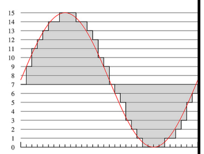
Penn Engineering

ESE



Lecture #3 – Compression

ESE 150 – DIGITAL AUDIO BASICS

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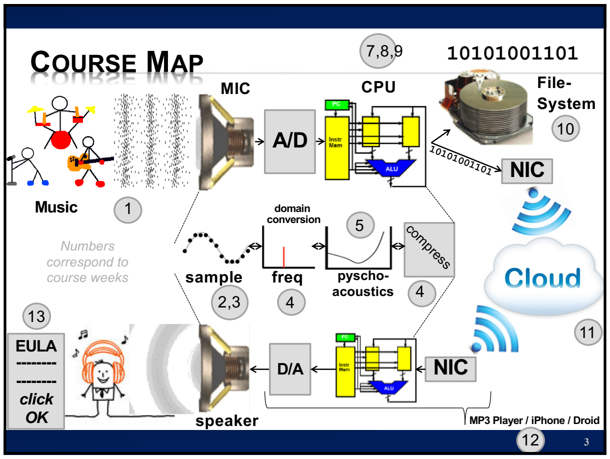
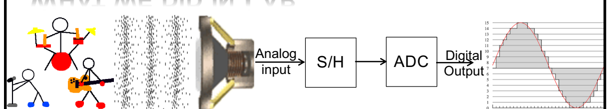
LECTURE TOPICS

- Where are we on course map?
- What we did in lab last week
  - How it relates to this week
- **Lossless Compression**
  - What is it, examples, classifications
  - Probability-based lossless compression
    - Huffman Encoding
  - Entropy
    - Shannon Limits
- Next Lab
- References

COURSE MAP

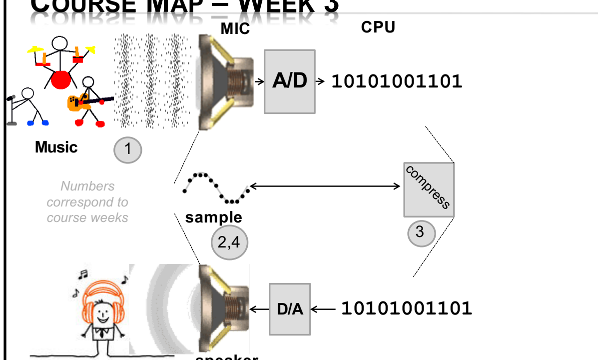


WHAT WE DID IN LAB…



- **Week 1: Converted Sound to analog voltage signal**
  - a "pressure wave" that changes air molecules w/ respect to time
  - a "voltage wave" that changes amplitude w/ respect to time
  - Sample: Break up independent variable, take discrete 'samples'
  - Quantize: Break up dependent variable into n-levels (need 2^n bits to digitize)
- **Week 2: Reconstructed analog signal from digital**

COURSE MAP – WEEK 3



PRECLASS

PRECLASS

- Tell me and I forget, teach me and I may remember, involve me and I learn
  - -- Benjamin Franklin
- **73 symbols** (fancy, more general term for letters)
- **19 unique (ignoring case)**
  - (A, B, C, D, E, F, G, H, I, L, M, N, O, R, T, V, Y, space, comma)
  - How many bits to represent each symbol?
- **How many bits to encode quote?**

PRECLASS

- **Tell me and I forget, teach me and I may remember, involve me and I learn**
  - -- Benjamin Franklin
- **73 symbols**
- **19 unique (ignoring case)**
- **If symbols occurrence equally likely, how many occurrences of each symbol should we expect in quote?**
- **How many e's are there in the quote?**

PRECLASS

- **Tell me and I forget, teach me and I may remember, involve me and I learn**
  - -- Benjamin Franklin
- **73 symbols**
- **19 unique (ignoring case)**
- **Conclude**
  - Symbols do not occur equally
  - Symbol occurrence is not uniformly random

