University of Pennsylvania
Department of Electrical and System Engineering
Medical Devices Lab

ESE3400, Fall 2022 — Quiz 2: 60 minutes — Monday, November 21

- 4 Problems with weightings shown. All 4 problems must be completed.
- Calculators allowed.
- 8.5"x11" note sheet allowed

Name: Answers

Grade:

Q1	
Q2	
Q3	
Q4	
Total	

Complex Identities ($j = \sqrt{-1}$):

$$e^{j(\omega \pm 2\pi)} = e^{j\omega}$$

$$e^{j \cdot 0} = 1$$
$$e^{j \cdot \frac{\pi}{2}} = j$$
$$e^{j \cdot \pi} = -1$$
$$e^{j \cdot \frac{3\pi}{2}} = -j$$

Trigonometric Identities:

$e^{j\Theta} = cos(\Theta) + jsin(\Theta)$	$cos(\Theta) = \frac{1}{2}(e^{j\Theta} + e^{-j\Theta})$	$sin(\Theta) = \frac{1}{2j}(e^{j\Theta} - e^{-j\Theta})$

Geometric Series:

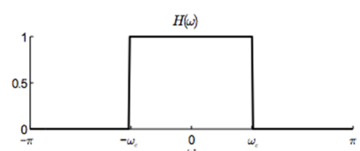
$$\sum_{n=0}^{N} r^n = \frac{1-r^{N+1}}{1-r}$$
$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, |r| < 1$$

DTFT Equations:

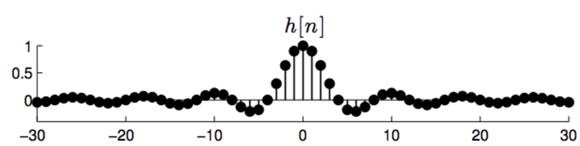
$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

Ideal Low Pass Filter:

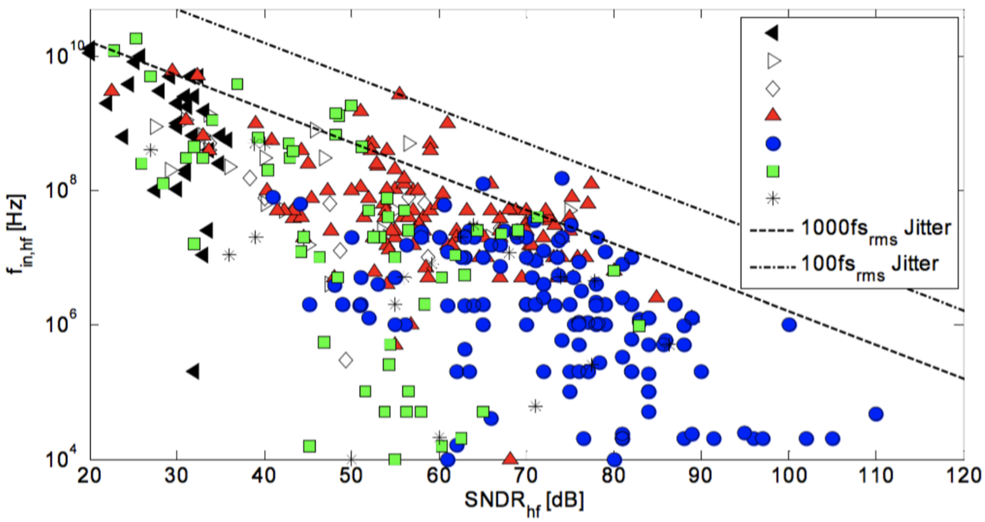
$$H(\omega) = \begin{cases} 1 & -\omega_c \le \omega \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$



$$h[n] \;=\; 2\omega_c \frac{\sin(\omega_c n)}{\omega_c n}$$



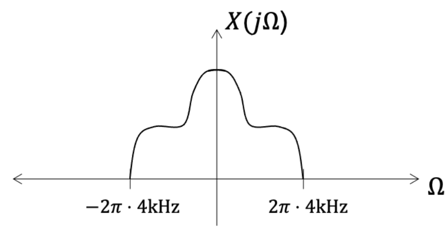
1. ADC (18 points). Below is the ADC survey data of journal published ADC state-of-the-art designs shown in class, where the different colored shapes represent different ADC architecture types. The three architectures we talked about in class are the Flash, Successive Approximation Register (SAR), and ΔΣ (Delta-Sigma) ADC architectures.



