Early Effect & BJT Biasing

- Early Effect
- DC BJT Behavior
- DC Biasing the BJT

Early Effect

Collector voltage has some effect on collector current – it increases slightly with increases in voltage. This phenomenon is called the "Early Effect" and is modeled as a linear increase in total current with increases in v_{CE}:

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} \left(1 + \frac{v_{CE}}{V_A}\right)$$

NMOS transistor

$$\lambda_n = \frac{1}{V_A}$$

$$i_D = i_{DS} \approx \frac{1}{2} k_n' \frac{W}{L} (v_{GS} - V_t)^2 (1 + \lambda_n v_{DS})$$

V_A is called the Early voltage and ranges from about 50 V. to 100 V.

Early Effect - Continued

Total (bias+signal) quantities: $i_C = I_S e^{\frac{v_{BE}}{V_T}} \left(1 + \frac{v_{CE}}{V_A}\right)$

$i_C = I_C + i_c$ $\quad v_{BE} = V_{BE} + v_{be}$ $\quad v_{CE} = V_{CE} + v_{ce}$

Consider dc (bias) condition (signal = 0):

$i_C = I_C$ $\quad v_{BE} = V_{BE}$ $\quad v_{CE} = V_{CE}$

$$i_C = I_C = I_S e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CE}}{V_A}\right) = I'_C \left(1 + \frac{V_{CE}}{V_A}\right)$$

Call the idealized collector bias current I'_C:

$$I'_C = I_S e^{\frac{V_{BE}}{V_T}}$$

Early Effect - Continued

Rearranging slightly:

$$I_C = I_C'\left(1+\frac{V_{CE}}{V_A}\right) = I_C' + \frac{V_{CE}}{\frac{V_A}{I_C'}}$$

We shall define: $r_o = \dfrac{V_A}{I_C'}$

$$I_C' = I_S e^{\frac{V_{BE}}{V_T}}$$

MOS transistor

$$r_o = \frac{V_A}{I_D} \qquad I_D = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2$$

The dc current due to both emitter and collector voltages is:

$$I_C = I_C' + \frac{V_{CE}}{r_o}$$

Early Effect - Continued

Although the bias current including the Early effect is better modeled as:

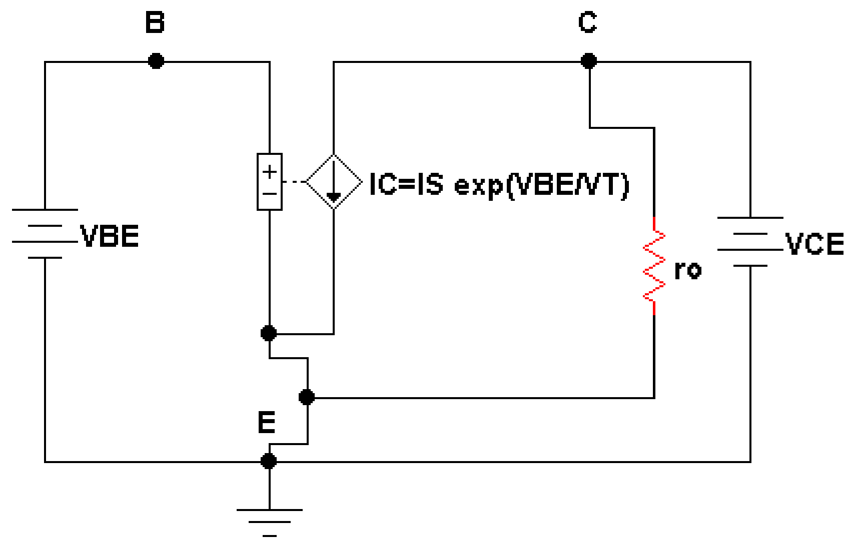
$$I_C = I_C^{'} + \frac{V_{CE}}{r_o}$$

We – almost always – will ignore the second term above and use our usual expression for the bias current:

$$\boxed{I_C \approx I_C^{'} = I_S e^{\frac{V_{BE}}{V_T}}}$$

Early Effect - Continued

The Early term adds r_o to the large signal model:



Early Effect - Continued

For typical operating conditions:

$$V_A \approx 50 - 100\,V.$$

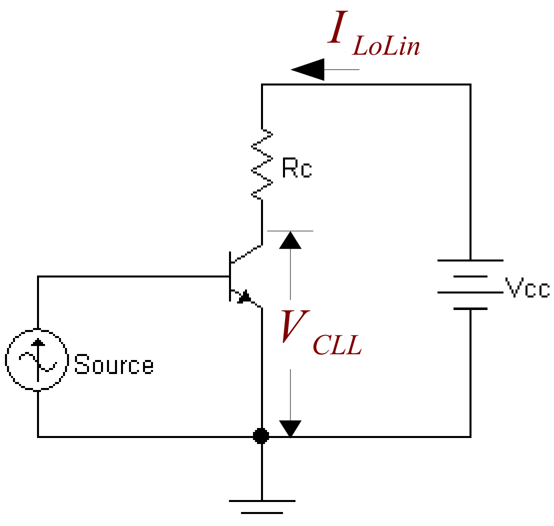
$$I_C \approx 1\,mA.$$

$$r_o = \frac{V_A}{I_C} \approx \frac{100}{10^{-3}} = 100\,k\,\Omega$$

We usually can ignore r_o since, in practice, r_o is in parallel with other resistors, which are much smaller than $100\,k\,\Omega$. For the time being, you will be told if you must include r_o in your pencil-and-paper circuit designs.

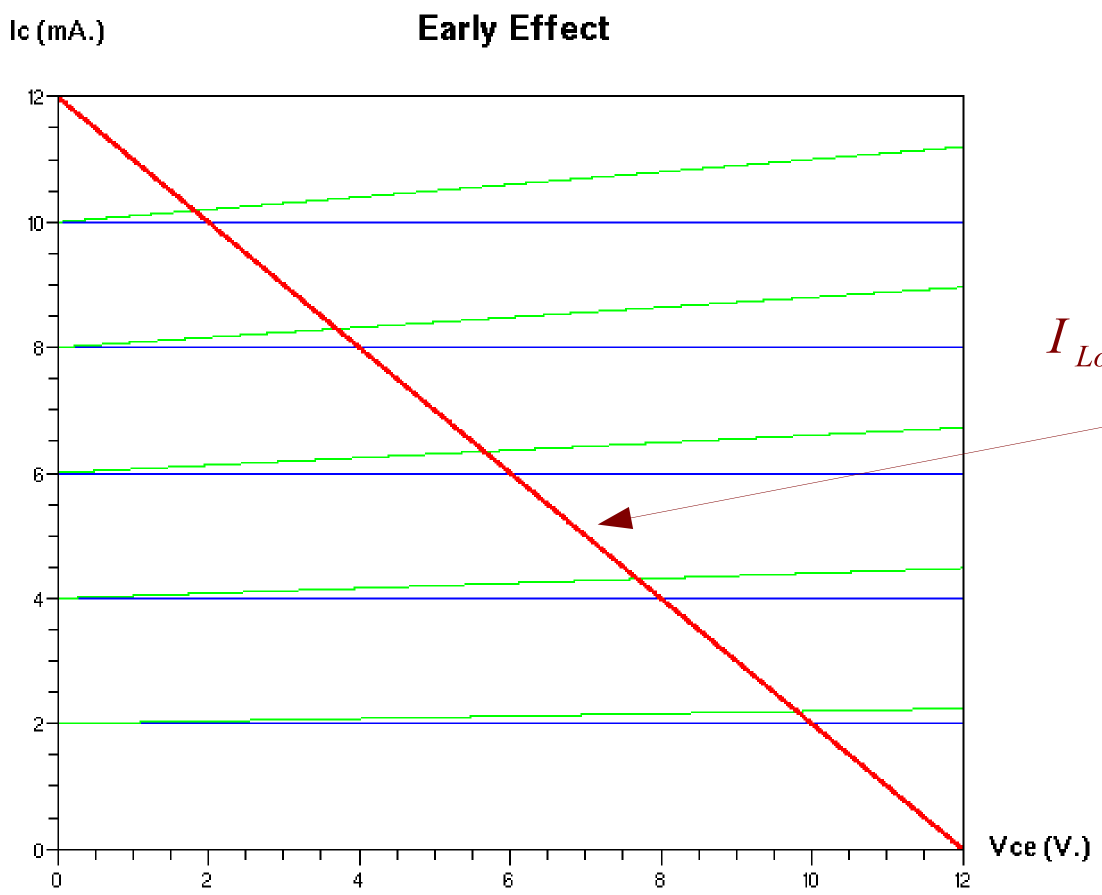
Early Effect Scilab Simulation

```
VsubA=100;
vCE=0.01:0.01:12; //array 0.01 to 12 in .01 V. steps
for ICprime = 2:2:10 //mA.
  ICp = ICprime*sign(vCE); //ICp value for each vCE one
  plot(vCE,ICp);    //Current in mA.
  ro=VsubA/ICprime;//Add bias Early effect
  IC=ICp+vCE/ro;
  plot(vCE,IC)
end
//Plot load line for 1 KOhm Rc - 12V Vcc
Rc=1;
Gc=1/Rc;
Vcc=12;
vCLL = 0:0.01:12;
ILoLin=Gc*Vcc-Gc*vCLL;
plot(vCLL,ILoLin)
```