PRECLASS

- **Tell me and I forget, teach me and I may remember, involve me and I learn**
  - -- Benjamin Franklin
- **Using fixed encoding (question 1)**
- **How many bits to encode first 10 symbols?**
- **How many bits using encoding given?**

PRECLASS

- **Tell me and I forget, teach me and I may remember, involve me and I learn**
  - -- Benjamin Franklin
- **Using fixed encoding (question 1)**
- **How many bits to encode first 24 symbols?**
- **How many bits using encoding given?**

PRECLASS

- **Tell me and I forget, teach me and I may remember, involve me and I learn**
  - -- Benjamin Franklin
- **Using fixed encoding (question 1)**
- **How many bits to encode al 73 symbols?**
- **How many bits using encoding given?**

Conclude

- **Can encode with (on average) fewer bits than $\log_2$(unique-symbols)**

Intro to Compression

Data Compression

- **What is compression?**
  - Encoding information using fewer bits than the original representation
- **Why do we need compression?**
  - Most digital data is not sampled/quantized/represented in the most compact form
    - It takes up more space on a hard drive/memory
    - It takes longer to transmit over a network
  - Why? Because data is represented in so that it is easiest to use
- **Two broad categories of compression algorithms:**
  - Lossless – when data is un-compressed, data is its original form
    - No data is lost or distorted
  - Lossy – when data is un-compressed, data is in **approximate** form
    - Some of the original data is lost

Representation of Data

Letter	Numeric Encoding	Letter	Numeric Encoding
A	0	N	13
B	1	O	14
C	2	P	15
D	3	Q	16
E	4	R	17
F	5	S	18
G	6	T	19
H	7	U	20
I	8	V	21
J	9	W	22
K	10	X	23
L	11	Y	24
M	12	Z	25

How to encode alphabet?

Easy to map/encode:
A→0 and Z→25

How many Bits to represent all letters?

Letter	Binary Encoding	Letter	Binary Encoding
A	00000	N	01101
B	00001	O	01110
C	00010	P	01111
D	00011	Q	10000
E	00100	R	10001
F	00101	S	10010
G	00110	T	10011
H	00111	U	10100
I	01000	V	10101
J	01001	W	10110
K	01010	X	10111
L	01011	Y	11000
M	01100	Z	11001

Including upper and lower case?
…and numbers, how many bits?

ASCII Encoding (7-bit encoding)

Letter	ASCII Code	Binary	Letter	ASCII Code	Binary
a	097	01100001	A	065	01000001
b	098	01100010	B	066	01000010
c	099	01100011	C	067	01000011
d	100	01100100	D	068	01000100
e	101	01100101	E	069	01000101
f	102	01100110	F	070	01000110
g	103	01100111	G	071	01000111
h	104	01101000	H	072	01001000
i	105	01101001	I	073	01001001
j	106	01101010	J	074	01001010
k	107	01101011	K	075	01001011
l	108	01101100	L	076	01001100
m	109	01101101	M	077	01001101
n	110	01101110	N	078	01001110
o	111	01101111	O	079	01001111
p	112	01110000	P	080	01010000
q	113	01110001	Q	081	01010001
r	114	01110010	R	082	01010010
s	115	01110011	S	083	01010011
t	116	01110100	T	084	01010100
u	117	01110101	U	085	01010101
v	118	01110110	V	086	01010110
w	119	01110111	W	087	01010111
x	120	01111000	X	088	01011000
y	121	01111001	Y	089	01011001
z	122	01111010	Z	090	01011010

ASCII:
American Standard Code for Information Interchange
2^7=128 combinations

Standard encoding, developed in the 1960's

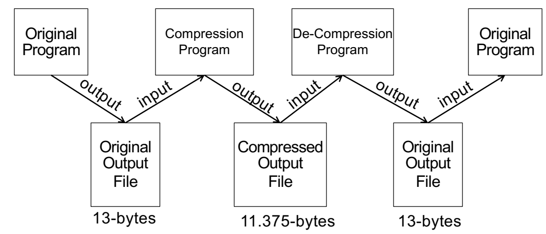
Didn't take into account international standards!

