Early Effect & BJT Biasing

- Early Effect
- DC BJT Behavior
- DC Biasing the BJT
Early Effect

Ideal NPN BJT Transfer Characteristic

<table>
<thead>
<tr>
<th>Mode</th>
<th>$V_{BE}$</th>
<th>$V_{BC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward-Active</td>
<td>$&gt; 0$</td>
<td>$\leq 0$</td>
</tr>
<tr>
<td>Reverse-Active</td>
<td>$\leq 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>Cutoff</td>
<td>$\leq 0$</td>
<td>$\leq 0$</td>
</tr>
<tr>
<td>Saturation</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
</tbody>
</table>

$V_{BC} = -V_{CB}$

Not Useful!
Early Effect - Continued

Collector voltage has some effect on collector current – it increases slightly with increases in voltage. This phenomenon is called the “Early Effect” and is modeled as a linear increase in total current with increases in $v_{CE}$:

$$i_C = I_S e^{\frac{v_{BE}}{V_T}} \left( 1 + \frac{v_{CE}}{V_A} \right)$$

is called the Early voltage and ranges from about 15 to 150 V.

$$V_A = \frac{\Delta V_{CE}}{V \Delta I_C} I_C$$

NMOS transistor

$$i_D = i_{DS} \approx \frac{1}{2} k_n \frac{W}{L} (v_{GS} - V_t)^2 (1 + \lambda_n v_{DS})$$

$$\lambda_n = \frac{1}{V_A}$$
Early Effect - Continued

Observed by James Early from BTL

15 V \leq V_A \leq 150 V
**Early Effect - Continued**

Total (bias+signal) quantities:

\[ i_C = I_S e^{\frac{V_{BE}}{V_T}} \left( 1 + \frac{V_{CE}}{V_A} \right) \]

\[ i_C = I_C + i_c \quad v_{BE} = V_{BE} + v_{be} \quad v_{CE} = V_{CE} + v_{ce} \]

Consider dc (bias) condition (signal = 0):

\[ i_C = I_C \quad v_{BE} = V_{BE} \quad v_{CE} = V_{CE} \]

\[ I_C = I_S e^{\frac{V_{BE}}{V_T}} \left( 1 + \frac{V_{CE}}{V_A} \right) = I_C' \left( 1 + \frac{V_{CE}}{V_A} \right) \]

Let's call the idealized collector bias current (no Early Effect) \( I_C' \), i.e.

\[ I_C' = I_S e^{\frac{V_{BE}}{V_T}} \]
We shall define:

\[ r_o = \frac{V_A}{I'_C} \]

\[ I'_C = I_S e^{\frac{V_{BE}}{V_T}} \Rightarrow r_o = f(V_{BE}) \]

The dc current due to both \( V_{BE} \) and \( V_{CE} \) is:

\[ I_C = I'_C + \frac{V_{CE}}{r_o} \]
Early Effect - Continued

Although the bias current is better modeled by including the Early effect

\[ I_C = I'_C + \frac{V_{CE}}{r_o} \]

We – almost always – will ignore the second term above in hand calculations and use our ideal expression for the bias current:

\[ I_C \approx I'_C = I_S e^{\frac{V_{BE}}{V_T}} \]
Early Effect - Continued

The Early term adds $r_o$ to the large signal model:

\[
V_{CE} = (I_C - I'_C) r_o
\]

\[
I_C = I'_C + \frac{V_{CE}}{r_o}
\]
Early Effect - Continued

For typical operating conditions:

\[ V_A \approx 50 \text{ } to \text{ } 100 \text{ } V. \]

\[ I_C' \approx 1 \text{ } mA. \]

\[ r_o = \frac{V_A}{I_C'} \approx \frac{100 \text{ } V}{10^{-3} \text{ } A} = 100 \text{ } k\Omega \]

We usually can ignore \( r_o \) since, in practice, \( r_o \) is in parallel with other resistors, which are much smaller than 100 kΩ. For the time being, you will be specifically told if you must include \( r_o \) in your circuit analyses and designs.
Simulation Results

**Early Effect**

\[ \text{slope} = -\frac{1}{R_C} \]

\[ I_C = \frac{1}{R_C} (V_{CC} - V_{CE}) \]

Note: \( r_o \) is in parallel with \( R_C \).

