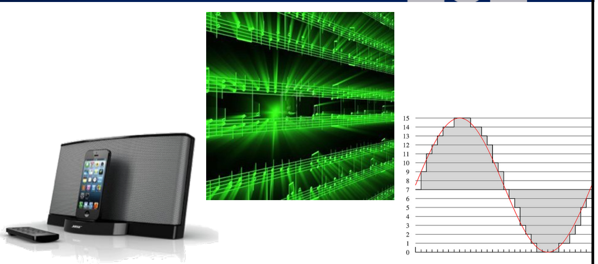

Lecture #2 – Nyquist-Shannon Sampling Theorem

ESE 150 – Digital Audio Basics

Lecture Topics

- Where are we on course map?
- What we did in lab last week
  - How it relates to this week
- Sampling/Quantization Review
- **Nyquist Shannon Sampling Rate**
- Next Lab
- References

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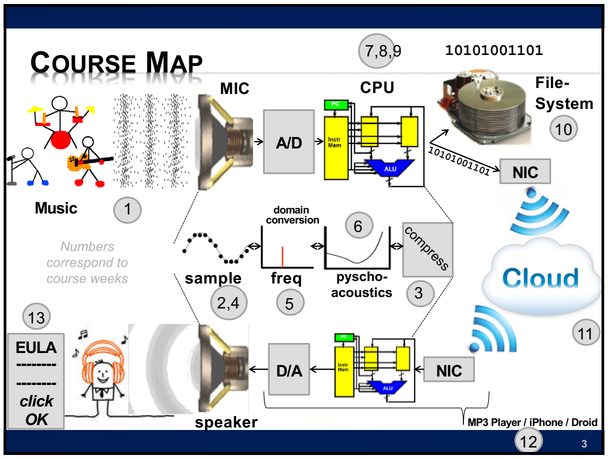
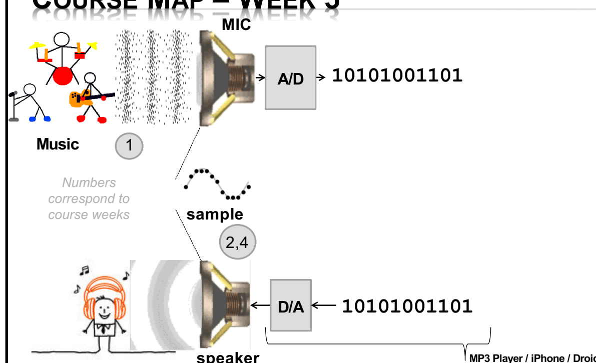
Course Map



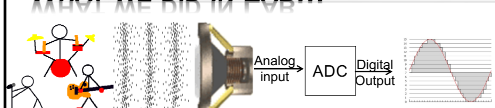
3

Course Map – Week 3



4

What we did in Lab…



- **Week 1: Converted Sound to analog voltage signal**
  - a "pressure wave" that changes air molecules w/ respect to time
  - a "voltage wave" that changes amplitude w/ respect to time
    - Sample: Break up independent variable, take discrete 'samples'
    - Quantize: Break up dependent variable into n-levels (need 2^n bits to digitize)

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Sampling vs Quantization Review

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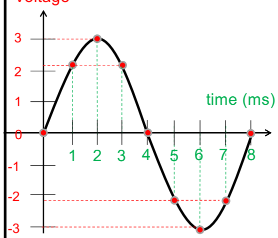
ADC – Sampling & Quantization

- **Analog-to-Digital (ADC) Conversion**
  - Converting analog (continuous) signal to digital signal
  - Digitization process has two important aspects:
    - *1) Sampling*
      - Converting ***independent*** variable of signal from continuous to discrete
      - e.g.: breaking continuous *time* down into intervals
    - *2) Quantization*
      - Converting ***dependent*** variable of signal from continuous to discrete
      - e.g.: breaking continuous *voltage* down into levels

ADC – Sampling

- **<u>Sampling</u>: breaking independent variable (time) into intervals**
- **<u>Quantization</u>: breaking dependent variable (voltage) into levels**