UNICODE
8-bit encoding
2^8=256 possibilities!

Example of *Lossless* Compression

- A simple form of compression would be the following:
  - ORIGINAL TEXT (13-characters): `I Love ESE150`
  - ASCII Encoding (13-bytes = 104 bits):
  - `01001001 00100000 01001100 01101111 01110110 01100101 00100000 01000101 01010011 01000101 00110010 00110101 00110000`
  - Convenient to write programs that read/write files 1-byte at a time
  - But, since ASCII only needs 7-bits (not 8):
    - *We could write a compression program that strips the leading 0*
  - Output of Compression Program (91 bits ~ 11.375 bytes):
  - `1001001 0100000 1001100 1101111 1110110 1100101 0100000 1000101 1010011 1000101 0110010 0110101 0110000`
  - Compression ratio: 104 bits in / 91 bits out = **1.14 :1**
  - ***Lossless*** because we can easily restore exact original

Compression Process

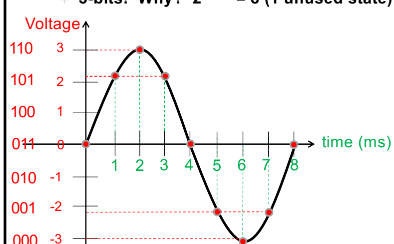


•Why not compress all the time?

- Inconvenient; expensive in terms of microprocessor cycles
- Leads to tradeoff: compute vs. storage

Recall ADC Process?

- **Analog-to-Digital (ADC) Conversion**
  - **We have 7 discrete voltages, # of bits to represent 7 things?**
  - **3-bits! Why? $2^{3\text{-bits}}$ = 8 (1 unused state)**



Samples: / Binary Encoding:

{ 0 ms, 0 Volts } → 011
{ 1 ms, 2 Volts } → 101
{ 2 ms, 3 Volts } → 110
{ 3 ms, 2 Volts } → 101
{ 4 ms, 0 Volts } → 011
{ 5 ms, -2 Volts } → 001
{ 6 ms, -3 Volts } → 000
{ 7 ms, -2 Volts } → 001
{ 8 ms, 0 Volts } → 011

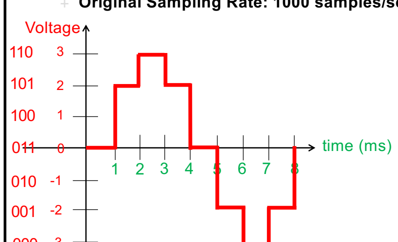
Encoding: mapping data from one form to another (not always conversion)

Example of Lossy Compression

- Sample Rate: **1000 samples/sec**, Resolution: **3-bits** per sample
- Our Sampled Signal: {0, 2.2V, 3V, 2.2V, 0, -2.2V, -3, -2.2V, 0}
- Our Quantized Signal: {0, 2V, 3, 2V, 0, -2, -3, -2, 0}
- Our 3-bit Digitized Data: {011, 101, 110, 101, 011, 001, 000, 001, 011}
  - space required to store/transmit: **27 bits**
- ADC related compression algorithm:
  - CS&Q (Coarser Sampling AND/OR Quantization)
    - Either reduce number of bits per sample AND/OR discard a sample completely
  - Example with our digitized data:
  - Our 3-bit Digitized Data: {011, 101, 110, 101, 011, 001, 000, 001, 011}
  - If we drop the sampling rate by a factor of 2, how effect number of bits needed?
  - ***Lossy*** because we **cannot** restore exact original

De-Compression of Signal:

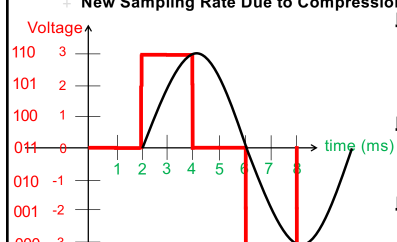
- **Decompression & DAC Process**
  - **Original digital signal: {011, 101, 110, 101, 011, 001, 000, 001, 011}**
  - **Original Sampling Rate: 1000 samples/sec**



Original Signal (recall ADC/DAC is approximation at best anyway!)

