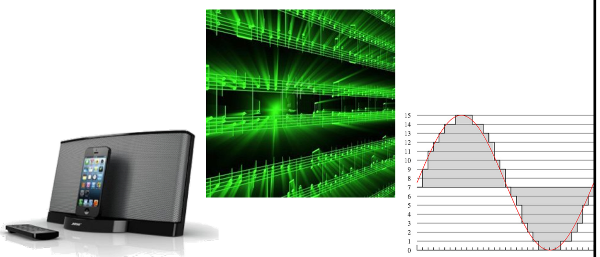


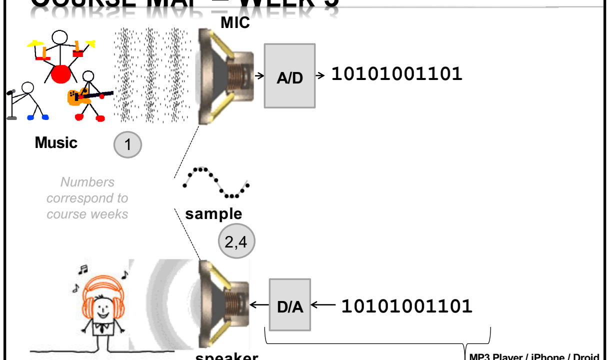
Lecture #5 – Anti-Aliasing

ESE 150 – Digital Audio Basics

Lecture Topics

- Where are we on course map?
- **Review**
  - **Nyquist-Shannon Sampling Rate**
  - Aliasing
- **Multi-frequency signals**
- **Anti-Alias Filtering**
- References

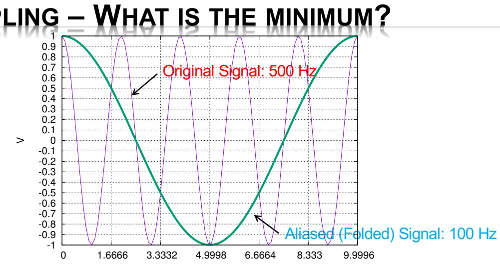
Course Map – Week 3



Nyquist Sampling

- **Sample at twice the maximum frequency**
  - Can reconstruct perfectly
- **If have frequencies > SampleRate/2**
  - Will get aliasing … as high frequencies fold

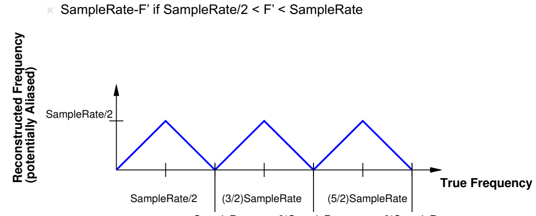
Sampling – What is the Minimum?



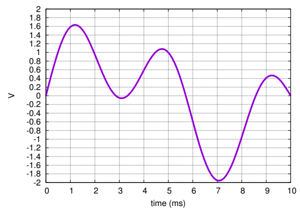
- **What frequency does aliasing occur?**
  - Original Signal's Frequency: 500 Hz
    - Sampling Rate: **600 Hz**
  - Aliasing occurs at: **600 Hz** – 500 Hz = 100 Hz
    - Also referred to as "Folding" – signal has "folds over" as if it were lower frequency

Sampling – What is the Minimum?

- **Generalize**
  - F' = frequency mod SampleRate (subtract out integer 2π terms)
  - Alias frequency is
    - F' if F'<SampleRate/2
    - SampleRate-F' if SampleRate/2 < F' < SampleRate

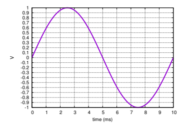


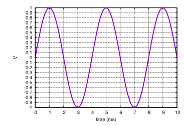
What about Multirate Signals?

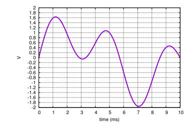


- Fourier's Theorem (week 4 preview!):
  - We can decompose continuous signal in terms of a sum of sines and cosines at different frequencies

Preclass 1



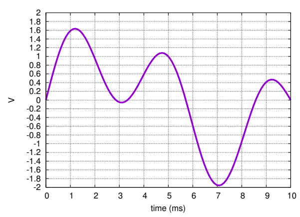




	0	1	2	3	4	5	6	7	8	9	10	f
L												
M												
R												

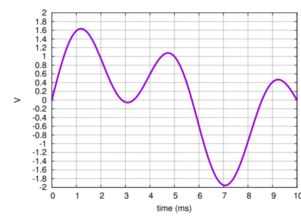
Properly sampled? Relation?

