Feedback Basics

- Stability
- Feedback concept
- Feedback in emitter follower
- One-pole feedback and root locus
- Frequency dependent feedback and root locus
- Gain and phase margins
- Conditions for closed loop stability
- Frequency compensation
Circuit/System Stability

1. Simply put, a linear system such as an electronic circuits is “stable” if it has no poles (denominator roots or zeros) in the right-half of the complex s-plane.

2. Right-half plane poles correspond to growing exponential terms in the solution of the system/circuit differential equation.

3. To determine stability, we need to check for RH plane poles. This requires either factoring the denominator polynomial or employing a number of other techniques developed to assist in the pencil and paper design of linear systems.
Operational Amplifier From Another View

\[ v_o = A \left( v_i - \Gamma_F v_o \right) \]
\[ (1 + A \Gamma_F) v_o = A v_i \]

\[ v_o = \frac{A}{1 + A \Gamma_F} v_i \]

\[ v_f = \frac{R_2}{R_1 + R_2} v_o = \Gamma_F v_o \]

Classic feedback structure
Feedback Block Diagram Point of View

Fundamental block diagram

We can split the ideal op amp and resistive divider into 2 separate blocks because they do not interact with, or “load,” each other.

1. The op amp output is an ideal voltage source.

2. The op amp input draws no current.

Fundamental feedback equation

\[ v_o = \frac{A}{1 + AF} v_i \]
Comments on the Feedback Equation

The quantity \( A \Gamma_F \) = *loop-gain* and \( A_{cl} = *closed-loop gain*.

The quantity \( A = *open-loop gain* and \( \Gamma_F = *feedback gain*.

If the loop gain is much greater than one, i.e.
\[
A \Gamma_F \gg 1
\]
(and the system is stable – a topic to be discussed later!) the *closed-loop gain* approximates \( 1/\Gamma_F \), and is independent of “\( A \)”!

\[
A_{v-cl} = \frac{v_o}{v_i} \approx \frac{1}{\Gamma_F}
\]
Feedback in the Emitter Follower

Emitter current equation:

\[ i_e = \frac{1}{\frac{R_s}{\beta + 1} + r_e + R_E} v_i \]

\[ v_o = R_E i_e = \frac{R_E}{\frac{R_s}{\beta + 1} + r_e + R_E} v_i \]

Create an “artificial” feedback equation, multiply numerator & denominator by \((\beta + 1) / R_s\):

\[ v_o = \frac{R_E(\beta + 1)}{R_s} v_i = \frac{A}{1 + A \Gamma_F} v_i \]

\[ 1 + \frac{R_E(\beta + 1)(r_e + R_E)}{R_s} \]
**Emitter Follower Emitter Current**

The forward gain open-loop term, $A$:

$$A = \frac{R_E(\beta+1)}{R_s}$$

The feedback term, $\Gamma_F$:

$$\Gamma_F = \frac{r_e + R_E}{R_E}$$

$$A \Gamma_F = \frac{R_E(\beta+1)}{R_s} \frac{r_e + R_E}{R_E} = (\beta+1) \frac{r_e + R_E}{R_s} \gg 1$$

If the “loop gain” is large, $A \Gamma_F \gg 1$

$$v_o \approx \frac{1}{\Gamma_F} \frac{A}{1 + A \Gamma_F} v_i = \frac{R_E}{r_e + R_E} v_i$$

Dependence on $\beta$ is eliminated.
Loop Gain Sensitivities For:

\[ A_{cl} = \frac{A}{1 + A \Gamma_F} \]

Open-loop gain:

\[ \frac{dA_{cl}}{dA} = \frac{1}{\left(1 + A \Gamma_F\right)^2} \]

\[ \frac{dA_{cl}}{A_{cl}} = \frac{1}{\left(1 + A \Gamma_F\right)^2} \frac{1 + A \Gamma_F}{A} dA \]

\[ \frac{dA_{cl}}{A_{cl}} = \frac{1}{\left(1 + A \Gamma_F\right)} \frac{dA}{A} \rightarrow 0 \quad \text{as } A \Gamma_F \rightarrow \infty \]