$$I_{LoLin}=\frac{1}{R_C}(V_{CC}-V_{CLL})$$

Simulation Results



$$I_{LoLin}=\frac{1}{R_C}(V_{CC}-V_{CLL})$$

Active Mode Conditions

Base-emitter diode forward biased:

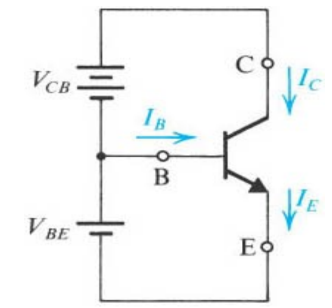
$$v_{BE} \geq 0.7\,V$$

Base-collector diode back biased:

$$v_{BC} = v_{BE} - v_{CE} \leq 0.5\,V$$

$$-v_{CE} \leq 0.5 - v_{BE} \Rightarrow v_{CE} \geq 0.2\,V$$

$$v_{CE} \geq 0.2\,V$$



Forward-Active (ideal cond.)
$V_{BE} > 0$
$V_{BC} < 0$
$i_E = i_C + i_B$
$v_{CE} = v_{CB} + v_{BE}$

Amplifier Biasing Goals

We wish to set a stable value of I_C so that we can apply a signal voltage or current signal to the emitter-base circuit and obtain an amplified (undistorted) version of the signal between the collector and ground.

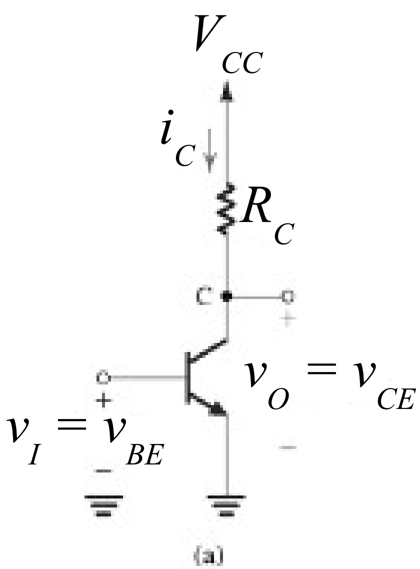
The transistor cannot *saturate* during operation, i.e.