De-Compression of Signal:

- **Decompression & DAC Process**
  - **Original compressed signal: {011, , 110, , 011, , 000, , 011}**
  - **New Sampling Rate Due to Compression: 500 samples/sec**



Effect of CS&Q compression
Lowered Sampling Rate
Added "noise" to signal
Listeners might not notice!

Lossy Compression:
One can achieve high Compression ratios

Frequently used for Audio:
MP3 format uses lossy compression algorithm

Lossy: Compression removed every other sample

Two forms of classification

Compression Algorithms

Lossy — Lossless

Compression Algorithms

Fixed Group Size — Variable Group Size

Examples of **Fixed** Group Size:
Take in 2 samples: (6-bits) always spit out: (3-bits)
Take in 8-bit ASCII character (group), spit out 7-bit ASCII character (group)

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Interlude (time permitting)

- **SNL – 5 minute University**
  - Father Guido Sarducci
- https://www.youtube.com/watch?v=kO8x8eoU3L4
- **What form of compression here?**

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For Computer Engineering?

- **Make the common case fast**
- **Make the frequent case small**

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Probability-Based Lossless Compression

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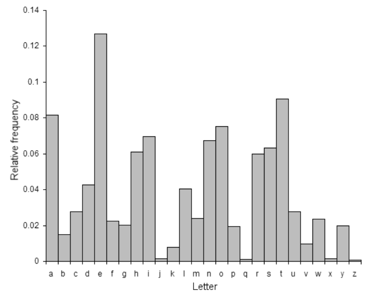
Information Content

- **Does each character contain the same amount of "information"?**

Statistics

- **How often does each character occur?**
  - Capital letters versus non-capitals?
  - How many e's in a preclass quote?
  - How many z's?
  - How many q's?

English Letter Frequency

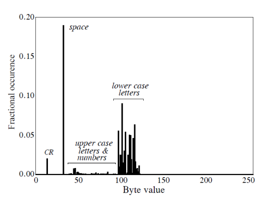


http://en.wikipedia.org/wiki/File:English-slf.png

Huffman Encoding

- **Developed in 1950's (D.A. Huffman)**
- **Takes advantage of frequency of stream of bits occurrence in data**
  - Can be done for ASCII (8-bits per character)
    - Characters do not occur with equal frequency.
    - How can we exploit statistics (frequency) to pick character encodings?
  - But can also be used for anything with symbols occurring frequently
    - E.g., Music (drum beats…frequently occurring data)
  - Example of **variable length** compression algorithm
    - Takes in fixed size group – spits out variable size replacement

Huffman Encoding – The Basics

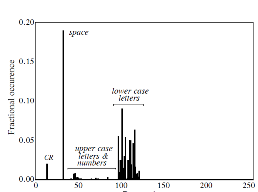


- **Example: more than 96% of file consists of 31 characters**
- **Idea: Assign frequently used characters fewer bits**
  - 31 common characters get 5b codes 00000--11110
  - Rest get 13b: 11111+original 8b code
- **How many bits do we need on average per original byte?**

Calculation

- **Bits = #5b-characters * 5 + #13b-character * 13**
- **Bits=#bytes*0.96*5 + #bytes*0.04*13**
- **Bits/original-byte = 0.96*5+0.04*13**
- **Bits/original-byte = 5.32**

Huffman Encoding – More Advanced



Example Encoding Table

letter	probability	Huffman code
A	.154	1
B	.110	01
C	.072	0010
D	.063	0011
E	.059	0001
F	.015	000010
G	.011	000011

- **Huffman goes further: Assign MOST used characters least # of bits:**
  - Most frequent: A= 1, least frequent: G=00011, etc.
  - **Example:** original data stream: C E G A D F B E A…

Huffman encoded: 0010 0001 000011 1 0011 000010 01 0001 1…

Preclass Encoding

symbol	encode	occur
(space)	00	15
A	1110	
B	100100	
C	100101	
D	10110	
E	110	11
F	011010	
G	011011	
H	011000	
I	0111	

symbol	encode	occur
L	0100	
M	1111	
N	1010	
O	10011	
R	0101	
T	10111	
V	10000	
Y	011001	
,	10001	