Feedback gain:

\[ \frac{dA_{cl}}{d \Gamma_F} = \frac{-A^2}{\left(1 + A \Gamma_F\right)^2} \]

\[ \frac{dA_{cl}}{A_{cl}} = \frac{-A^2}{\left(1 + A \Gamma_F\right)^2} \frac{1 + A \Gamma_F}{A} d \Gamma_F \]

\[ \frac{dA_{cl}}{A_{cl}} = \frac{- A \Gamma_F}{\left(1 + A \Gamma_F\right)} \frac{d \Gamma_F}{\Gamma_F} \rightarrow - \frac{d \Gamma_F}{\Gamma_F} \quad \text{as } A \Gamma_F \rightarrow \infty \]

High loop gain makes system insensitive to \( A \), but sensitive to \( \Gamma_F \)!
Feedback - One-pole $A(s)$

Consider the case where:

$$A(s) = \frac{a K_0}{s + a}$$

open-loop pole

$$A_{cl}(s, \Gamma_F) = \frac{a K_0}{s + a} \frac{1}{1 + \Gamma_F a K_0}$$

closed-loop pole

Where $\Gamma_F$ and $K_0$ are positive real quantities.

$$A_{cl} = \frac{v_o}{v_i} = \frac{A}{1 + A \Gamma_F}$$

pole(s) of $A_{cl}(s)$ are the roots of $1 + A \Gamma_F = 0$, or

$$A(s) \Gamma_F = -1 = 1 \ast e^{j \pi \pm 2k\pi}$$

for $k = 0, 1, ...$

Root Locus Plot for $A_{cl}(s, \Gamma_F)$

stable for all $\Gamma_F$!
Feedback - One-pole $A(s)$ cont.

$$A(s) = \frac{a K_0}{s + a}$$

open-loop (OL)

$$A_{cl}(s) = \frac{a K_0}{s + a + \Gamma_F a K_0} = \frac{N(s)}{D(s)}$$

closed-loop (CL)

Gain - dB

$20 \log_{10} K_0$

$20 \log_{10} \frac{K_0}{1 + \Gamma_F K_0}$

0

frequency ($\omega$)

$-20$ dB /dec

$a$

$a(1 + \Gamma_F K_0)$

$\text{GBW}_{OL} = \text{GBW}_{CL} = a K_0$
**One-pole Feedback - Root Locus**

\[
A(s) = \frac{a K_0}{s + a} \quad \text{and} \quad A_{cl}(s) = \frac{a K_0}{s + a + \Gamma_F a K_0} = \frac{N(s)}{D(s)}
\]

\[A(s) \Gamma_F = -1 = 1 \cdot e^{j \pi \pm 2k \pi}\]

Pole of \(A_{cl}(s)\):

\[D(s) = s + a + \Gamma_F a K_0 = 0 \quad \text{or} \quad s = -(a + \Gamma_F a K_0)\]

\[s + a + \Gamma_F a K_0 = 0 \Rightarrow \Gamma_F a K_0 = -s = 1 = 1 \cdot e^{j \pi \pm 2k \pi} \quad \text{for} \quad k = 0, 1, ...\]

\(r_1\) is a root of \(D(s) = 0\) or \(1 + A(s) \Gamma_F = 0\) iff

\[-\phi_{(s+r_1)} = \pi \pm 2k \pi \quad \text{and} \quad \frac{\Gamma_{F1} a K_0}{|r_1 + a|} = 1 \quad \text{for p.r.} \Gamma_{F1}\]

\[\phi_{(r_1+a)} = \pi \quad \text{and} \quad \phi_{(r_1+a)} = 0 \neq \pi \pm 2k \pi\]

\[\infty > \Gamma_F K_0 \geq 0 \quad \text{Not allowed}\]
Frequency-Dependent Feedback