Match these three architecture type with each symbol below:

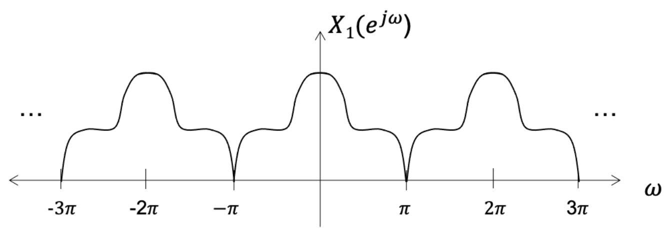
Black Triangle ◄	Flash
Blue Circle ●	ΔΣ
Green Square ■	SAR

2. Sampling (28 points). Consider a continuous time signal, $x(t)$, with the frequency spectrum, $X(j\Omega)$ shown below (the Ω-axis is in units of radians per second):

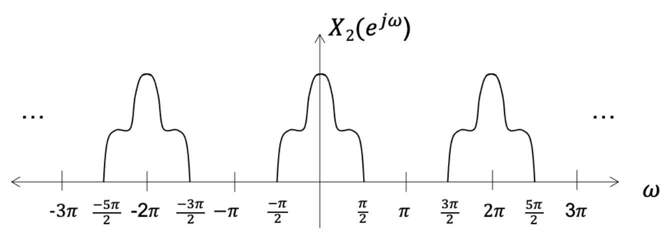


(a) What is the Nyquist rate, Ω_{Nyq}, in radians per second?
$\Omega_{Nyq} = 2\pi \cdot 4kHz \cdot 2 = 16000\pi$

(b) If we sampled at the Nyquist rate, we would get the discrete time signal, $x_1[n]$, with the frequency spectrum, $X_1(e^{j\omega})$, below:



However, there was an error and instead it was sampled at some other rate, Ω_S, resulting in the discrete time signal $x_2[n]$ with the following frequency spectrum:



What is Ω_S in radians per second?

$\Omega_S = 32000\pi$

(c) Is there aliasing with this new sampling rate?

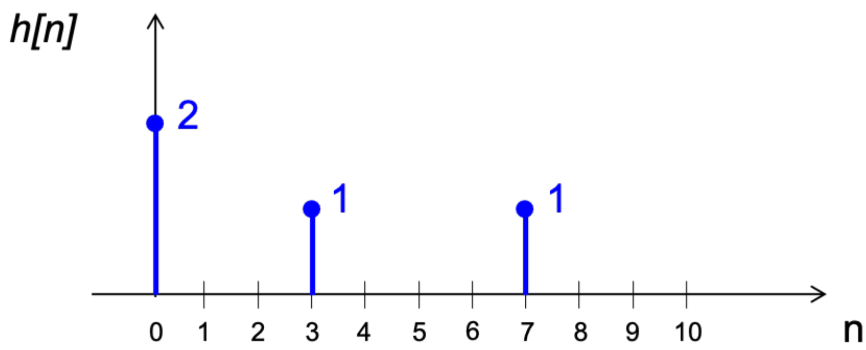
No. There is no corruption from overlap of the original frequency spectrum.

3. LTI Systems (36 points). Consider the following definition of an LTI system with an input, $x[n]$, and output, $y[n]$:

$$y[n] = 2x[n] + x[n-3] + x[n-7] \tag{1}$$

(a) What is the impulse response, $h[n]$, of the LTI system? Give the expression for $h[n]$ and plot it on the discrete-time axis, n.

$h[n] = 2\delta[n] + \delta[n-3] + \delta[n-7]$



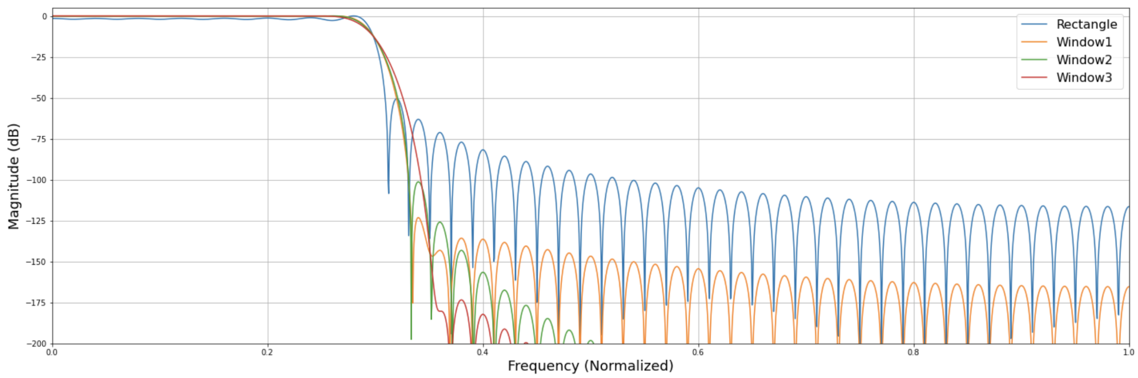
(b) Derive an expression for the frequency response, $H(e^{j\omega})$, of the LTI system.