Preclass Encoding

symbol	encode	occur
(space)	00	15
A	1110	6
B	100100	
C	100101	
D	10110	
E	110	11
F	011010	
G	011011	
H	011000	
I	0111	4

symbol	encode	occur
L	0100	4
M	1111	6
N	1010	5
O	10011	
R	0101	4
T	10111	
V	10000	
Y	011001	
,	10001	

Preclass Encoding

symbol	encode	occur
(space)	00	15
A	1110	6
B	100100	
C	100101	
D	10110	3
E	110	11
F	011010	
G	011011	
H	011000	
I	0111	4

symbol	encode	occur
L	0100	4
M	1111	6
N	1010	5
O	10011	2
R	0101	4
T	10111	3
V	10000	2
Y	011001	
,	10001	2

Preclass Encoding

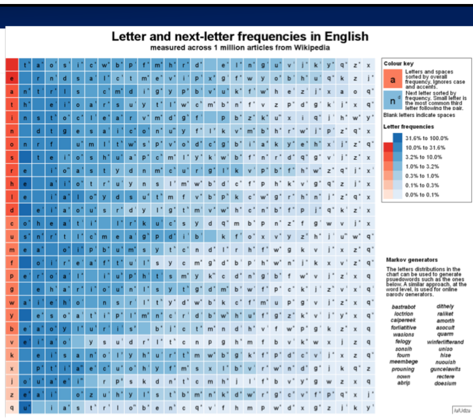
symbol	encode	occur
(space)	00	15
A	1110	6
B	100100	1
C	100101	1
D	10110	3
E	110	11
F	011010	1
G	011011	1
H	011000	1
I	0111	4

symbol	encode	occur
L	0100	4
M	1111	6
N	1010	5
O	10011	2
R	0101	4
T	10111	3
V	10000	2
Y	011001	1
,	10001	2

See how variable length encoding saved bits?
Questions?

Many Types of Frequency

- **Previous example:**
  - Simply looked at letters in isolation, determined frequency of occurrence
- **More advanced models:**
  - Predecessor context: What's probability of a symbol occurring, given: PREVIOUS letter.
- **Ex: What's most likely character to follow a T?**



Found: https://www.reddit.com/r/dataisbeautiful/comments/6rk2yr/letter_and_nextletter_frequencies_in_english_oc/

Models

- **Many models for compression**
  - Context-independent letter frequency
  - Context-dependent on previous letter
  - Context-dependent on multiple previous letters
  - Previous occurs of multi-letter strings…
- **Compressibility will depend on model employed**
  - More context
    - More compressibility
    - Must hold on to more state
    - More complex to encode/decode

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Compressibility

- **Compressibility depends on non-randomness (uniformity)**
  - Structure
  - Non-uniformity
- **If every character occurred with same freq:**
  - There's no common case
  - To which character do we assign the shortest encoding?
    - No clear winner
  - For everything we give a short encoding,
    - Something else gets a longer encoding
- **The less uniformly random data is…**
  - the more opportunity for compression

Common Case

- **Big idea in optimization engineering**
  - Make the common case inexpensive
- **Shows up throughout computer systems**
  - Computer architecture
    - Caching, instruction selection, branch prediction, …
  - Networking and communication, data storage
    - Compression, error-correction/retransmission
  - Algorithms and software optimization
  - User Interfaces
    - Where things live on menus, shortcuts, …
    - How you organize your apps on screens

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Entropy

Is there a lower bound for compression?

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How low can we go with compression?

What is the least # of bits required to encode information?