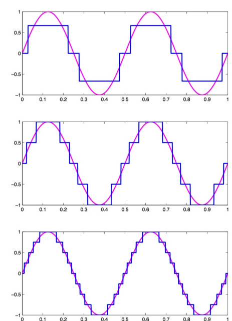
Samples @ 1ms intervals:	Quantized into 7 levels	Levels digitized into 3-bits
{ 0 ms, 0 Volts }	{ 0 ms, 0 Volts }	011
{ 1 ms, 2.2 Volts }	{ 1 ms, 2 Volts }	101
{ 2 ms, 3 Volts }	{ 2 ms, 3 Volts }	110
{ 3 ms, 2.2 Volts }	{ 3 ms, 2 Volts }	101
{ 4 ms, 0 Volts }	{ 4 ms, 0 Volts }	011
{ 5 ms, -2.2 Volts }	{ 5 ms, -2 Volts }	001
{ 6 ms, -3 Volts }	{ 6 ms, -3 Volts }	000
{ 7 ms, -2.2 Volts }	{ 7 ms, -2 Volts }	001
{ 8 ms, 0 Volts }	{ 8 ms, 0 Volts }	011

Two Knobs

1. **Quantization level (bits/sample)**
2. **Sampling rate (samples/second)**

- **Impact Quality of sound**
- **Impact costs (resources -- #bits needs to store)**

Effect of Increasing Quantization



- **Dividing dependent variable up into more levels**
  - Increasing resolution at each sample
  - Doesn't change the # of samples itself!

Effect of Increasing Sampling Rate

- **Increasing how often we take samples also helps**
  - Much like quantization…
    - 1 bit was too few, 16 bits was more than enough
    - Is there a sweet spot for the sampling rate?

Both (Quantization, Sampling) impact storage

- **How many bytes for 3 minute song sampled at 8b precision and 1000 samples/s?**
- **at 2000 samples/s?**
- **16b precision at 2000 samples/s?**

Key Question

- **What sampling rate should we use?**

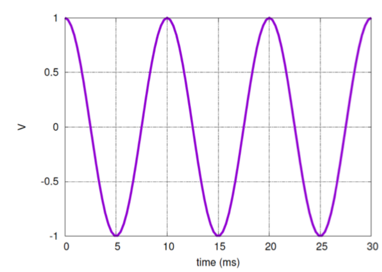
Definition of Good Sampling

- **Definition of proper sampling:**
  - Let's say you've sampled an analog signal…
  - If you can exactly reconstruct the analog signal from the samples
    - You have done the sampling properly!
  - Essentially: if you can reverse the process…
    - You've capture enough information about the signal
- **Can we formalize this a bit more?**
  - Yes, next few slides will try….

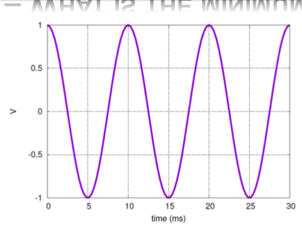
Preclass

- **Identify frequencies**
- **Samples**
- **What's indistinguishable at various sample rates?**

Sampling – What is the minimum?

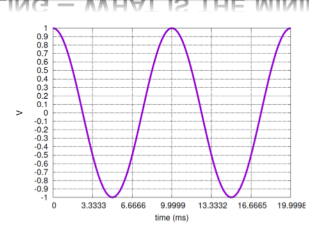


Sampling – What is the minimum?



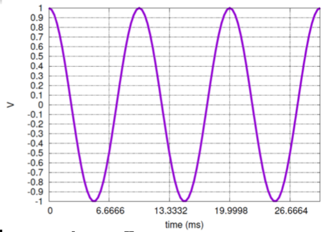
- **How much do we need to capture to reconstruct it?**
  - If we sample at 200 Hz, capture peaks & troughs of signal
  - Sample rate: 2 x frequency = 200 Hz

Sampling – What is the minimum?



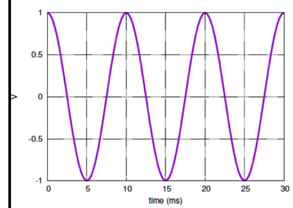
- **How much do we need to capture to reconstruct it?**
  - If we sample at 3 x frequency or 300 Hz
    - more than enough samples to capture it!
  - We are actually wasting space!
  - …more samples…more bits per sample…more storage required

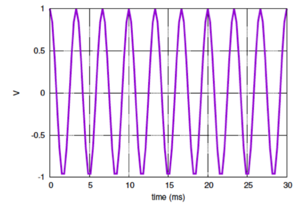
Sampling – What is the minimum?



