1. Let $\Sigma = \{a, b\}$. Consider the language $L$ consisting of strings $w$ such that $w$ has at least one occurrence of the substring $aba$.

   (a) Give a DFA that accepts $L$. 4 pts

   (b) Prove that the DFA of part (a) accepts $L$. That is, show that for every string $w$, $\hat{\delta}(q_0, w)$ is an accepting state precisely when $w$ has at least one occurrence of the substring $aba$. 6 pts

2. (Sipser 1.14)

   (a) Show that if $M$ is a DFA recognizing language $B$, swapping the accept and nonaccept states in $M$ yields a new DFA $\bar{M}$ recognizing $\bar{B}$, the complement of $B$. Conclude that the class of regular languages is closed under complement. 5 pts

   (b) Show by example that if $M$ is an NFA recognizing language $B$, $\bar{M}$ does not necessarily recognize $\bar{B}$. 5 pts

3. (Sipser 1.31) For any string $w = w_1 w_2 \cdots w_n$, the reverse of $w$ is $w^R = w_n \cdots w_2 w_1$. For any language $L$, let $L^R = \{w^R \mid w \in L\}$. Show that $L$ is regular $\iff L^R$ is regular. 10 pts

4. (Sipser 1.36) Let $B_n = \{a^k \mid k$ is a multiple of $n\}$. Show that, for each natural number $n \geq 1$, the language $B_n$ is regular. 5 pts

5. (Sipser 1.48) Given alphabet $\Sigma = \{a, b\}$, prove that 5 pts

   $$L = \{w \mid w$ contains an equal number of occurrences of the substrings $ab$ and $ba\}$$

   is regular.

6. (Sipser 1.51) Let $x$ and $y$ be strings in $\Sigma^*$. As discussed in class, we say that $x$ and $y$ are distinguishable by a language $L$ if some string $z \in \Sigma^*$ exists such that exactly one of strings $xz$ and $yz$ is a member of $L$. Otherwise, for every string $z$ we have $xz \in L$ if and only if $yz \in L$ and we say that $x$ and $y$ are indistinguishable by $L$, written $x \equiv_L y$. Prove that $\equiv_L$ is an equivalence relation. 10 pts

7. Since we proved that $\equiv_L$ is an equivalence relation in Question 6, it is natural to consider the equivalence classes induced by $\equiv_L$. Recalling that given language $L$ and string $x$, the equivalence class to which $x$ belongs is the set of strings $\{y \mid x \equiv_L y\}$. $\equiv_L$ therefore partitions $L$ into some number of equivalence classes.

   Prove that if $L$ has an infinite number of equivalence classes under $\equiv_L$ then it cannot be recognized by a DFA. 10 pts
8. Given alphabet $\Sigma = \{a, b\}$, show that any DFA accepting the language

$$L = \{w \in \Sigma^* \mid \text{count}(w, a) \equiv 2 \text{ or } 4 \mod 5\}$$

must have at least two final states.

9. Given alphabet $\Sigma = \{a, b\}$ and language $L = \{w \mid \text{count}(w, a) \geq k\}$ for some natural number $k$, what is the minimum number of states of a DFA $M$ accepting $L$? Prove your answer.

10. Log into Automata Tutor and complete the NFA construction problems.