z;
  } else {
    e = y;
  }
}
return x;
```



# Example Liveness Analysis

Each iteration update:

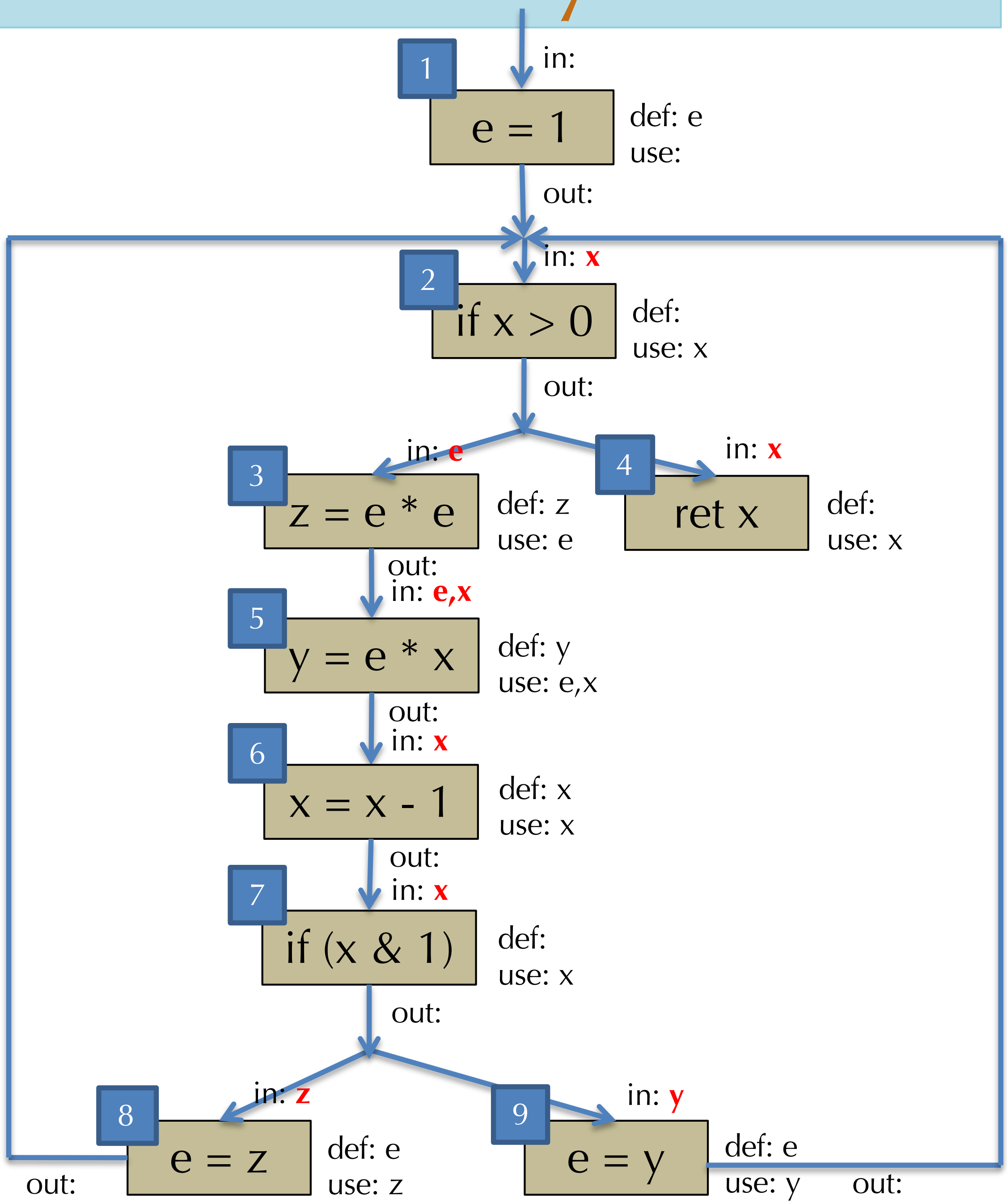
out[n] := $\cup_{n' \in succ[n]} in[n']$

in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 1:

in[2] = x
in[3] = e
in[4] = x
in[5] = e,x
in[6] = x
in[7] = x
in[8] = z
in[9] = y

(showing only updates that make a change)



