1. (Sipser 1.20) Given alphabet Σ = \{a, b\}, for each of the following languages provide two strings that are in the language and two that are not. 10 pts
   
   (a) \(a^*b^*\)
   (b) \(a(ba)^*b\)
   (c) \(a^* \cup b^*\)
   (d) \((aaa)^*\)
   (e) \(\Sigma^*a\Sigma^*b\Sigma^*a\Sigma^*\)

2. Given alphabet Σ, a homomorphism is a function \(h: \Sigma \rightarrow \Sigma'^*\) that maps symbols of Σ to strings in another alphabet Σ'. We can extend homomorphisms from symbols to strings by \(h(w_1w_2\cdots w_n) = h(w_1)h(w_2)\cdots h(w_n)\).
   
   Prove that, if \(L\) is a regular language, then \(h(L) = \{h(w) \mid w \in L\}\) is also a regular language (hint: regular expressions).

3. Prove or disprove the following claims regarding languages \(L\) and \(M\) over alphabet Σ = \{0, 1\}. 12 pts
   
   (a) If \(L\) is regular then \(L \cup M\) is also regular.
   (b) If \(L\) is regular then \(L \cap M\) is also regular.
   (c) If neither \(L\) nor \(M\) is regular then \(L \cup M\) is not regular.
   (d) If neither \(L\) nor \(M\) is regular then \(L \cap M\) is not regular.

4. (Sipser 1.28) Convert the following regular expressions into equivalent NFAs. You don’t need to prove the correctness of the NFAs. 10 pts
   
   (a) \(a(ab)^* \cup \emptyset\)
   (b) \(aa^* \cup ab(ab)^*\)

5. Given alphabet Σ = \{a, b, c\}, is language \(L = \{w = a^kb^mc^n \mid k + m + n\text{ is even}\}\) regular? Either way, prove it. 10 pts

6. Given alphabet Σ = \{1\}, prove that the language
   
   \(L = \{1^p \mid p \text{ is prime}\}\)
   
   is not regular.
7. Given alphabet $\Sigma = \{1\}$, prove that the language

$$L = \{1^n \mid n \text{ is not a perfect square}\}$$

is not regular.

8. (Sipser 1.46) Given alphabet $\Sigma = \{0, 1\}$, prove that the following languages are not regular.

(a) $L = \{0^n1^m0^n \mid m, n \geq 0\}$

(b) $L = \{w \mid w \text{ is not a palindrome}\}$ where a palindrome is any string $w$ that is the same backwards and forwards, e.g. $0100010$.

9. Log into Automata Tutor and complete the regular expression construction problems.