- Could we go lower?
  - If we sample at rate 1.5 x frequency or 150 Hz
    - We aren't capturing all peaks/troughs of signal
  - Yes, we lose information
    - **but it gets worse!**

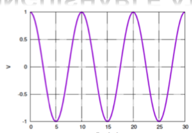
200 Hz Sample

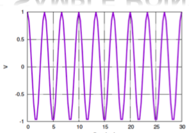


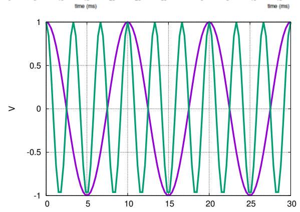


- **What happened here?**

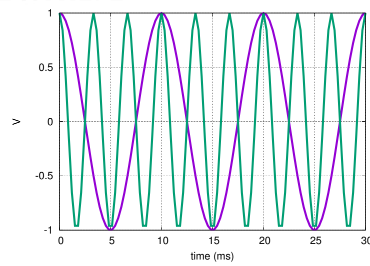
Indistinguishable at Sample Points





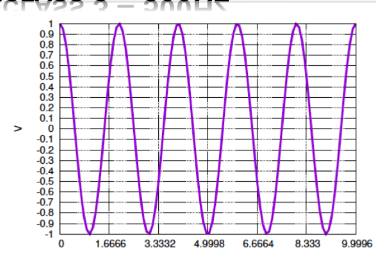


200 Hz Sample



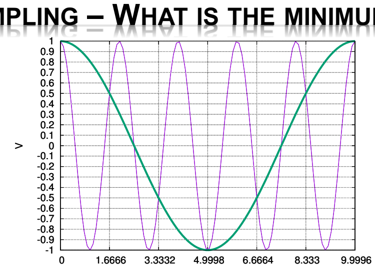
- **Cannot let signal "wiggle" around between samples**
- **Sample too infrequently, can miss signal behavior**

Preclass 3 – 500Hz



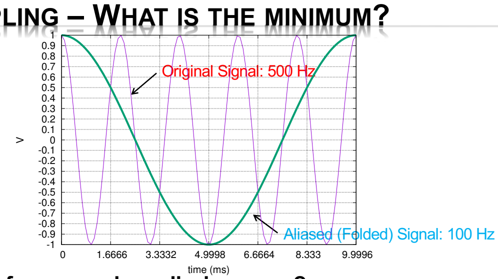
- **Is this properly sampled?**
- **What did we get?**

Sampling – What is the minimum?



- **Cannot sample lower without reconstruction error**
  - We not only lose information…
    - …but when we 'reconstruct' the signal from the samples alone…
    - We will reconstruct at a lower frequency!
    - This phenomenon is called: aliasing

Sampling – What is the minimum?



- What frequency does aliasing occur?
  - Original Signal's Frequency: 500 Hz
    - Sampling Rate: **600 Hz**
  - Aliasing occurs at: **600 Hz** – 500 Hz = 100 Hz
    - Also referred to as "Folding" – signal has "folds over" as if it were lower frequency

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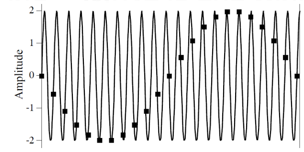
Sampling – What is the minimum?



- **Another example/effect of aliasing**
  - The dots represent the samples, we can see an inverse sine-wave
  - Not only has the frequency of the original signal changed…
    - But phase of the signal has changed too!
    - Original signal: sine wave + 0° phase
    - Aliased signal: sine wave w/different frequency + 180° phase shift!

Figures from reading: *The Scientist and Engineer's Guide to Digital Signal Processing*, By Steven W. Smith

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Sampling Rate

- **Established (by counterexamples) that we can sample too infrequently**
  - **Necessary** to sample at 2x highest frequency present
- **Haven't shown clearly that 2x is sufficient**
  - (won't in this class)
  - Just giving you intution

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Sampling – What is the minimum?

- **Harry Nyquist**
  - Electronic Engineer for AT&T from 1917 to 1954
  - Published paper in 1928 defining the: Sampling Theorem
    - **Nyquist Sampling Rate** = 2 x frequency of signal
      - Anything less: *under-sampling* – leads to aliasing
      - Anything more: *over-sampling* – waste of space?