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Claude Shannon

- **Father of Information Theory, brilliant mathematician**
- **While at AT&T Bell Labs, landmark paper in 1948**
- **Determined exactly how low we can go with compression!**

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Shannon's Entropy

- **What is entropy?**
  - Chaos/Disorganization/Randomness/Uncertainty
- **Shannon's Famous Entropy Formula:**

$$H = -\sum p(x) \log p(x)$$

Shannon's Entropy *(measured in bits)*

Negative Sum Of:

Probability of each outcome X $\log_2$ of (probability of each outcome)

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Estimating Entropy of English Language

- **27 Characters (26 letters + space)**
- **If we assume all characters are equally probable:**
  - $p(each\ character) = \frac{1}{27}$
- **Information Entropy per character:**

$$H = -\sum p(x) \log p(x)$$

$$H = -27\left(\frac{1}{27}\right)\log\left(\frac{1}{27}\right) = -\log\left(\frac{1}{27}\right) = +4.75\ bits$$

Same thing we got when we said we needed $\log_2$(unique_things) bits

Shannon Entropy

- **Essentially says**
  - Should be able to encode with log(1/p) bits

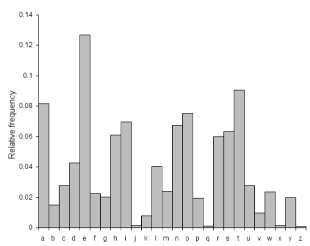
$$\text{Average Bits} = \sum_i p_i \times \text{bits}(i)$$

$$H = -\sum_i p_i \times \log_2(p_i)$$

Where did we calculate Average Bits earlier in lecture?

Recall:

Not all letters are equally probable in English Language



http://en.wikipedia.org/wiki/File:English-slf.png

Shannon Entropy English Letters

$$H = -\sum_i p_i \times \log_2(p_i)$$

letter	p	$-\log_2(p)$	$-p*\log_2(p)$
a	8.17%	3.61	0.30
b	1.49%	6.07	0.09
c	2.78%	5.17	0.14
d	4.25%	4.56	0.19
e	12.70%	2.98	0.38
f	2.23%	5.49	0.12
z	0.07%	10.40	0.01
sum	100.00%		4.18

Shannon Entropy Preclass Quote

$$H = -\sum_i p_i \times \log_2(p_i)$$

Symbol	Bits	Occur	P	-log2(p)	H	p*bits
(space)	2	15	0.21	2.28	0.47	0.41
A	4	6	0.08	3.60	0.30	0.33
B	6	1	0.01	6.19	0.08	0.08
C	6	1	0.01	6.19	0.08	0.08
D	5	3	0.04	4.60	0.19	0.21
E	3	11	0.15	2.73	0.41	0.45
,	5	2	0.03	5.19	0.14	0.14
				sum	3.74	3.77

Summing it up: Shannon & Compression

- **Shannon's Entropy represents a lower limit for lossless data compression**
  - It tells us the minimum amount of bits that can be used to encode a message without loss (according to a particular model)
- **Shannon's Source Coding Theorem:**
  - A lossless data compression algorithm cannot compress messages to have (on average) more than 1 bit of Shannon's Entropy per bit of encoded message

To consider

- Assumed know statistics
- What if you don't?
- What if it changes?
- How could we adapt the code to changing statics?

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This week in Lab

- Implement Compression!
  - Implement Huffman Compression
- Remember:
  - Lab 2 report is due on canvas on Friday.
- Office Hours:
  - Moved T2:30 → R4:30
  - Moved R2:30 → R7:00

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Big Ideas

- Lossless Compression
  - Exploit non-uniform statistics of data
  - Given short encoding to most common items
- Common Case
  - Make the common case inexpensive
- Shannon's Entropy
  - Gives us a formal tool to define lower bound for compressibility of data

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Learn More

- ESE 301– Probability
  - Central to understanding probabilities
    - What cases are common and how common they are
- ESE 674 – Information Theory
- Most all computer engineering courses
  - Deal with common-case optimizations
  - CIS240, CIS371, CIS380, ESE407, ESE532….

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References

- S. Smith, "The Scientists and Engineer's Guide to Digital Signal Processing," 1997.
- Shannon's Entropy (excellent video) http://www.youtube.com/watch?v=JnJq3Py0dyM
  - Used heavily in the creation of entropy slides

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