Big Idea (Week 3): Nyquist Sampling Rate

The *Nyquist-Shannon Sampling Theorem* tells us that band-limited signals (those with no frequency harmonics above a particular rate) can be **exactly** reconstructed from sufficiently frequently collected samples (*i.e.*, collected at twice the highest frequency harmonic present). Given that human audition is insensitive to high frequency acoustic phenomena (roughly above 22KHz as you will empircally verify in lab), this raises the hope that we might be able to capture and represent audio data in such a manner that its reconstruction is indistinguishable from the original by any human listener. This explains why compact discs use a (roughly) 44KHz sample rate.

However, samples collected insufficiently frequently will be contaminated by spurious "spill-over" information from the more rapidly varying components of a signal. That is, if we sample below the maximum frequency in a source, the contribution from the higher frequency components cannot be distinguished from lower frequency signals. This phenomena is known as *aliasing* since the higher frequency components map to lower frequency noise. These noise artifacts are unavoidable in any attempted reconstruction. Even though humans may not be able to hear frequencies above 22KHz, if we simply sampled pressure waves at 44KHz and reconstructed them assuming there were no higher harmonics, it would be possible to introduce aliasing artifacts that we can hear.

If we can remove enough high frequency components by appropriate "low pass" filtering of the received signal (*i.e.*, *anti-aliasing*), then reconstructing the sound from the remaining frequency domain representation is governed by the Nyquist-Shannon Sampling Theorem and yields a signal that sounds like the original.

This phenomena is not limited to sound and can impact any discretely sampled signal, including spatially-sampled images (digital photos) and time-sampled images (video).