Consider the case where the open-loop gain is:

\[ A(s) = K \frac{(s+a)}{(s+b)(s+c)} \quad a < b < c \]

**dc Gain:**

\[ A(s)_{s=0} = K \frac{(0+a)}{(0+b)(0+c)} = K \frac{a}{bc} = K_0 \]

A sketch of the Bode plot would look something like:
Frequency-Dependent Feedback

\[ A_{cl}(s) = \frac{K_0(s+a)}{(s+b)(s+c)} \]

1 + \gamma_F K_0 \frac{(s+a)}{(s+b)(s+c)}

Rationalizing this expression leads to:

\[ A_{cl}(s) = \frac{K_0(s+a)}{(s+b)(s+c)+\gamma_F K_0(s+a)} = \frac{N(s)}{D(s)} \]

The numerator is factored, but the denominator is not. We have a new quadratic polynomial \( D(s) \) for \( A_{cl}(s) \). \( D(s) \) can be factored using the quadratic formula.

\[ D(s) = (s+b)(s+c)+\gamma_F K_0(s+a) = (s+r_1)(s+r_2) = 0 \]

where \( r_1 \) and \( r_2 \) may be complex conjugate
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all $\Gamma_F$?

$$A_{cl}(s) = \frac{K_0 \frac{(s+a)}{(s+b)(s+c)}}{1 + \Gamma_F K_0 \frac{(s+a)}{(s+b)(s+c)}} = K_0 \frac{(s+a)}{(s+b)(s+c) + \Gamma_F K_0 (s+a)}$$

$a < b < c$
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all $\Gamma_F$?

$$(s+b)(s+c) + \Gamma_F K_0 (s+a) = 0$$

Since the poles of $A_{cl}(s)$ will not equal any of the poles of $A(s)$, we can divide by $(s+b)(s+c)$ and obtain:

$$1 + \Gamma_F K_0 \frac{(s+a)}{(s+b)(s+c)} = 0$$

The “loop-gain” terms, $\Gamma_F$ and $K_0$, are positive real numbers, so for a root, say $s = -r_i$ to exist, the value of the frequency-dependent terms must be real and negative when evaluated at $s = -r_i$, where $r_i$ is in general complex.

$$\Gamma_F K_0 \frac{(s+a)}{(s+b)(s+c)} = -1 = 1 \times e^{j \pi \pm 2k \pi} \quad \text{for } k = 0, 1, \ldots$$
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all $\Gamma_F$?

$$\Gamma_F K_0 \frac{(s + a)}{(s + b)(s + c)} = -1 = 1 \times e^{j\pi \pm 2k\pi} \quad \text{where } k = 0, 1, \ldots$$

Working with the complex numbers in polar form:

$$\phi(r_i + a) - \phi(r_i + b) - \phi(r_i + c) = \pi \pm 2k\pi \quad \text{and} \quad \Gamma_{Fi} \frac{K_0 |r_i + a|}{|r_i + b||r_i + c|} = 1 \quad \text{for p.r. } \Gamma_{Fi}$$

To test a potential root at say $s = -r_i$, we first add the angles for zeros to the point “$r_i$” in the complex s-plane and subtract the angles for poles.
What's the Root Locus for $A_{cl}(s)$?

**Is $A_{cl}(s)$ Stable for all $\Gamma_F$ ?**

Only $D(s)$ root locations where the angles with respect to open-loop pole/zero locations of $A(s)$ are odd multiples of $\pm 180^\circ$ are candidates.

$$D(s=r_i) = (r_i + b)(r_i + c) + \Gamma_F K_0 (r_i + a) = 0 \quad \text{for } i = 1, 2$$

$$\phi_{tot} = \phi_{(r_i+a)} - \phi_{(r_i+b)} - \phi_{(r_i+c)} = \pi \pm 2k\pi$$

1. $\phi_{(r_i+a)} = 0; \phi_{(r_i+b)} = 0; \phi_{(r_i+c)} = 0 \Rightarrow \phi_{tot} = 0 \neq \pi \pm 2k\pi$