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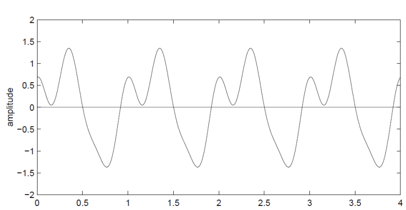
What about Multirate Signals?



- **Fourier's Theorem (week 4 preview!):**
  - We can decompose continuous signal in terms of a sum of sines and cosines at different frequencies
  - This waveform: sum of sine waves at 1 Hz, 2 Hz, 3 Hz
    - What's the Nyquist Sampling Rate then?

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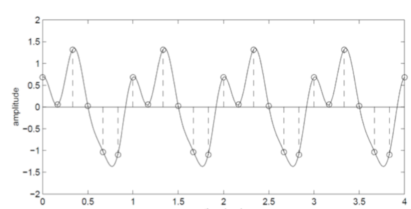
What about Multirate Signals?



- **Fourier's Theorem & Nyquist Rate:**
  - Highest component's frequency: 3 Hz
  - What is Nyquist Sampling Rate?
    - 2 x highest frequency contained in the signal = 6 Hz
    - Sampling at this rate: avoids aliasing problem

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Nyquist Rate vs Frequency

- **Nyquist Sampling Rate:**
  - f_s = 2 x highest frequency component of signal
    - Minimum sampling rate that satisfies: Nyquist Sampling Criterion for a given signal or family of signals
    - Minimum sampling rate that avoids aliasing
    - Property of a continuous-time signal
- **Nyquist Frequency:**
  - ½ f_s = ½ sampling rate
    - Highest frequency that can be recovered from samples
    - Property of a discrete-time signal

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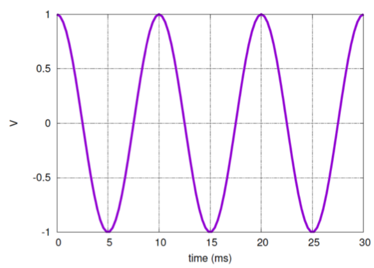
Sample at Frequency

- **What happens if we sample 100Hz signal at 100Hz?**



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Interlude

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Video

- **How many frames/second for video (TV, Film?)**
- http://www.youtube.com/watch?v=jHS9JGkEOmA

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Aliasing in Movies

- **Called visual aliasing**
  - See it all the time on TV/Film
    - Wheels tend to move backwards on moving cars…why?
  - What is it?
    - Primer: Movies are just pictures (frames) flying by quickly
    - Movies "sample" real life at roughly 24 frames per second
  - What do we know from Nyquist Sampling Theorem?
    - Aliasing will occur if changes occur faster than ½ f_s
    - Film Example:
      - If **light to dark transitions** occur faster than ½ f_s aka: 12 frame/sec
      - Aliasing will occur…

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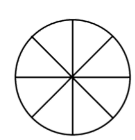
The "Wagon Wheel" Effect

- **Consider a wagon with 8 spokes:**



  - Let's say it turns at a rate of 3 revolutions per second clockwise
    - That's 180 rpm
  - On film this wheel will appear to stand still! Why?

$$\frac{\left(3\ \frac{revolutions}{sec}\right)\times\left(8\ \frac{spokes}{revolution}\right)}{\left(24\ \frac{frames}{sec}\right)} = 1\ \frac{spoke}{frame}$$

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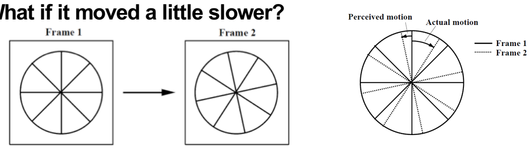
THE "WAGON WHEEL" EFFECT

- **What if it moved a little slower?**



  - Let's say it turns at a rate of 2.5 revolutions per second clockwise

$$\frac{\left(2.5\,\frac{revolutions}{sec}\right)\times\left(8\,\frac{spokes}{revolution}\right)}{\left(24\,\frac{frames}{sec}\right)} = .83\,\frac{spoke}{frame}$$

  - Our brain could interpret this in two possible ways:
    - Wheel has moved clockwise by 83% of spoke interval in clockwise direction
    - OR: wheel has moved counter-clockwise by 17%
      - Our brains prefer this view! So we see the wheel moving backwards! *(thanks aliasing!)*