\( r_o \neq \infty \)

\( r_o = \infty \)

load-line
dictated by circuit

\( V_{CE} \) (V)

\( I_C \) (mA)
Active Mode Conditions

Base-emitter diode forward-biased:

\[ V_{BE} \geq 0.7 \, V \]

Base-collector diode reverse-biased:

\[ V_{BC} = V_{BE} - V_{CE} \leq 0.5 \, V \]

\[-V_{CE} \leq 0.5 - V_{BE} \Rightarrow V_{CE} \geq 0.2 \, V \]

\[ V_{CE} \geq 0.2 \, V \]
**Amplifier Biasing Goals**

We wish to set a stable value of $I_C$ so that we can apply a signal voltage or signal current to the emitter-base circuit and obtain an amplified (undistorted) version of the signal between the collector and ground.

The transistor cannot *saturate* during operation, i.e.

$$v_{CE} > 0.2 \, V.$$  

And it cannot *cut off* during operation, *i.e.*

$$i_C > 0 \, mA.$$
Amplifier DC Bias Problem

\[ i_C = I_C + i_c \]
\[ v_{BE} = V_{BE} + v_{be} \]
\[ v_{CE} = V_{CE} + v_{ce} \]
Amplifier Action

- **Base current source:**
  - A small ac change in base current results in a large ac collector current \((\beta i_b)\).
  - This yields a large change in the ac collector voltage \(v_{ce}\).

- **Base voltage source:**
  - A small ac change in base voltage results in a large change in the ac collector current \((i_c = I_S \exp(v_{be}/V_T))\).
  - This yields a large change in the ac collector \(v_{ce}\) voltage.
Voltage Source Input With Collector Load

Solution of the simultaneous equations exists where the two curves: the exponential \((i_C, v_{BE})\) and the straight line \((i_C, v_{CE})\) intersect:

\[
\begin{align*}
i_C &= I_S e^{\frac{v_{BE}}{V_T}} \\
i_C &= \frac{V_{CC} - v_{CE}}{R_C}
\end{align*}
\]

BJT

Circuit

Load Line
Scilab Plot of NPN Characteristic

//Calculate and plot npn BJT collector
//characteristic using active mode model
VT=0.025;
VTinv=1/VsubT;
IsubS=1E-14;
vCE=0:0.01:10;
for vBE=0.58:0.01:0.63
    iC=IsubS*exp(VTinv*vBE);
    plot(vCE,1000*iC); //Current in mA.
end
VCC=10;
Rc=10000;
vLoad=0:0.01:10;
iLoad=(VCC-vLoad)/Rc;
plot(vLoad,1000*iLoad);
Plot Output $i_C \ (mA)$

NPN Transistor Load Line

$\Delta v_{BE} = 0.04V$

$V_{CC} = 10V$

$R_C = 10k \ \Omega$

$\Delta v_{CE} \approx 7V$

$v_{BE} = 0.63V.$

$v_{BE} = 0.62V.$

$v_{BE} = 0.60V.$

$k$
Amplifier Action

Note that as $v_{BE}$ varies from about $0.59\,V$ to $0.63\,V$, $v_{CE}$ varies from about $1\,V$ to $8\,V$!

A $0.04\,V$ peak-to-peak swing of $v_{BE}$ results in an $7\,V$ peak-to-peak swing in $v_{CE}$ - a voltage-gain ratio of $7/0.04$, or about $175$.