# Example Liveness Analysis

Each iteration update:

out[n] := $\cup_{n' \in succ[n]}$ in[n']

in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 2:

out[1]= x

in[1] = x

out[2] = e,x

in[2] = e,x

out[3] = e,x

in[3] = e,x

out[5] = x

out[6] = x

out[7] = z,y

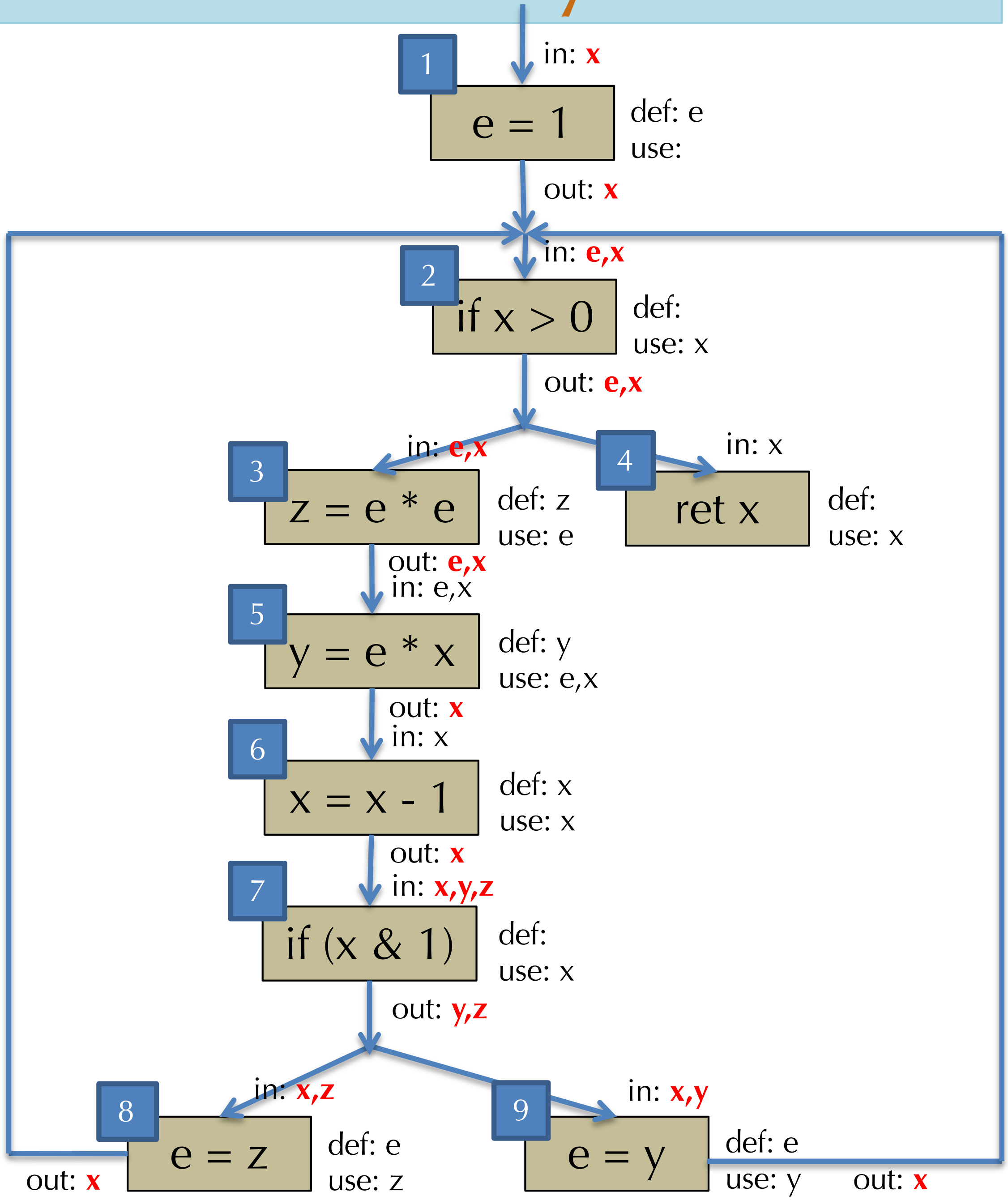
in[7] = x,z,y

out[8] = x

in[8] = x,z

out[9] = x

in[9] = x,y



# Example Liveness Analysis

Each iteration update:

out[n] := $\cup_{n' \in succ[n]}$ in[n']

in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 3:

out[1]= e,x

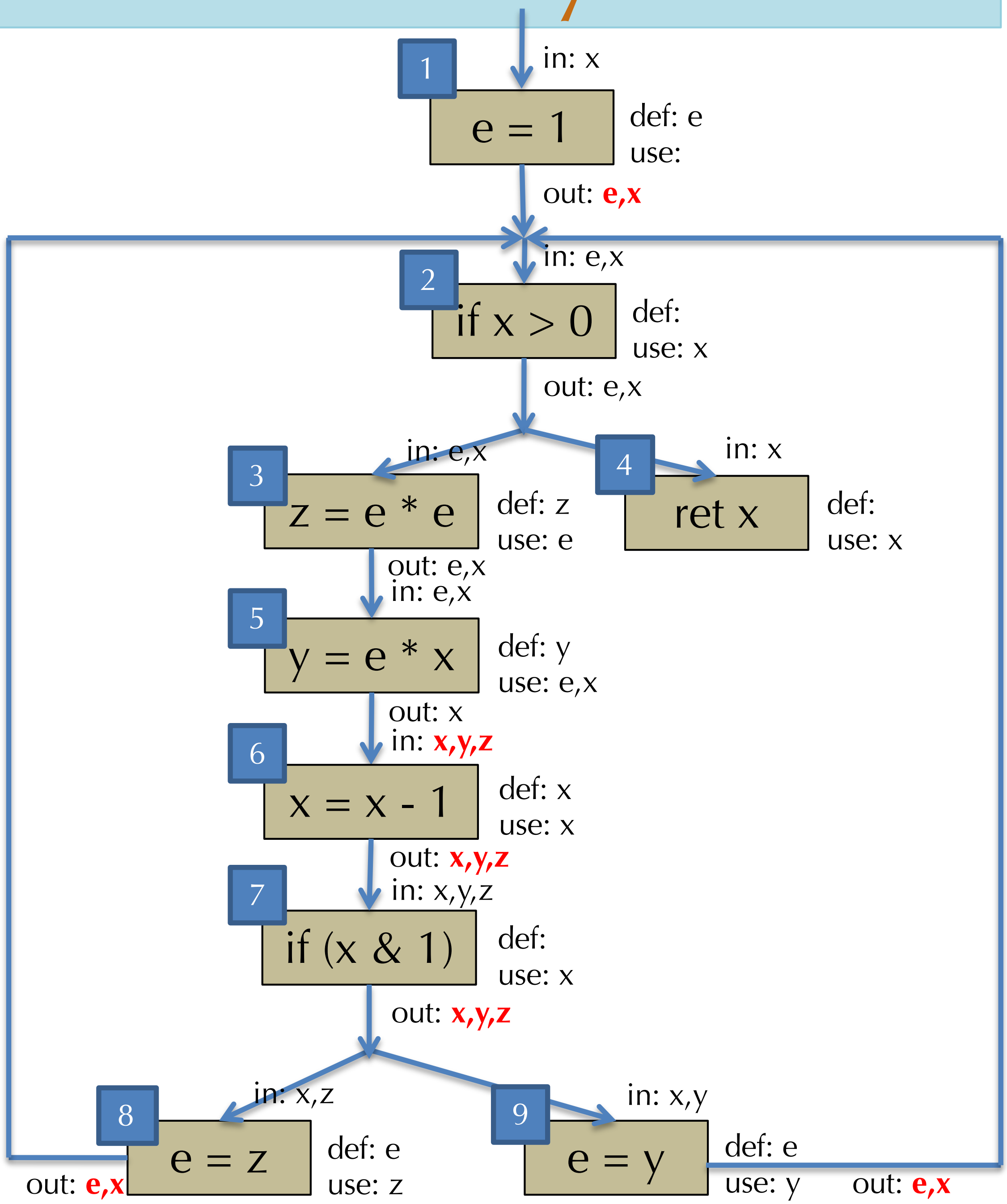
out[6]= x,y,z

in[6]= x,y,z

out[7]= x,y,z

out[8]= e,x

out[9]= e,x



# Example Liveness Analysis

Each iteration update:

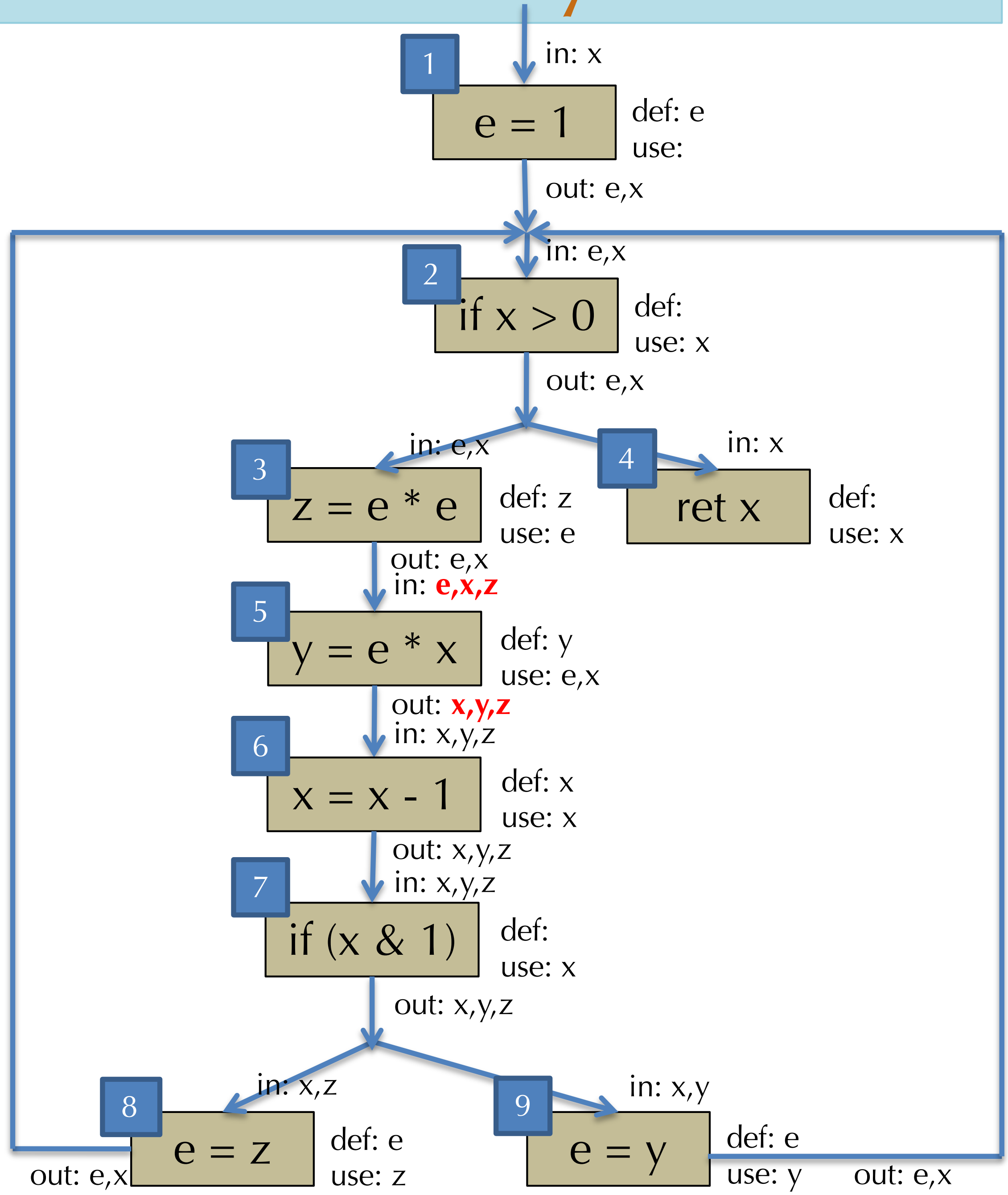
out[n] := $\cup_{n' \in succ[n]}$ in[n']