What about Multirate Signals?



- Fourier's Theorem (week 4 preview!):
  - We can decompose continuous signal in terms of a sum of sines and cosines at different frequencies
  - This waveform: sum of sine waves at 100Hz and 250Hz
    - What's the Nyquist Sampling Rate then?

What about Multirate Signals?

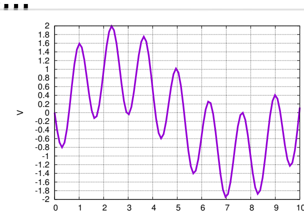


- Fourier's Theorem & Nyquist Rate:
  - Highest component's frequency?
  - What is Nyquist Sampling Rate?
    - 2 x **highest** frequency contained in the signal
    - Sampling at this rate: avoids aliasing problem

Effects of Aliasing

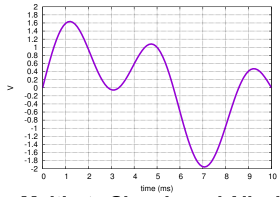
Aliasing in Music…

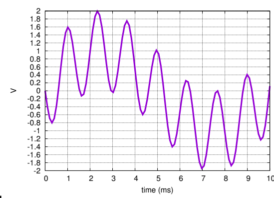
Preclass 2



- Multirate Signals and Aliasing
  - Imagine the above is a music signal (100Hz + 750Hz)
    - What happens if we undersample? Should sample at 1500 Hz, but instead 1000 Hz
    - The 100 Hz signal will be sampled just fine (as 200 Hz is *2 x* 100 Hz)
    - What happens to 750 Hz Signal?

Aliasing in Music…





- **Multirate Signals and Aliasing**
  - Imagine the above is a music signal (100 Hz and 750 Hz chord)
  - Where will 750Hz appear as?
    - folding occurs at: **1000 Hz** – 750 Hz = 250 Hz
    - Sample rate – frequency = aliasing/folding frequency

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Anti-Aliasing

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How do we fix this?

- **It's simple…sample at the Nyquist Rate**
  - But…what if your rate is fixed? Like 24 frames/sec?
  - Or our eye's sampling rate: 60 cycles/degree
    - Spatial variations finer than this are undetectable!

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Why do we need the antialias filter?

- **If we can't hear anything above 20kHz…**
  - Why do we need to filter it out?
    - Dog's can hear from 40 Hz to 60 kHz
      - so clearly there are sounds above 20 kHz
  - Let's imagine a high frequency noise in music studio
    - Let's say it's a vibration occurring at 25 kHz
      - No human can hear it, why filter it out?
    - Because of aliasing:
      - Frequency aliasing/folding will occur:
      - Sample rate – frequency = aliasing/folding frequency
      - 40 kHz - 25 kHz = 15 kHz
      - The 25 kHz vibration will fold-over to a 15 kHz "hum" or audible noise
        - It will ruin our recording and source of noise wouldn't be obvious!

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How to Avoid Aliasing with Digital Music?

- **If we simply sample at *2 x highest frequency of signal*…**
  - (AKA: Nyquist Rate)
  - …we won't encounter aliasing!
- **But how do we guarantee highest frequency of our signal?**
  - Audio: this is easy!
    - We know the range of human ear: 20 Hz to 20 kHz…
    - The highest frequency component in music is then: 20 kHz
    - …so, before sound goes into ADC, we apply a filter!
      - Blocks any frequency above 20 kHz from going into ADC
    - Essentially, we are fixing our sampling rate & 'blurring' or filtering our incoming signal

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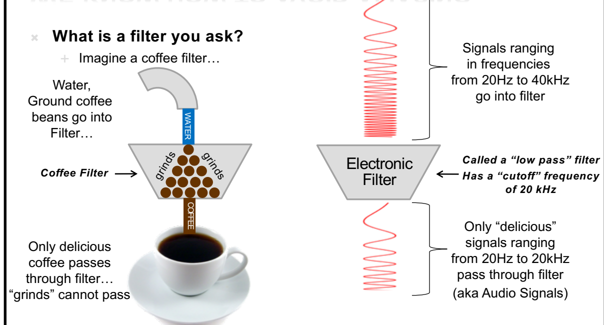
We know how to avoid aliasing…