2. $\phi_{(r_2+a)} = \pi; \phi_{(r_2+b)} = 0; \phi_{(r_2+c)} = 0 \Rightarrow \phi_{tot} = \pi$

3. $\phi_{(r_3+a)} = \pi; \phi_{(r_3+b)} = \pi; \phi_{(r_3+c)} = 0 \Rightarrow \phi_{tot} = 0 \neq \pi \pm 2k\pi$

4. $\phi_{(r_4+a)} = \pi; \phi_{(r_4+b)} = \pi; \phi_{(r_4+c)} = \pi \Rightarrow \phi_{tot} = \pi$

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1. All roots of $D(s)$ must lie along the “green” loci.
2. Bandwidth of $A_{cl}(s)$ > bandwidth of $A(s)$
3. If an $r_i = -a$ there is a zero/pole cancellation
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all $\Gamma_F$?

$$A_{cl}(s) = \frac{K_0\frac{(s+a)}{(s+b)(s+c)}}{1+\Gamma_F K_0\frac{(s+a)}{(s+b)(s+c)}} = K_0\frac{(s+a)}{(s+b)(s+c)+\Gamma_F K_0(s+a)} \quad a < b < c$$

1. zeros of $A_{cl}(s) = $ zeros of $A(s)$ and are independent of feedback
2. poles of $A_{cl}(s) \neq $ poles of $A(s)$ are a function of the feedback

$A_{cl}(s)$ IS stable for all $\Gamma_F$!
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all $\Gamma_F$?

$$A(s) = \frac{K_0 bc}{(s+b)(s+c)}$$

two-pole $b < c$

$$A_{cl}(s) = \frac{K_0 bc}{(s+b)(s+c) + \Gamma_F K_0 bc} = \frac{N(s)}{D(s)}$$
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all $\Gamma_F$?

$$A(s) = \frac{K_0 bc}{(s+b)(s+c)}$$

$$A_{cl}(s) = \frac{K_0 bc}{(s+b)(s+c) + \Gamma_F K_0 bc} = \frac{N(s)}{D(s)} \quad b < c$$

Complex conjugate poles of $A_{cl}(s)$

$A_v(s)$ IS stable for all $\Gamma_F$!
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all $\Gamma_F$?

$$A(s) = \frac{K_0 bcd}{(s+b)(s+c)(s+d)}$$
$$b < c < d$$

$$A_{cl}(s) = \frac{K_0 bcd}{(s+b)(s+c)(s+d) + \Gamma_F K_0 bcd} = \frac{N(s)}{D(s)}$$
What's the Root Locus for $A_{cl}(s)$?

Is $A_{cl}(s)$ Stable for all $\Gamma_F$?

$$A(s) = \frac{K_0 bcd}{(s+b)(s+c)(s+d)}$$

$$b < c < d$$

$$A_{cl}(s) = \frac{K_0 bcd}{(s+b)(s+c)(s+d) + \Gamma_F K_0 bcd} = \frac{N(s)}{D(s)}$$

$A_{cl}(s)$ NOT stable for all $\Gamma_F$!
The Root Locus Method

This graphical method for finding the roots of a polynomial is known as the root locus method. It was developed before computers were available. It is still used because it gives valuable insight into the behavior of feedback systems as the loop gain is varied. Matlab (control systems toolbox) will plot root loci.

In the frequency-dependent feedback (two-pole & one-zero) example, we noted that increasing feedback increases the CL bandwidth – i.e. the low & high frequency break points moved in opposite directions as $\Gamma_F$ increases. Higher $\Gamma_F$, as a trade-off, reduces the closed-loop mid-band gain.
Stability - Gain and Phase Margins

\[ A_{cl}(j \omega) = \frac{V_o(j \omega)}{V_i(j \omega)} = \frac{A(j \omega)}{1+A(j \omega) \Gamma_F} \]

loop-gain
\[ A(j \omega) \Gamma_F = 1 e^{j \pi \pm 2k \pi} \Rightarrow \text{oscillation or instability} \]