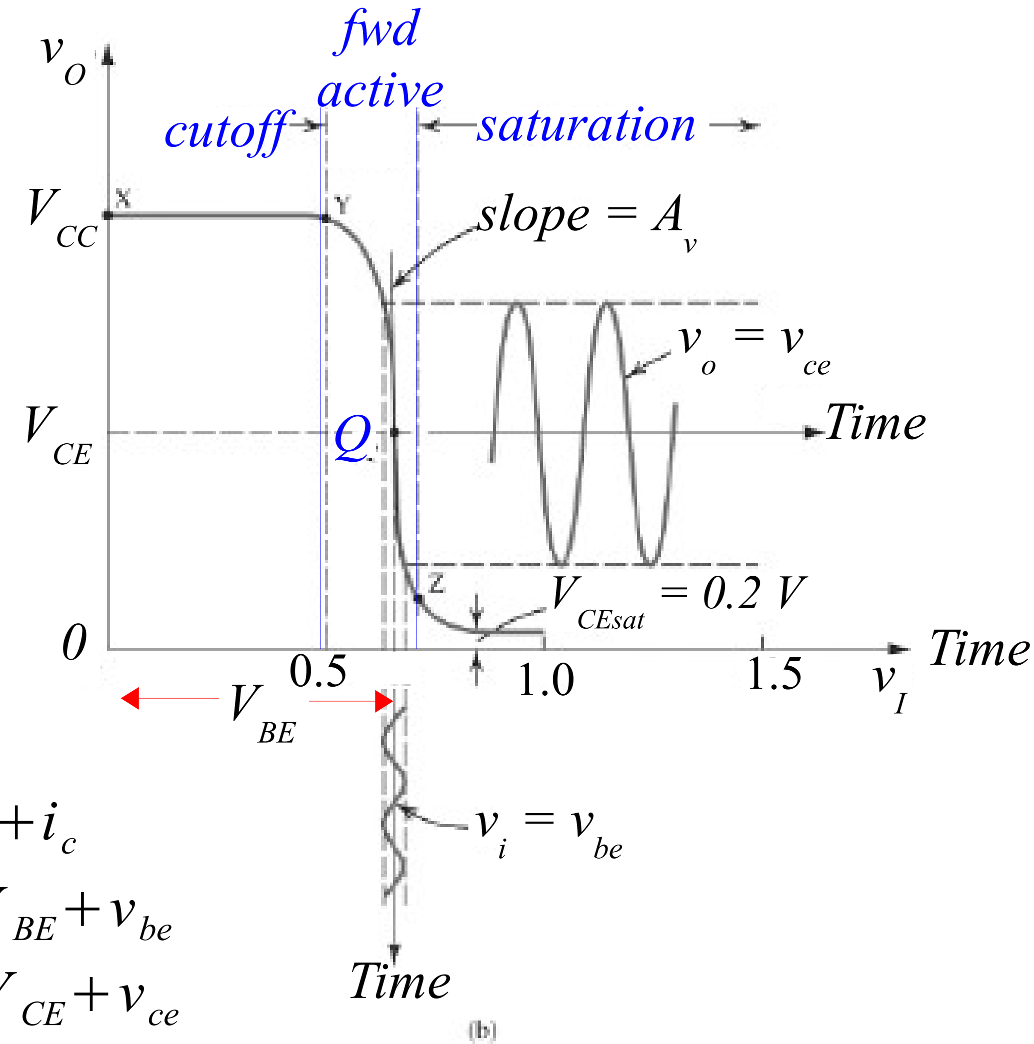
$$v_{CE} > 0.2\,V.$$

And it cannot *cut off* during operation, *i.e.*

$$i_C > 0\,mA.$$

Amplifier DC Bias Problem



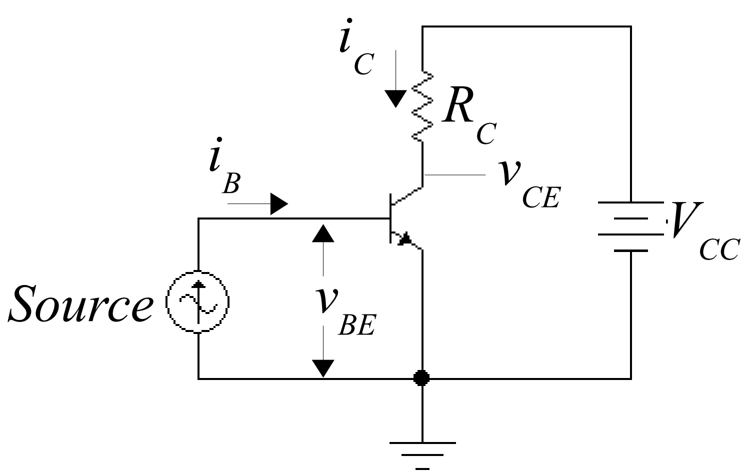


$$i_C = I_C + i_c$$

$$v_{BE} = V_{BE} + v_{be}$$

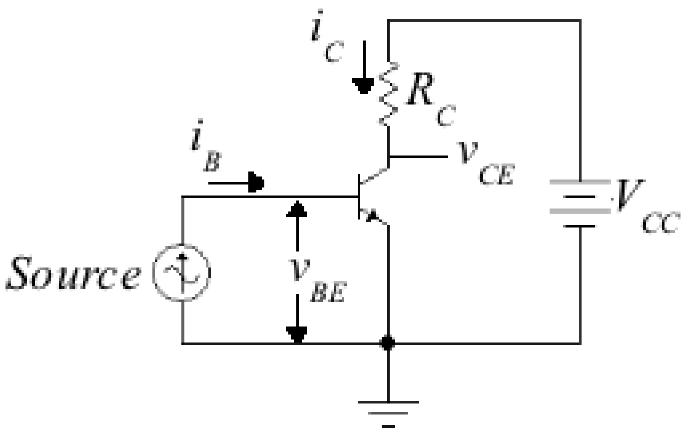
$$v_{CE} = V_{CE} + v_{ce}$$

Amplifier Action



- **Base current source:**
  - A small change in base current results in a large collector current (βi_b).
  - This yields a large change in collector voltage.
- **Base voltage source:**
  - A small change in base voltage results in a large change in collector current ($i_C = I_S \, exp(v_{BE}/V_T)$).
  - This yields a large change in collector voltage.

Voltage Source Input With Collector Load



Solution of the simultaneous equations exists where the two curves: the exponential (i_C, v_{BE}) and the straight line (i_C, v_{CE}) intersect:

$$\frac{V_{CC} - v_{CE}}{R_C} = I_S e^{\frac{v_{BE}}{V_T}}$$

$$i_C = I_S e^{\frac{v_{BE}}{V_T}}$$ BJT

$$i_C = \frac{V_{CC} - v_{CE}}{R_C}$$ Circuit

Load Line

Scilab Plot of NPN Characteristic

```
//Calculate and plot npn BJT collector
//characteristic using active mode model
VsubT=0.025;
VTinv=1/VsubT;
IsubS=1E-14;
vCE=0:0.01:10;
for vBE=0.58:0.01:0.63
  iC=IsubS*exp(VTinv*vBE);
  plot(vCE,1000*iC);    //Current in mA.
end
Vcc=10;
Rc=10000;
vLoad=0:0.01:10;
iLoad=(Vcc-vLoad)/Rc;
plot(vLoad,1000*iLoad);
```

