Take Away (Week 4): Time-Frequency Representation and Transformation

The Time-Frequency lecture was ambitious. There are a set of concepts you will need to carry forward from this lecture. There were also many details in the lecture that we include to make the story more complete, but we don't expect you to master for this course.

Essential

- **There are two different representations we can use for sound waveforms (signals): time domain and frequency domain.** As we move forward, we will begin to reason about human hearing in terms of frequency components and how important each component is to human perception. Removing frequency components that do not contribute will be a key element of sound compression.
- **In the frequency domain, we can represent a signal as a weighted sum of sines and cosines.**
- **It is possible to convert between the two representations using a set of mathematical operation.** The most important thing is that you accept there is a way to perform this computation on computers. We will depend on algorithms (written by someone else) that give us the frequency components to setup the optimization to remove inessential frequency components, and we will depend on algorithms to convert the saved frequency components back to time samples of sound for reproduction.

Useful to Appreciate

- **In particular, we can perform a set of multiplications and additions in the form of dot products to perform the conversion in either direction.** This goes a step further and says there are particular dot products that make up the conversion. We'll ask you to perform the dot products, and we'll reason about the computation required to perform the conversion based on the dot products. You can appreciate this even if you cannot reproduce the particular coefficients of the dot product from your own memory or first principles.

Bonus for Greater Depth and Future

- **The vector used in the dot product to determine the amount of a particular frequency component (e.g. sine wave at a given frequency) is the frequency component itself.** It's not a big step from the dot products to the particular coefficients involved.
- **This works because the sine and cosine waves form an orthogonal basis, and we are performing a change of basis operation.** Why this is true is beyond what we prove to you in this course. We share the rationale to, perhaps, connect to things you already know, and to plant a conceptual seed for you to build from in other courses.
- **The transformation equations can be stated compactly as a set of complex exponential summations over frequency components or time samples.** We share this mostly because you will see it stated this way in other sources. We're hoping that making the connection will help you recognize it when it shows up. The routines we use in MATLAB will frame it in the exponential notation.