For stable closed-loop amplifier:

\[ |A(j \omega_{180})\Gamma_F| < 1 \]

or

\[ 20 \log |A(j \omega_{180})\Gamma_F| < 0 \text{ dB} \]

or

\[ \phi(j \omega_1) = \arg[A(j \omega_1) \Gamma_F] > -180^\circ \]

mutually consistent stability conditions

\[ GM = 0 \text{ dB} - 20 \log_{10} |A(j \omega_{180})\Gamma_F| \]

\[ GM > 0 \text{ dB} \Rightarrow \text{stable amplifier} \]

\[ PM = \phi(j \omega_1) - (-180^\circ) = \phi(j \omega_1) + 180^\circ \]

\[ PM > 0^\circ \Rightarrow \text{stable amplifier} \]
\[ A_{cl}(j\omega) = \frac{v_o}{v_i} = \frac{A(j\omega)}{1 + A(j\omega)\cdot \Gamma_F} \]

\[ \rho = \text{damping ratio} \]

\[ \rho \cdot \omega_0 \]

For acceptable overshoot: \( PM > 50^\circ \)

Effect of PM on closed-loop gain
overshoot or peaking

-12 dB/octave

(\( \text{maximally flat response} \))

\[ \rho = 0.5 \]

\[ \rho = 0.707 \]

\[ \rho = 3.33 \]

\[ \rho = 1 \]
Effect of PM on closed-loop gain

-12 dB/octave (maximally flat response)

For best compromise of overshoot vs. $t_{\text{rise}}$ & $t_{\text{settle}}$.

$70^\circ > \text{PM} > 55^\circ$
**Alternative Stability Analysis**

1. Investigating stability for a variety of feedback gains $\Gamma_F$ by constructing Bode plots for the loop-gain $A(j\omega)\Gamma_F$ can be tedious and time consuming.

2. A simpler approach involves constructing Bode plots for $A(j\omega)$ and $\Gamma_F$ (or $1/\Gamma_F$) separately.

$$20\log|A(j\omega)\Gamma_F| = 20\log|A(j\omega)| - 20\log\frac{1}{\Gamma_F}$$

when

$$20\log|A(j\omega)|_{\omega=\omega_1} = 20\log\frac{1}{\Gamma_F} \Rightarrow 20\log|A(j\omega_1)\Gamma_F| = 0 \text{ dB} \Rightarrow |A(j\omega_1)\Gamma_F| = 1$$

**RECALL:** $A(j\omega)\Gamma_F \gg 1 \Rightarrow A_{cl}(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)\Gamma_F} \approx \frac{1}{\Gamma_F}$

closed-loop gain
Alternative Stability Analysis - cont.

Stable iff

1. \( 20 \log \left| A(jf) \right| - 20 \log \left| \frac{1}{\Gamma_F} \right| > 60 \text{ dB} \)
   
2. \( 20 \log \left| A(jf) \right| - 20 \log \left| \Gamma_F \right| < -60 \text{ dB} \)
   
3. \( \Gamma_F < 0.001 \)
   
=> \( f_1 < f_{180} \)
Alternative Stability Analysis - cont.

\[
A(jf) = \frac{10^5}{(1 +jf/10^5)(1 +jf/10^6)(1 +jf/10^7)}
\]

\[
20 \log |A(jf)| = 20 \log |A(jf_1)| \Gamma_F| = 0
\]

\[
20 \log \left| \frac{A(jf)}{\Gamma_F} \right| = \phi(jf_1) - 180^\circ
\]

\[
\phi(jf_1) = -108^\circ \rightarrow -90^\circ
\]

\[
PM = \phi(jf_1) + 180^\circ = 72^\circ
\]

stable!
Alternative Stability Analysis - cont.

\[ A(jf) = \frac{10^5}{(1 + jf/10^5)(1 + jf/10^6)(1 + jf/10^7)} \]

\[ 20 \log |A(jf)| = 20 \log |A(jf)| - 20 \log \frac{1}{\Gamma_F} \]

\[ GM = \Phi(jf) = -210^\circ \]

\[ PM = \Phi(jf) + 180^\circ = -30^\circ \quad \text{unstable!} \]
Alternative Stability Analysis - cont.

stable iff

\[ \frac{1}{\Gamma_F} > 60 \text{ dB} \]

or

\[ 20 \log \Gamma_F < -60 \text{ dB} \]

or

\[ \Gamma_F < 0.001 \]
Alternative Stability Analysis - cont.