in[n] := use[n] ∪ (out[n] - def[n])

- Iteration 4:

out[5]= x,y,z

in[5]= e,x,z



# Example Liveness Analysis

Each iteration update:

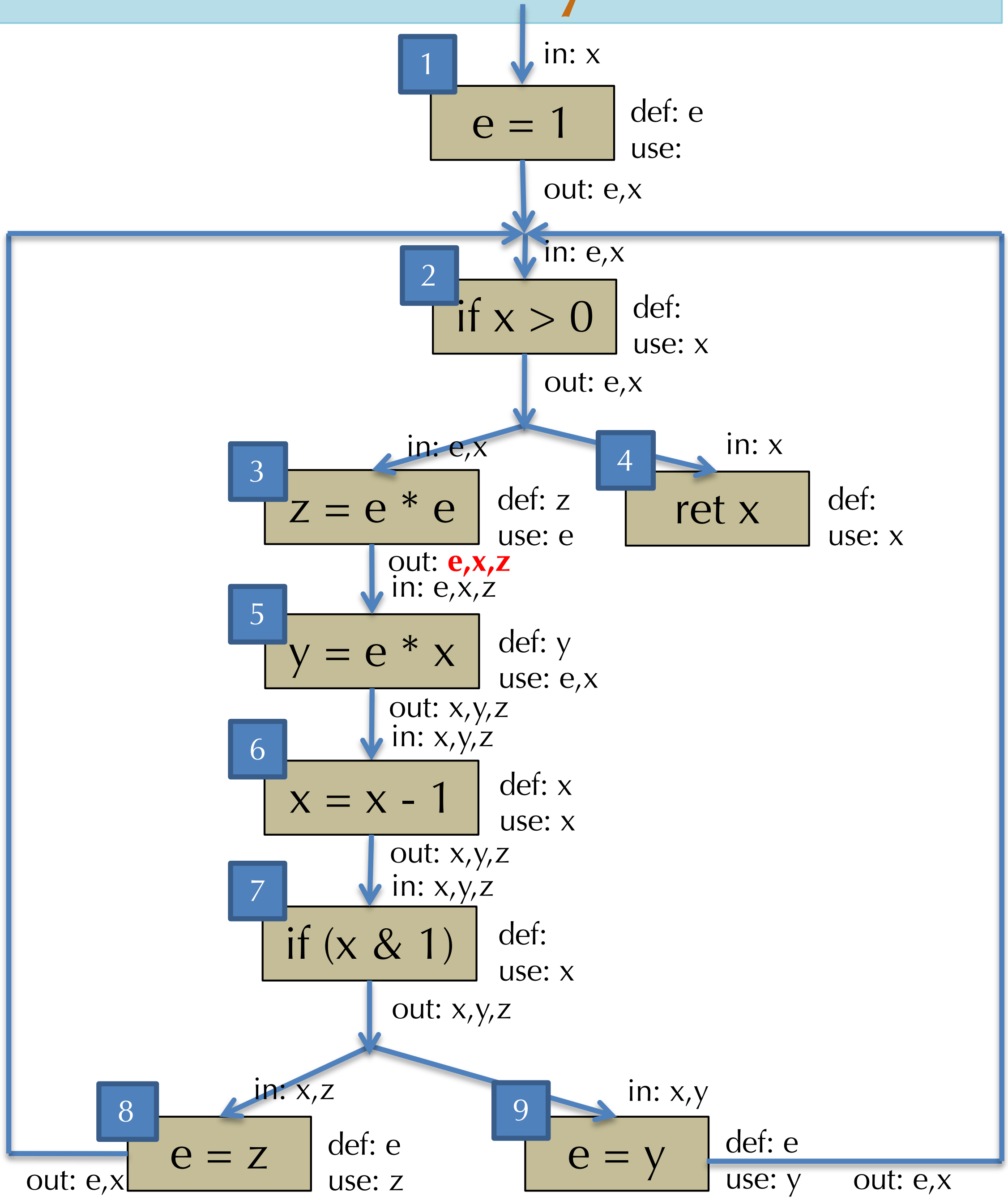
$out[n] := \cup_{n' \in succ[n]} in[n']$

$in[n] := use[n] \cup (out[n] - def[n])$

- Iteration 5:

out[3]= e,x,z

Done!



# Improving the Algorithm

- Can we do better?
- Observe: the only way information propagates from one node to another is using: $out[n] := \bigcup_{n' \in succ[n]} in[n']$
  - This is the only rule that involves more than one node
- If a node's successors haven't changed, then the node itself won't change.
- Idea for an improved version of the algorithm:
  - Keep track of which node's successors have changed

# A Worklist Algorithm

- Use a FIFO queue of nodes that might need to be updated.