The input signal has two components: a dc one called the bias voltage, and an ac one called the (small) signal voltage. For proper operation, let:

$$v_{BE} = V_{BIAS} = \left( v_{BE(MAX)} + v_{BE(MIN)} \right)/2 = 0.61\,V$$

$$v_{be} = v_{signal} = \left( v_{BE(MAX)} - v_{BE(MIN)} \right)/2 = 0.02\,V\,peak$$
**Candidate Bias Configurations**

- **Base current source**
- **Base voltage source**
- **Emitter current source**
Drive Base With a Base Current Source

For this collector current:

\[ V_{CE} = V_{CC} - R_C I_C \]
\[ V_{CE} = 10 - 10^4 \cdot 0.5 \cdot 10^{-3} = 5 \text{ V} \]

Assume: \( \beta = 100 \)

\[ I_C = \beta I_B = 100 \cdot 5 \cdot 10^{-6} \]

\[ I_C = 0.5 \text{ mA}. \]

The transistor is almost right in the center of the desired operating region!
Current Bias Beta Dependence

Unfortunately, $\beta$ is often poorly controlled and may easily vary from 100 to 200. And $\beta$ is also temperature dependent!

For $\beta = 100$:
\[ I_C = 100 \cdot 5 \cdot 10^{-6} = 0.5 \, mA. \]

\[ V_{CE} = V_{CC} - R_C I_C \]

\[ V_{CE} = 10 - 10^4 \cdot 0.5 \cdot 10^{-3} = 5 \, V \]

The BJT with a $V_{CE} = 5 \, V$

For $\beta = 200$:
\[ I_C = 200 \cdot 10 \cdot 5^{-6} = 1.0 \, mA. \]

\[ V_{CE} = 10 - 10^4 \cdot 1 \cdot 10^{-3} = 0 \, V \]

The BJT is saturated!

Base current source biasing $\rightarrow$ BIAS POINT IS UNSTABLE.
Drive Base with a Base Voltage Source

Given: \( I_S = 10^{-14} \ A \)

and: \( I_C = 0.5 \cdot 10^{-3} \ A \)

\[
V_{BE} = 0.025 \ln \left( 0.5 \cdot 10^{11} \right)
\]

\[
V_{BE} = 0.025 \cdot 24.635 = 0.616 \ V
\]

Since \( V_{CE} = 5 \ V \) the transistor is nearly at the center of the desired operating region!

OK. Apply 0.616 volts to the base and we have the desired collector current!
Voltage Bias $I_S$ and $V_{CE}$ Dependence

Unfortunately, $I_S$ is highly temperature-dependent, doubling for every 5$^\circ$C increase in temperature.

If the base-emitter voltage is chosen to give $I_C = 0.5 \text{ mA}$ at 20$^\circ$C (68$^\circ$F), it will be 2x at 25$^\circ$C and 0.5x at 15$^\circ$C.

$I_C$ is also highly sensitive to $V_{BE}$. Consider two values $I_C$ and $10I_C$:

\[
\frac{10I_C}{I_C} = \frac{V_{BE10}}{V_T} \quad \frac{I_S e^{V_{BE1}}}{V_T} = V_T \ln(10)
\]

\[
V_{BE10} - V_{BE1} = 0.025 \cdot 2.3025 = 0.058 \text{ V}.
\]

Less than a 60 mV change in $V_{BE}$ voltage increases $I_C$ by an order of magnitude (10X). BIAS POINT IS UNSTABLE.
Emitter Current Source

This holds collector current close to its desired value since:

\[ I_C = \alpha I_E \]

Changes in \( I_C \) due to variations in \( \alpha \) in the range determined by the extremes of \( \beta \) are negligible, i.e.

\[
100 < \beta < 200 \Rightarrow \frac{100}{101} < \alpha < \frac{200}{201} \Rightarrow 0.990 < \alpha < 0.995
\]

\[
\alpha = \frac{\beta}{1 + \beta}
\]

There is considerable variation in base current, however, but this is usually of no consequence.

\[
I_B = \frac{I_E}{\beta + 1} \Rightarrow \frac{I_E}{101} < I_B < \frac{I_E}{201}
\]
Conclusion

Biasing a BJT poses potential large bias stability problems, since its characteristics are highly sensitive to temperature and since its electrical properties (principally $\beta$) can vary widely from one device to another!

The next lecture sequence will cover some techniques for stabilizing the BJT bias.