Fool your brain: http://www.youtube.com/watch?v=jHS9JGkEOmA

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EFFECTS OF ALIASING

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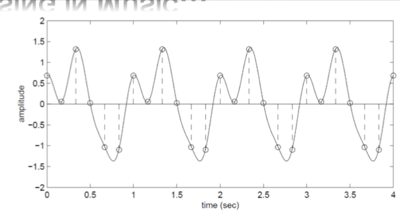
ALIASING IN MUSIC…



- **Multirate Signals and Aliasing**
  - Imagine the above is a music signal (1 Hz, 2 Hz and 3 Hz chord)
    - What happens if we undersample? Should sample at 6 Hz, but instead 4 Hz
    - The 1Hz & 2Hz signals will be sampled just fine (as 4 Hz is *2 x* 2Hz)
    - But what happens to 3 Hz Signal?
      - Fold baby fold!

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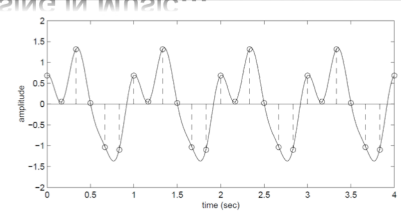
ALIASING IN MUSIC…



- **Multirate Signals and Aliasing**
  - Imagine the above is a music signal (1 Hz, 2 Hz and 3 Hz chord)
    - Where will folding occur?
      - folding occurs at: **4 Hz** – **3 Hz** = 1 Hz
      - Sample rate – frequency = aliasing/folding frequency
    - But what happens to 3 Hz signal? It will "fold over" and sound like a 1 Hz signal!
    - To us that can sound like a low frequency noise in our music

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ANTI-ALIASING

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HOW DO WE FIX THIS?

- **It's simple…sample at the Nyquist Rate**
  - But…what if your rate is fixed? Like 24 frames/sec?
  - Or our eye's sampling rate: 60 cycles/degree
    - Spatial variations finer than this are undetectable!

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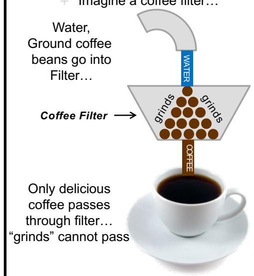
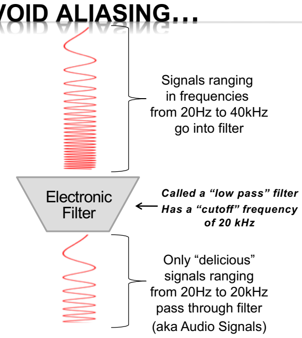
How to Avoid Aliasing with Digital Music?

- **If we simply sample at *2 x highest frequency of signal*…**
  - (AKA: Nyquist Rate)
  - …we won't encounter aliasing!
- **But how do we guarantee highest frequency of our signal?**
  - Audio: this is easy!
    - We know the range of human ear: 20 Hz to 20 kHz…
    - The highest frequency component in music is then: 20 kHz
    - …so, before sound goes into ADC, we apply a filter!
      - Blocks any frequency above 20 kHz from going into ADC
    - Essentially, we are fixing our sampling rate & 'bluring' or filtering our incoming signal

We know how to avoid aliasing…

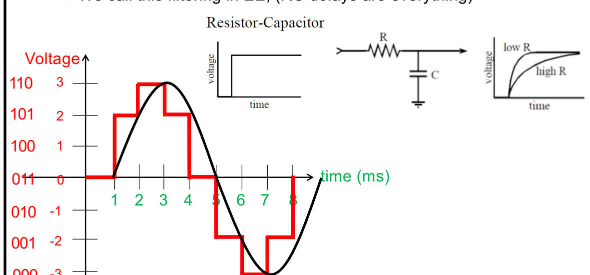
- **What is a filter you ask?**
  - Imagine a coffee filter…





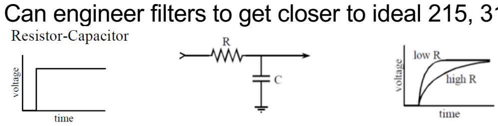
DAC – Filtering (from last lecture)

- **Digital-to-Analog (DAC) Conversion**
  - We call this filtering in EE, (RC delays are everything)