```
for all n, in[n] := Ø, out[n] := Ø
w = new queue with all nodes
repeat until w is empty
    let n = w.pop()                       // pull a node off the queue
     old_in = in[n]                       // remember old in[n]
     out[n] := ∪_{n'∈succ[n]} in[n']
     in[n] := use[n] ∪ (out[n] - def[n])
     if (old_in != in[n]),                // if in[n] has changed
        for all m in pred[n], w.push(m)   // add to worklist
end
```

# OTHER DATAFLOW ANALYSES

# Generalizing Dataflow Analyses

- The kind of iterative constraint solving used for liveness analysis applies to other kinds of analyses as well.
  - Reaching definitions analysis
  - Available expressions analysis
  - Alias Analysis
  - Constant Propagation
  - These analyses follow the same 3-step approach as for liveness.

- To see these as an instance of the same kind of algorithm, the next few examples to work over a canonical intermediate instruction representation called *quadruples*
  - Allows easy definition of def[n] and use[n]
  - A slightly "looser" variant of LLVM's IR that doesn't require the "static single assignment" – i.e. it has *mutable* local variables
  - We will use LLVM-IR-like syntax

# Def / Use for SSA

- Instructions n: def[n] use[n] description

| Instructions n: | def[n] | use[n] | description |
|---|---|---|---|
| a = op b c | {a} | {b,c} | arithmetic |
| a = load b | {a} | {b} | load |
| store a, b | Ø | {a,b} | store |
| a = alloca t | {a} | Ø | alloca |
| a = bitcast b to u | {a} | {b} | bitcast |
| a = gep b [c,d, …] | {a} | {b,c,d,…} | getelementptr |
| a = $f(b_1,\ldots,b_n)$ | {a} | $\{b_1,\ldots,b_n\}$ | call w/return |
| $f(b_1,\ldots,b_n)$ | Ø | $\{b_1,\ldots,b_n\}$ | void call (no return) |

- Terminators

| | | | |
|---|---|---|---|
| br L | Ø | Ø | jump |
| br a L1 L2 | Ø | {a} | conditional branch |
| return a | Ø | {a} | return |

# REACHING DEFINITIONS

# Reaching Definition Analysis

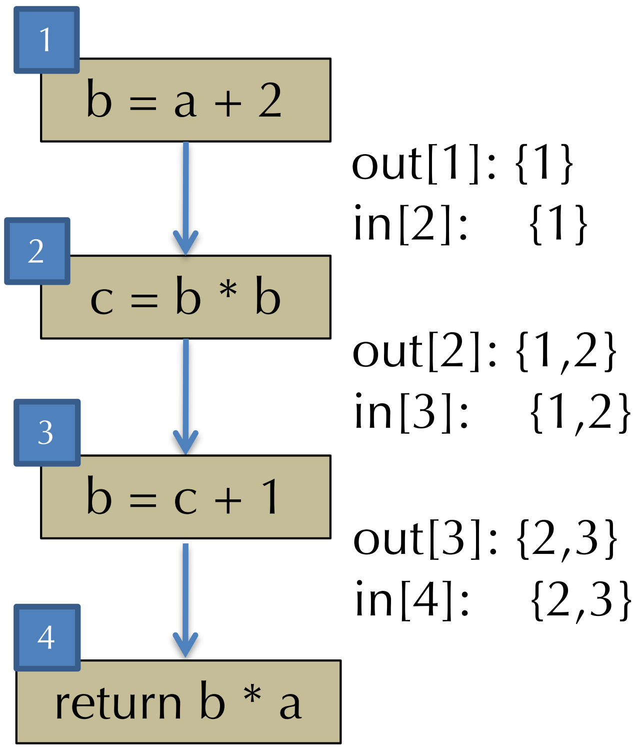
- Question: what uses in a program does a given variable definition reach?

- This analysis is used for constant propagation & copy prop.
  - If only one definition reaches a particular use, can replace use by the definition (for constant propagation).