Using “Rule of Thumb”:

\[
\text{arg} \left( A(jf_1) \Gamma_F \right) > -135^\circ
\]

\[
\Rightarrow \quad PM > 45^\circ
\]
Rate of Closure (RC)

\[ RC = \text{slope } A(j2\pi f) \text{ dB/dec} - \text{slope} \left( \frac{1}{\Gamma_F} \right) \text{ dB/dec} \]

\[ RC = -20 \text{ dB/dec} - 0 \text{ dB/dec} = -20 \text{ dB/dec} \]

\[ RC = -20 \text{ dB/dec} - 0 \text{ dB/dec} = -40 \text{ dB/dec} \]

RECALL:
“Rule of Thumb” – Closed-loop amplifier will be stable if \( 20 \log |A(jf)| \) line intersects \( 20 \log |A(jf)| \) curve on the -20 dB/dec segment.

“Equivalent Rule of Thumb” – Closed-loop amplifier will be stable if

\[ RC \geq -20 \text{ dB/dec} \]
Frequency Dependent Feedback

\[ RC = \text{slope} \ A(j2\pi f)_{\text{dB/dec}} - \text{slope} \left( \frac{1}{\Gamma_F} \right)_{\text{dB/dec}} \]

- \[ RC_1 = -20 \text{ dB/dec} - (+20 \text{ dB/dec}) = -40 \text{ dB/dec} \]
- \[ RC_2 = -20 \text{ dB/dec} - (0 \text{ dB/dec}) = -20 \text{ dB/dec} \]
- \[ RC_3 = -40 \text{ dB/dec} - (0 \text{ dB/dec}) = -40 \text{ dB/dec} \]
- \[ RC_4 = -40 \text{ dB/dec} - (-20 \text{ dB/dec}) = -20 \text{ dB/dec} \]

“Equivalent Rule of Thumb” – Closed-loop amplifier will be stable if

\[ RC \geq -20 \text{ dB/dec} \]
**Frequency Compensation**

"Rule of Thumb" – Closed-loop amplifier will be stable if the $20 \log \frac{1}{\Gamma_F}$ line intersects the $20 \log |A(j\omega)|$ curve on the -20 dB/dec segment.

Using “Rule of Thumb”:

$$\arg (A(j\omega) \Gamma_F) > -135^\circ$$

=> $\text{PM} > 45^\circ$

Ex: LM 741 op amp is frequency compensated to be stable with $60^\circ \text{ PM}$ for

$$20 \log |A_{cl}(j\omega)| = 20 \log \frac{1}{\Gamma_F} = 0 \text{ dB}.$$  

frequency compensation – modifying the open-loop $A(s)$ so that the closed-loop $A_{cl}(s)$ is stable for any desired value of $|A_{cl}(j\omega)|$ by extending the -20 dB/dec segment.

desired implementation - minimum on-chip or external components.
Compensation – What if 1st pole is shifted lower?

\[
A(jf) = \frac{10^5}{(1 + jf/10^5)(1 + jf/10^6)(1 + jf/10^7)}
\]

\[
A_{\text{comp}}(jf) \approx \frac{10^5}{(1 + jf/10^3)(1 + jf/10^6)(1 + jf/10^7)}
\]