- **What is a filter you ask?**
  - Imagine a coffee filter…



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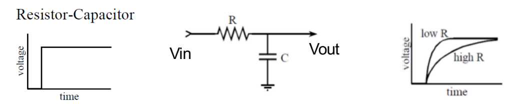
Low Pass Analog Filtering

- **Can limit rate of change**
  - Set a minimum time period for a value to charge output
- **If signal tries to move too fast (high frequency)**
  - The input change won't be reflected in the output
- **Ideal:** fast inputs "erased" from output
- **In practice:** magnitude reduced
- Can engineer filters to get closer to ideal 215, 319

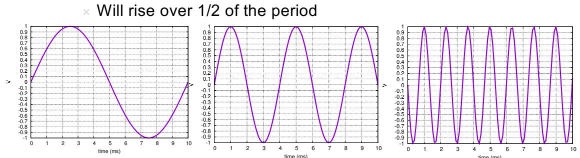
Low Pass Analog Filtering

- Transfer Vin into Capacitor Vout
- Think of Resistor as straw or hose
  - Limits rate of electron flow into Capacitor
- Think of Capacitor as a glass or bucket
  - Must fill with electrons through Resistor to have Voltage (water) level rise
- Circuit limits rate of change time at Vout

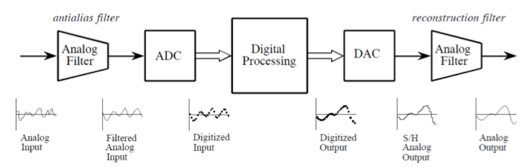


Frequency and Signal Change

- **There's a direct relationship between frequency of a signal and rate at which signal changes**
  - Higher frequency → higher $\Delta V / \Delta T$
  - Limit rise time → limit frequency can pass through filter
  - Maximum frequency → limits the rise-time will have
    - Will rise over 1/2 of the period



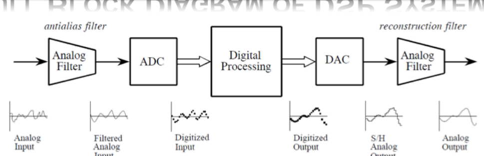
Full Block Diagram of DSP System



- **Before ADC, we put music signal through *antialias* filter**
  - Filter blocks any signals higher than 20 kHz (prevents aliasing!)

Figures from reading: *The Scientist and Engineer's Guide to Digital Signal Processing*, By Steven W. Smith

Full Block Diagram of DSP System



- **Before ADC, we put music signal through *antialias* filter**
  - Filter blocks any signals higher than 20 kHz (prevents aliasing!)
  - Then our ADC can safely sample at 2 x 20 kHz without aliasing
    - What is our Nyquist Rate?
      - $f_s = 2 \times$ 20 kHz = 40 kHz, or 40 thousand samples per second!
    - What is our Nyquist Frequency?
      - $\frac{1}{2} f_s$ = 20 kHz
  - Cutoff frequency of our filter? Has to be the Nyquist Frequency

Big Ideas

- **Sample at twice the maximum frequency**
  - Can reconstruct perfectly
- **If have frequencies > sample_freq/2**
  - Will get aliasing … as high frequencies fold
- **Avoid aliasing with analog Anti-Alias (low-pass) prefilter before sampling**
  - Eliminate high frequencies

Learn More

- **ESE215 – include analog filtering**
- **ESE319 – active analog filtering**
- **ESE224 – Signal Processing**

Admin

- **Remember feedback**
  - Including on Lab 2
- **Lab 3 posted**
  - Prelab contains a longer MATLAB tutorial
    - And need to install MATLAB
  - Plan time for it over weekend

References

- S. Smith, "The Scientists and Engineer's Guide to Digital Signal Processing," 1997.
- http://en.wikipedia.org/wiki/Nyquist_frequency
- http://en.wikipedia.org/wiki/Nyquist_rate
- http://en.wikipedia.org/wiki/Oversampling
- http://en.wikipedia.org/wiki/Sampling_rate
- http://en.wikipedia.org/wiki/Hearing_range
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- B. Olshausen, "Aliasing", PSC 129 – Sensory Processes Course Notes, UC Davis