  - Copy propagation additionally requires that the copied value still has its same value – computed using an *available expressions* analysis (next)

- Input: Quadruple CFG
- Output: in[n] (resp. out[n]) is the set of nodes defining some variable such that the definition may reach the beginning (resp. end) of node n

# Example of Reaching Definitions

- Results of computing reaching definitions on this simple CFG:



# Reaching Definitions Step 1

- Define the sets of interest for the analysis
- Let defs[a] be the set of *nodes* that define the variable a
- Define gen[n] and kill[n] as follows:
- Quadruple forms n:

| Quadruple forms n: | gen[n] | kill[n] |
|---|---|---|
| a = b op c | {n} | defs[a] - {n} |
| a = load b | {n} | defs[a] - {n} |
| store b, a | Ø | Ø |
| a = $f(b_1,\ldots,b_n)$ | {n} | defs[a] - {n} |
| $f(b_1,\ldots,b_n)$ | Ø | Ø |
| br L | Ø | Ø |
| br a L1 L2 | Ø | Ø |
| return a | Ø | Ø |

# Reaching Definitions Step 2

- Define the constraints that a reaching definitions solution must satisfy.
- $out[n] \supseteq gen[n]$
  "The definitions that reach the end of a node at least include the definitions generated by the node"
- $in[n] \supseteq out[n']$ if $n'$ is in $pred[n]$
  "The definitions that reach the beginning of a node include those that reach the exit of *any* predecessor"
- $out[n] \cup kill[n] \supseteq in[n]$
  "The definitions that come in to a node either reach the end of the node or are killed by it."
  - Equivalently: $out[n] \supseteq in[n] - kill[n]$

# Reaching Definitions Step 3

- Convert constraints to iterated update equations:
- $in[n] := \bigcup_{n' \in pred[n]} out[n']$
- $out[n] := gen[n] \cup (in[n] - kill[n])$

- Algorithm: initialize in[n] and out[n] to Ø
  - Iterate the update equations until a fixed point is reached

- The algorithm terminates because in[n] and out[n] increase only *monotonically*
  - At most to a maximum set that includes all variables in the program
- The algorithm is precise because it finds the *smallest* sets that satisfy the constraints.

# AVAILABLE EXPRESSIONS

# Available Expressions

- Idea: want to perform common subexpression elimination:
  - a = x + 1 → a = x + 1
    … → …
    b = x + 1 → b = a
- This transformation is safe if x+1 means computes the same value at both places (i.e. x hasn't been assigned).
  - "x+1" is an *available expression*

- Dataflow values:
  - in[n] = set of nodes whose values are available on entry to n
  - out[n] = set of nodes whose values are available on exit of n

# Available Expressions Step 1

- Define the sets of values
- Define gen[n] and kill[n] as follows:
- Quadruple forms n:

| Quadruple forms n | gen[n] | kill[n] |
| --- | --- | --- |
| a = b op c | {n} - kill[n] | uses[a] |
| a = load b | {n} - kill[n] | uses[a] |
| store b, a | Ø | uses[ [x] ] (for all x that may equal a) |
| br L | Ø | Ø |
| br a L1 L2 | Ø | Ø |
| $a = f(b_1,\ldots,b_n)$ | Ø | uses[a]∪ uses[ [x] ] (for all x) |
| $f(b_1,\ldots,b_n)$ | Ø | uses[ [x] ] (for all x) |
| return a | Ø | Ø |

Note the need for "may alias" information…

Note that functions are assumed to be impure…

# Available Expressions Step 2

- Define the constraints that an available expressions solution must satisfy.
- out[n] $\supseteq$ gen[n]
  "The expressions made available by n that reach the end of the node"
- in[n] $\subseteq$ out[n′]    if n′ is in pred[n]
  "The expressions available at the beginning of a node include those that reach the exit of *every* predecessor"
- out[n] $\cup$ kill[n] $\supseteq$ in[n]
  "The expressions available on entry either reach the end of the node or are killed by it."
  - Equivalently: out[n] $\supseteq$ in[n] - kill[n]

Note similarities and differences with constraints for "reaching definitions".

# Available Expressions Step 3

- Convert constraints to iterated update equations:
- $in[n] := \bigcap_{n' \in pred[n]} out[n']$
- $out[n] := gen[n] \cup (in[n] - kill[n])$

- Algorithm: initialize in[n] and out[n] to {set of all nodes}
  - Iterate the update equations until a fixed point is reached

- The algorithm terminates because in[n] and out[n] *decrease* only *monotonically*
  - At most to a minimum of the empty set
- The algorithm is precise because it finds the *largest* sets that satisfy the constraints.