Graph showing dB vs. frequency with curves at -20 dB/dec, -40 dB/dec, and -60 dB/dec.
Frequency Compensation Using Miller Effect

\[ V_o = \frac{(sC_{comp} - g_m) R_1 R_2}{1 + s[C_1 R_1 + C_2 R_2 + C_{comp}(g_m R_1 R_2 + R_1 + R_2)] + s^2[C_1 C_2 + C_{comp}(C_1 + C_2)] R_1 R_2} \]

\[ = \frac{(sC_{comp} - g_m) R_1 R_2}{(1 + \frac{s}{\omega_{p1c}})(1 + \frac{s}{\omega_{p2c}})} = \frac{(sC_{comp} - g_m) R_1 R_2}{1 + s\left(\frac{1}{\omega_{p1c}} + \frac{1}{\omega_{p2c}}\right) + \frac{s^2}{\omega_{p1c} \omega_{p2c}}} \]

where \( \omega_{p2c} \gg \omega_{p1c} \)

with \( C_{comp} = 0 \)

\[ \omega_{p1} = \frac{1}{R_1 C_1} \]

\[ \omega_{p2} = \frac{1}{R_2 C_2} \]

BJT Miller Capacitance

\[ C_1 = C_{\pi} + C_{\mu}(1 + g_m R_2) \]

\[ R_1 = R_B || r_\pi \]

\[ R_2 = R_C || r_o || R_{in2} \]
Compensation Using Miller Effect - cont.

\[
\frac{V_o}{I_i} = \frac{(sC_{\text{comp}} - g_m) R_1 R_2}{1 + s\left(\frac{1}{\omega_{p1c}} + \frac{1}{\omega_{p2c}}\right) + s^2 \frac{1}{\omega_{p1c} \omega_{p2c}}} 
\approx \frac{(sC_{\text{comp}} - g_m) R_1 R_2}{1 + s\left(\frac{1}{\omega_{p1c}}\right) + s^2 \left(\frac{1}{\omega_{p1c} \omega_{p2c}}\right)}
\]

and

\[
\frac{V_o}{I_i} = \frac{(sC_{\text{comp}} - g_m) R_1 R_2}{1 + s \left[ C_1 R_1 + C_2 R_2 + C_{\text{comp}} (g_m R_1 R_2 + R_1 + R_2) \right] + s^2 \left[ C_1 C_2 + C_{\text{comp}} (C_1 + C_2) \right] R_1 R_2}
\]

\[
\omega_{p1c} = \frac{1}{C_1 R_1 + C_2 R_2 + C_{\text{comp}} (g_m R_1 R_2 + R_1 + R_2)} \approx \frac{1}{C_{\text{comp}} g_m R_2 R_1}
\]

\[
\omega_{p2c} = \frac{\omega_{p1c} \omega_{p2c}}{\omega_{p1c}} = \frac{C_1 R_1 + C_2 R_2 + C_{\text{comp}} (g_m R_1 R_2 + R_1 + R_2)}{[ C_1 C_2 + C_{\text{comp}} (C_1 + C_2) ] R_1 R_2} \approx \frac{C_{\text{comp}} g_m}{C_{\text{comp}} (C_1 + C_2)} \approx \frac{g_m}{C_1 + C_2}
\]

If \( C_{\text{comp}} (C_1 + C_2) \gg C_1 C_2 \)
Compensation Using Miller Effect - cont.

\[
A(j\omega) = \frac{10^5}{\left(1 + \frac{j\omega}{10^5}\right)\left(1 + \frac{j\omega}{10^6}\right)\left(1 + \frac{j\omega}{10^7}\right)} = \frac{10^5}{\left(1 + \frac{j\omega}{f_{p1}}\right)\left(1 + \frac{j\omega}{f_{p2}}\right)\left(1 + \frac{j\omega}{f_{p3}}\right)}
\]

\[
f_{p1} = \frac{\omega_{p1}}{2\pi} = \frac{1}{R_1C_1}
\]

\[
f_{p2} = \frac{\omega_{p2}}{2\pi} = \frac{1}{R_2C_2}
\]

