University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

ESE531, Spring 2017 Final Wednesday, May 3

- 4 Problems with point weightings shown. All 4 problems must be completed.
- Calculators allowed.
- Closed book = No text allowed.
- One two-sided 8.5x11 cheat sheet allowed.
- # Final answers here on the question sheet.
- Additional workspace in "blue" book. Note where to find work in "blue" book if relevant.
- Sign Code of Academic Integrity statement at back of "blue" book.

Name:

Grade:

Q1	
Q2	
Q3	
Q4	
Total	

Common z-transform pairs:

Sequence		z-transform
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\|z\|>1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\|z\|<1$
$\delta[n-m]$	z^{-m}	All z except 0 (if m>0) or ∞ (if m<0)
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\|z\|>\|a\|$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\|z\|<\|a\|$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|>\|a\|$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\|z\|<\|a\|$

Common DTFT pairs:

Sequence	DTFT
$\delta[n]$	1
$u[n]$	$\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k)$
1	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega+2\pi k)$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega-\omega_0+2\pi k)$
$\alpha^n u[n], \|\alpha\|<1$	$\frac{1}{1-\alpha e^{-j\omega}}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega})=\begin{cases}1, & \|\omega\|\le\omega_c\\ 0, & \omega_c<\|\omega\|<\pi\end{cases}$

Trigonometric Identity:

$$e^{jA} = cos(A) + jsin(A)$$

Geometric Series:

$$\sum_{n=0}^{N} r^n = \frac{1-r^{N+1}}{1-r}$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, |r| < 1$$

DTFT Equations:

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

DFT Equations:

N-point DFT of $\{x[n], n = 0, 1, ..., N-1\}$ is $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$, for $k = 0, 1, ..., N-1$

N-point IDFT of $\{X[k], k = 0, 1, ..., N-1\}$ is $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn}$, for $n = 0, 1, ..., N-1$

Upsampling/Downsampling:

Upsampling by L (↑L): $X_{up} = X(e^{j\omega L})$

Downsampling by M (↓M): $X_{down} = \frac{1}{M}\sum_{i=0}^{M-1} X(e^{j(\frac{\omega}{M}-\frac{2\pi}{M}i)})$

1. (30 pts) A continuous-time filter has a system function given by

$$H_a(s) = \frac{2}{(s+1)(s+3)}$$

A discrete-time filter is designed using impulse invariance with an impulse response given as samples of the impulse response of the continuous-time filter: $h[n] = Th_a(nT)$, where T is the sampling period. The frequency response of the discrete-time filter is given as:

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_a\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right)$$

(a) Find the impulse response, $h[n]$, and system function, $H(z)$, of the discrete-time filter. Reminder: $\mathcal{L}\{e^{at}\} = \frac{1}{s+a}$

(b) What are the poles of the discrete-time filter?

(c) Is the discrete-time filter stable and causal?

(d) How should the sampling period, T, be chosen such that $|H(e^{j\pi})| \leq 0.1$? HINT: Think about writing $H(e^{j\omega})$ as a function of $H_a(s)$ for $\omega = \pi$.

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2. (25 points) Let $H(z)$ be the system function for a stable LTI system and given as:

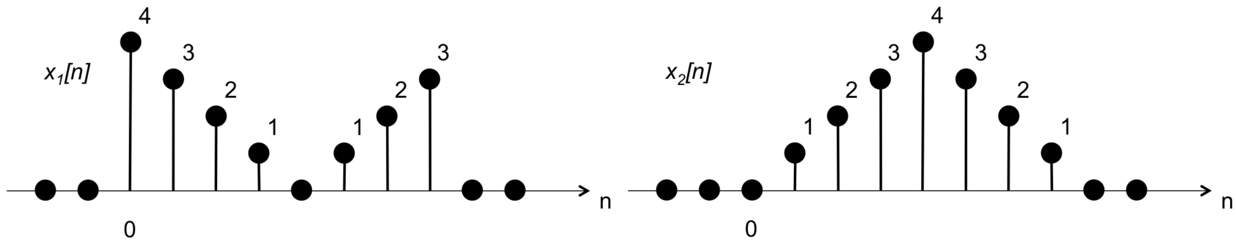
$$H(z) = \frac{(1-9z^{-2})(1+\frac{1}{3}z^{-1})}{1-\frac{1}{3}z^{-1}}$$

(a) $H(z)$ can be represented as a cascade of a minimum-phase system $H_{min}(z)$ and a unity-gain all-pass system $H_{ap}(z)$. Determine $H_{min}(z)$ and $H_{ap}(z)$ such that $|H_{ap}(z)| = 1$ for any z on the unit circle.

(b) Write an expression for the phase of $H_{min}(e^{j\omega})$. Is the minimum-phase system, $H_{min}(z)$, a generalized linear-phase system?

(c) If the minimum-phase system, $H_{min}(z)$, is not a generalized linear-phase system, can $H(z)$ be represented as a cascade of a generalized linear-phase system $H_{lin}(z)$ and an all-pass system $H_{ap2}(z)$? If your answer is yes, determine $H_{lin}(z)$ and $H_{ap2}(z)$. If your answer is no, explain why such representation does not exist.

3. (25 points) Let $x_1[n]$ and $x_2[n]$ be the two 8-point shown below:



(a) Let $X_1[k]$ and $X_2[k]$ be 8-point DFTs of $x_1[n]$ and $x_2[n]$ respectively. Write an expression for $X_2[k]$ in terms of $X_1[k]$. Hint: Think about how to write $x_2[n]$ in terms of $x_1[n]$.

(b) Find the sequence $h[n]$ which satisfies the relation $x_2[n] = x_1[n] \circledast_8 h[n]$ where $\circledast_8$ denotes the 8-point circular convolution.

4. (20 points) Let $x[n]$ be a signal of length N, $n = 0, 1, ..., N-1$. Let $y[n]$ be the $2N$ point signal created by repeating $x[n]$:

$$y[n] = \begin{cases} x[n] & \text{for } n = 0, 1, ..., N-1, \\ x[n-N] & \text{for } n = N, ..., 2N-1 \end{cases}$$

(a) Find a simple expression for the $2N$-point DFT of $y[n]$ in terms of the DFT of $x[n]$. Specify the length of the DFT of $x[n]$ in your expression.

(b) Approximate the minimum number of multiplications needed to calculate the 2^8-point DFT of $y[n]$. Describe the computations done to obtain the DFT. (I.e What DFT did you perform?)