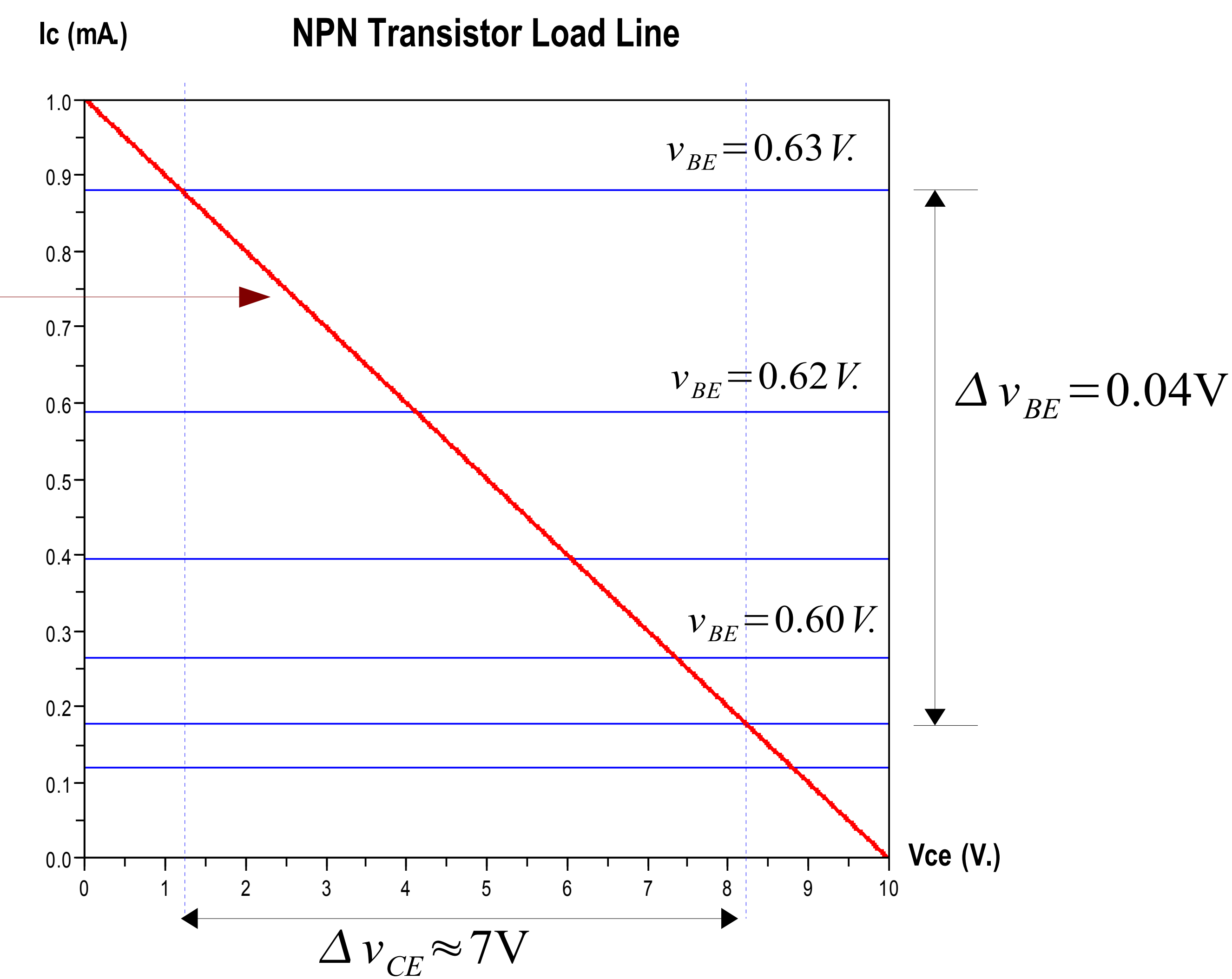
Plot Output

Load Line

$$i_C = \frac{V_{CC} - v_{CE}}{R_C}$$

$$V_{CC} = 10\text{V}$$

$$R_C = 10\text{k}\,\Omega$$



Amplifier Action

Note that as v_{BE} varies from about 0.59 V to O.63 V, v_{CE} varies from about 8 V to 1V!

A 0.04 V peak-to-peak swing of v_{BE} results in an 7 V peak-to-peak swing in v_{CE} - A ratio of 7/0.04, or about 175.