Using Miller effect compensation: \(f_{p1} \rightarrow f_{p1c} << f_{p1}\) and \(f_{p2} \rightarrow f_{p2c} >> f_{p2}\) (pole-splitting)

Also \(f_{p3}\), determined by another stage, is unaffected by the compensation \(\Rightarrow f_{p3c} = f_{p3}\)

\[
A_{comp}(j\omega) = \frac{10^5}{\left(1 + \frac{j\omega}{f_{p1c}}\right)\left(1 + \frac{j\omega}{f_{p2c}}\right)\left(1 + \frac{j\omega}{f_{p3}}\right)}
\]

\[
f_{p1c} = \frac{\omega_{p1c}}{2\pi} \approx \frac{1}{2\pi[C_{comp}g_mR_1R_2]}
\]

\[
f_{p2c} = \frac{\omega_{p2c}}{2\pi} \approx \frac{g_m}{2\pi[C_1+C_2]}
\]
**Miller Compensation Example**

Given:

\[
A(j\omega) = \frac{10^5}{(1 + \frac{j\omega}{R_1 C_1})(1 + \frac{j\omega}{R_2 C_2})(1 + \frac{j\omega}{10^7})} = \frac{10^5}{(1 + \frac{j\omega}{f_{p1}})(1 + \frac{j\omega}{f_{p2}})(1 + \frac{j\omega}{10^7})} = \frac{10^5}{(1 + \frac{j\omega}{10^5})(1 + \frac{j\omega}{10^6})(1 + \frac{j\omega}{10^7})}
\]

where \(C_1 = 100 \text{ pF}, C_2 = 5 \text{ pF}, g_m = 40 \text{ mS}, R_1 = 100/2\pi \text{ k\Omega} \) and \(R_2 = 200/2\pi \text{ k\Omega}\)

Design Objective: determine \(C_{\text{comp}}\) s.t. \(f_{p1c} = 100 \text{ Hz}\) and compute \(f_{p2c}\).

\[
A_{\text{comp}}(j\omega) = \frac{10^5}{(1 + \frac{j\omega}{10^2})(1 + \frac{j\omega}{f_{p2c}})(1 + \frac{j\omega}{10^7})} = \frac{10^5}{(1 + \frac{j\omega}{f_{p1c}f_{p2c}})(1 + \frac{j\omega}{f_{p2c}})(1 + \frac{j\omega}{10^7})}
\]

\[
f_{p1c} = 100 \text{ Hz} \approx \frac{1}{2\pi \left[C_{\text{comp}} g_m R_1 R_2\right]} = \frac{(2\pi)^2}{2\pi \left[C_{\text{comp}} (40 \times 10^{-3})(10^5)(2 \times 10^5)\right]} \Rightarrow C_{\text{comp}} = 78.5 \text{ pF}
\]

\[
f_{p2c} = \frac{g_m}{2\pi (C_1 + C_2)} \approx 61 \text{ MHz}
\]
Compensation Using Miller Effect - cont.

\[ 20 \log |A(jf)| \text{ dB} \]

\[ 20 \log |A_{\text{comp}}(jf)| \]

1-pole roll-off!

-20 dB /dec
-40 dB /dec
-60 dB /dec

\[ 20 \log 1/\Gamma_f = 0 \text{ dB} \]

\[ A_{\text{comp}}(jf) \approx \frac{10^5}{(1 + \frac{jf}{10^5})(1 + \frac{jf}{10^6})(1 + \frac{jf}{10^7})} \]

\[ PM > 45^\circ \text{ for } |A_{\text{cl}}(jf)| = \frac{1}{\Gamma_f} \geq 1 \]
Summary

Feedback has many desirable features, but it can create unexpected - undesired results if the full frequency-dependent nature (phase-shift with frequency) of the feedback circuit is not taken into account.

Feedback can be used to convert a well-behaved stable circuit into an oscillator. Sometimes, due to parasitics, feedback in amplifiers results in unexpected oscillations.

The root-locus method was used to show how feedback can create the conditions for oscillation and instability.