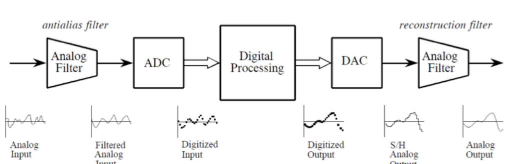


Low Pass Analog Filtering

- **Can limit rate of change**
  - Set a minimum time period for a value to charge output
- **If signal tries to move too fast (high frequency)**
  - The input change won't be reflected in the output
- **Ideal:** fast inputs "erased" from output
- **In practice:** magnitude reduced
- Can engineer filters to get closer to ideal 215, 319



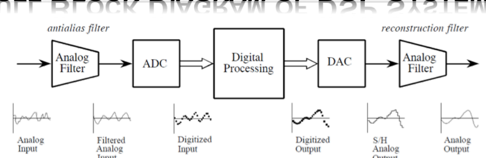
Full Block Diagram of DSP System



- **Before ADC, we put music signal through *antialias* filter**
  - Filter blocks any signals higher than 20 kHz (prevents aliasing!)

Figures from reading: *The Scientist and Engineer's Guide to Digital Signal Processing*, By Steven W. Smith

Full Block Diagram of DSP System



- **Before ADC, we put music signal through *antialias* filter**
  - Filter blocks any signals higher than 20 kHz (prevents aliasing!)
  - Then our ADC can safely sample at 2 x 20 kHz without aliasing
    - What is our Nyquist Rate?
      - $f_s = 2 \times 20$ kHz = 40 kHz, or 40 thousand samples per second!
    - What is our Nyquist Frequency?
      - $\frac{1}{2} f_s$ = 20 kHz
  - Cutoff frequency of our filter? Has to be the Nyquist Frequency

Why do we need the antialias filter?

- **If we can't hear anything above 20kHz…**
  - Why do we need to filter it out?
    - Dog's can hear from 40 Hz to 60 kHz
      - so clearly there are sounds above 20 kHz
  - Let's imagine a high frequency noise in music studio
    - Let's say it's a vibration occurring at 25 kHz
      - No human can hear it, why filter it out?
    - Because of aliasing:
      - Frequency aliasing/folding will occur:
      - Sample rate – frequency = aliasing/folding frequency
      - 40 kHz - 25 kHz = 15 kHz
      - The 25 kHz vibration will fold-over to a 15 kHz "hum" or audible noise
        - It will ruin our recording and source of noise wouldn't be obvious!

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Big Ideas

- **Sample at twice the maximum frequency**
  - Can reconstruct perfectly
- **If have frequencies > sample_freq/2**
  - Will get aliasing … as high frequencies fold
- **Avoid aliasing with analog Anti-Alias prefilter before sampling**

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This Week In Lab

- **Lab 2: D2A** – play back the samples you recorded last week
- Reminder: lab writeups due on Friday.

c

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Office Hour Times Poll

How many can make each?

TA	Office Hours	Where?
Aditya Hota	T 2:30-3:30pm	Detkin (South Side)
Lakshay Sharma	T 3:30-4:30pm	Detkin (South Side)
Han Yan	W 12-1pm	Detkin (South Side)
Vipula Sateesh	W 6-7pm	Ketterer
Andrew Merczynski-Hait	W 7-8pm	Detkin (South Side)
Andrew Butt	R 2:30-3:30pm	Detkin (South Side)
Caroline Leng	R 8-9pm	Detkin (South Side)

Note: Start this week.

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Piazza

- **Signup piazza**
  - Reminders and administrivia
  - Answer questions from lecture

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Learn More

- **ESE215 – include analog filtering**
- **ESE319 – active analog filtering**
- **ESE224 – Signal Processing**

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References

- S. Smith, "The Scientists and Engineer's Guide to Digital Signal Processing," 1997.
- http://en.wikipedia.org/wiki/Nyquist_frequency
- http://en.wikipedia.org/wiki/Nyquist_rate
- http://en.wikipedia.org/wiki/Oversampling
- http://en.wikipedia.org/wiki/Sampling_rate
- http://en.wikipedia.org/wiki/Hearing_range
- http://electronics.howstuffworks.com/telephone6.htm
- B. Olshausen, "Aliasing", PSC 129 – Sensory Processes Course Notes, UC Davis

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