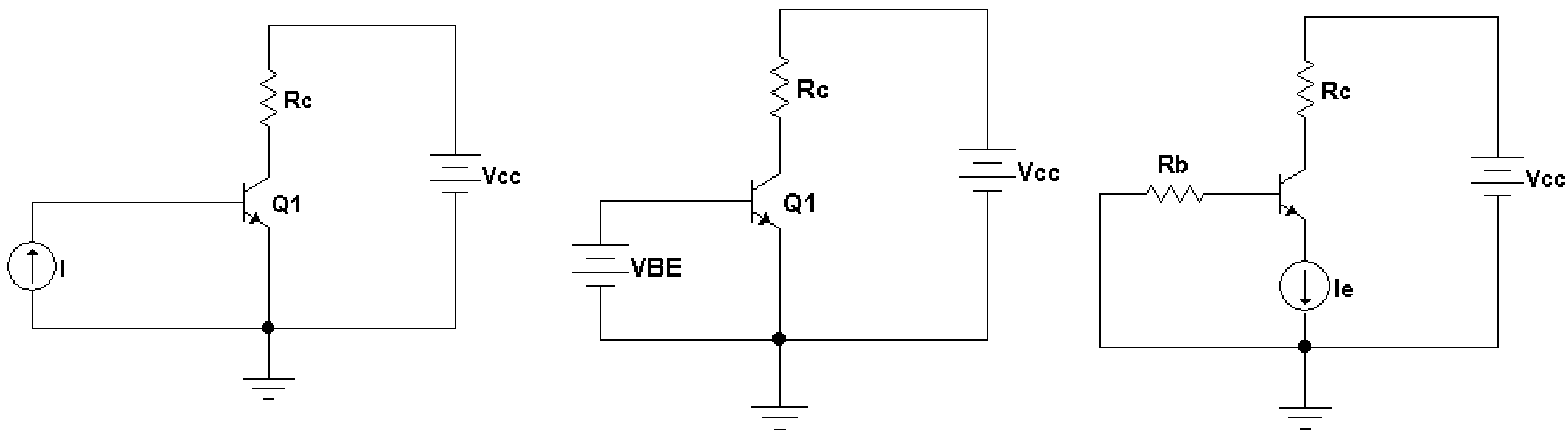
The output magnitude is about 175 times the input magnitude, for a gain of 175.

The input signal has two components: a DC one called the *bias voltage*, and an AC one called the (*small*) *signal voltage*. For proper operation, let:

$$V_{BIAS} = (V_{BE(MAX)} + V_{BE(MIN)})/2 = 0.61\,V$$

$$v_{SIGNAL} = (V_{BE(MAX)} - V_{BE(MIN)})/2 = 0.02\,V$$

Candidate Bias Configurations

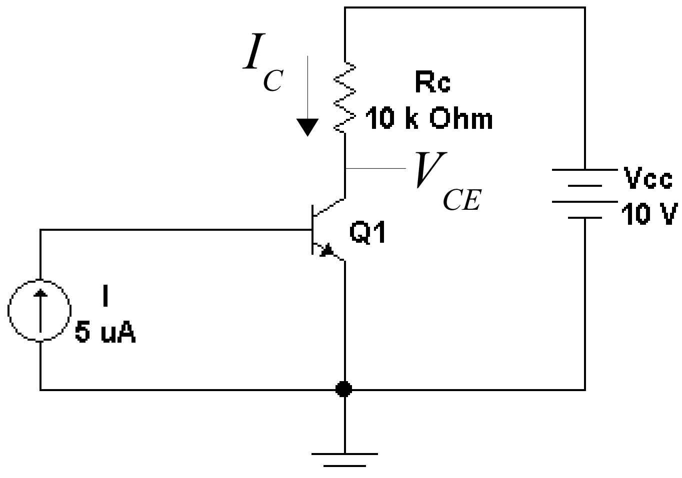


Base current source

Base voltage source

Emitter current source

Drive Base With a Current Source



For this collector current:

$$V_{CE}=V_{CC}-R_C I_C$$

$$V_{CE}=10-10^4\cdot 0.5\cdot 10^{-3}=5\,V$$

Assume: $\beta=100$

$$I_C=\beta I_B=100\cdot 5\cdot 10^{-6}$$

$$I_C=0.5\,mA.$$

The transistor is almost right in the center of the desired operating region!

Current Bias Beta Dependence

Unfortunately, beta is very poorly controlled and may easily vary from 50 to 200. And beta is also temperature dependent!

