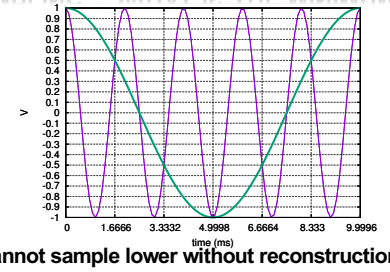
Sampling – What is the minimum?



- **Cannot sample lower without reconstruction error**
  - We not only lose information…
    - …but when we 'reconstruct' the signal from the samples alone…
    - We will reconstruct at a lower frequency!
    - This phenomenon is called: aliasing

1

Aliasing Mathematical Derivation

- **500Hz cosine:** $\cos(2\pi \cdot 500 \cdot t)$
- **Sampled at 600Hz**
  - Only look at t=I/600
  - I is the index for samples
- **So, our discrete version:** $\cos\left(2\pi \cdot 500 \cdot \left(\frac{I}{600}\right)\right)$
- **Simplify :** $\cos\left(2\pi \cdot \left(\frac{5}{6}\right) \cdot I\right)$
- **Rearrange :** $\cos\left(2\pi \cdot I - 2\pi \cdot \left(\frac{1}{6}\right) \cdot I\right)$

2

Mathematical Manipulation

- **500Hz cosine:** $\cos(2\pi \cdot 500 \cdot t)$
- **Sampled at 600Hz**
- **Now :** $\cos\left(2\pi \cdot I - 2\pi \cdot \left(\frac{1}{6}\right) \cdot I\right)$
  - I is an integer.
  - $\cos(x + 2\pi) = \cos(x)$
- **Apply:** $\cos\left(-2\pi \cdot \left(\frac{1}{6}\right) \cdot I\right)$
  - $\cos(-x) = \cos(x)$
- **Apply:** $\cos\left(2\pi \cdot \left(\frac{1}{6}\right) \cdot I\right)$

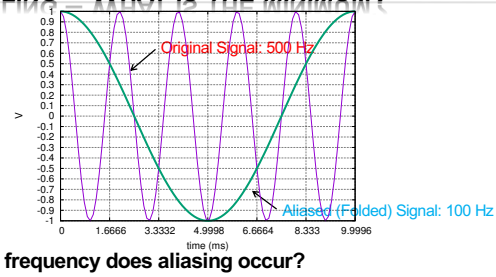
3

Aliasing Derivation

- **500Hz cosine:** $\cos(2\pi \cdot 500 \cdot t)$
- **Sampled at 600Hz**
- **Simplified to:** $\cos\left(2\pi \cdot \left(\frac{1}{6}\right) \cdot I\right)$
- **Same as:** $\cos\left(2\pi \cdot 100 \cdot \left(\frac{I}{600}\right)\right)$
  - Which would correspond to 100Hz signal

4

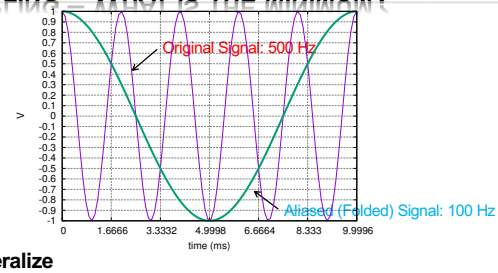
Sampling – What is the minimum?



- **What frequency does aliasing occur?**
  - Original Signal's Frequency: 500 Hz
    - Sampling Rate: 600 Hz
  - Aliasing occurs at: 600 Hz – 500 Hz = 100 Hz
    - Also referred to as "Folding" – signal has "folds over" as if it were lower frequency

5

Sampling – What is the minimum?



- **Generalize**
  - F' = frequency mod sample-rate (subtract out integer 2π terms)
  - Alias frequency is
    - F' if F'<sample-rate/2
    - Sample-rate-F' if sample-rate/2 < F' < sample-rate

(probably a better way to express this)

6