$$\begin{aligned} H(e^{j\omega}) &= \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k} \\ &= 2 \cdot e^{-j\omega \cdot 0} + e^{-j\omega \cdot 3} + e^{-j\omega \cdot 7} \\ &= 2 + e^{-j3\omega} + e^{-j7\omega} \\ &= 2 + e^{-j5\omega}(e^{j2\omega} + e^{-j2\omega}) \\ &= 2 + e^{-j5\omega}(2cos(2\omega)) \\ &= 2 + 2e^{-j5\omega}cos(2\omega) \end{aligned}$$

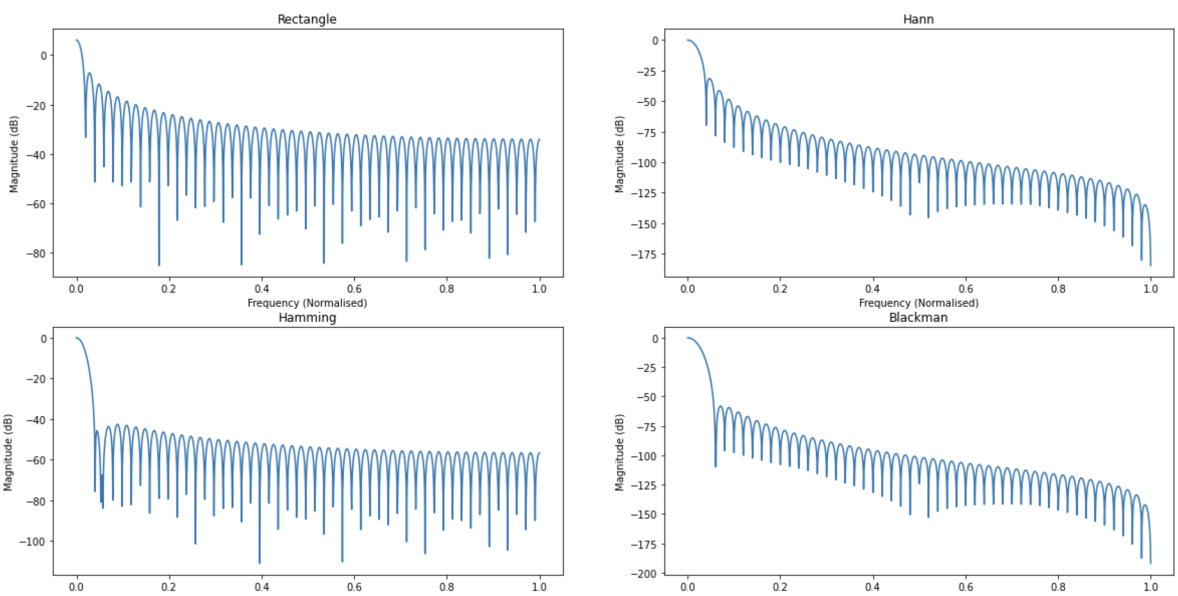
(c) An input, $x[n] = e^{j\frac{\pi}{2}n}$, results in an output $y[n]$. Use part (b) to compute the output $y[n]$.

$$\begin{aligned}
y[n] &= H(e^{j\omega})|_{\omega=\frac{\pi}{2}} \cdot e^{j\frac{\pi}{2}n} \\
&= \left[2 + 2e^{-j5\left(\frac{\pi}{2}\right)} cos\left(2\left(\frac{\pi}{2}\right)\right)\right] \cdot e^{j\frac{\pi}{2}n} \\
&= (2 + 2e^{-j\frac{5\pi}{2}})e^{j\frac{\pi}{2}n} \\
&= (2 - 2j)e^{j\frac{\pi}{2}n} \\
&= 2\sqrt{2}e^{-j\frac{\pi}{4}}e^{j\frac{\pi}{2}n} \\
&= 2\sqrt{2}e^{j\left(\frac{\pi}{2}n - \frac{\pi}{4}\right)}
\end{aligned}$$

4. Filters (18 points). Below is the magnitude of the frequency response of 4 different low pass filters (LPFs) created by windowing the sinc impulse response of the ideal LPF. (See equation sheet for more detail on the ideal LPF).



The LPF filter frequency response from using a rectangular window is labeled, and the other three filters are from applying 3 different windows to the sinc function: Blackman, Hamming, and Hann. All filters and windows are the same length. The frequency response of the four windows functions are given below. Note the different scales on the y-axis.



Match the window type used to design the LPF filters in the table below

Window 1	Hamming
Window 2	Hann
Window 3	Blackman