For beta of 50:

$$I_C = 50 \cdot 5 \cdot 10^{-6} = 0.25\, mA.$$

The device with a V_{CE} = 7.5 V is close to the I_C cutoff point.

For beta of 200:

$$I_C = 200 \cdot 10 \cdot 5^{-6} = 1.0\, mA.$$

The device is saturated, V_{CE} tries to be 0 V!

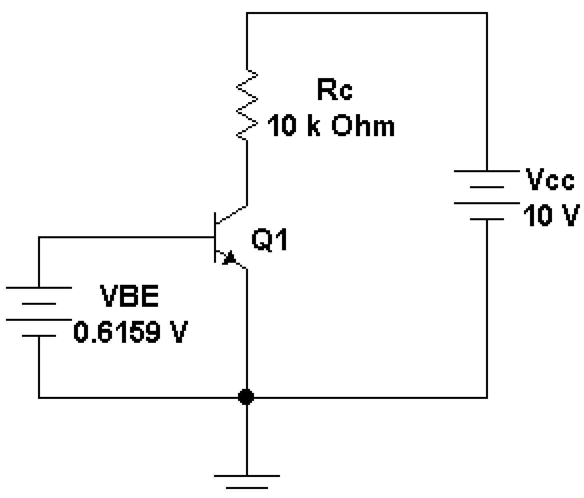
$$V_{CE} = V_{CC} - R_C I_C$$

$$V_{CE} = 10 - 10^4 \cdot 0.25 \cdot 10^{-3} = 7.5\, V$$

$$V_{CE} = 10 - 10^4 \cdot 1 \cdot 10^{-3} = 0\, V$$

There are huge problems associated with trying to drive a BJT with a base current source! BIAS POINT IS UNSTABLE.

Drive base with a voltage source



$$I_C = I_S e^{\frac{V_{BE}}{V_T}}$$

For an I_C of 0.5 mA:

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$$

Given: $I_S = 10^{-14} A$

and: $I_C = 0.5 \cdot 10^{-3} A$

$$V_{BE} = 0.025 \ln\left(0.5 \cdot 10^{11}\right)$$

$$V_{BE} = 0.025 \cdot 24.635 = 0.6159\, V$$

Again, the transistor is nearly at the center of the desired operating region!

OK. Apply 0.6159 volts to the base and we have the desired collector current!

Voltage Bias I_S and V_{CE} Dependence

Unfortunately, I_S is highly temperature-dependent, doubling for a 5 degree C increase in temperature. If the base-emitter voltage is chosen to give 0.5 mA collector current at 20 degrees C (68 F), it will double at 25 C and halve at 15 C.

I_C is also highly sensitive to V_{BE}. Consider two values of collector current, I_C and $10I_C$:

$$\frac{10\,I_C}{I_C}=\frac{I_S e^{\frac{V_{BE10}}{V_T}}}{I_S e^{\frac{V_{BE1}}{V_T}}}$$

$$V_{BE10}-V_{VE1}=V_T\ln(10)$$

$$V_{BE10}-V_{BE1}=0.025\cdot 2.3025=0.0576\,V.$$

Less than a 60 mV change in VBE voltage increases IC by an order of magnitude (10X). BIAS POINT IS UNSTABLE.

Emitter Current Source

This holds collector current close to its desired value since:

$$I_C = \alpha I_E$$

Changes in I_C for α in the range determined by the extremes of β are negligible

$$50 < \beta < 200 \Rightarrow \frac{50}{51} < \alpha < \frac{200}{201}$$

There is considerable variation in base current, however, but this is usually of no consequence.

$$I_B = \frac{I_E}{\beta + 1} \Rightarrow \frac{I_E}{51} < I_B < \frac{I_E}{201}$$

Conclusion

Biasing a BJT poses large stability problems, since its characteristics are highly sensitive to temperature and since its electrical properties (principally beta) vary widely from one device to another!

The next lecture will cover some techniques for stabilizing BJT operation.