ESE 531 Digital Signal Processing

December 11, 2015 Final Exam Closed Book/Notes 2 Hours

See information on p.3. ***<u>Give the derivations and/or reasoning leading to your answers</u>***.

Problem 1

An LTI system has *real* unit-sample response $h_A[n] = \begin{cases} a^n, & n = 0,1,\ldots,L-1 \\ 0, & otherwise \end{cases}$, $a > 1$

(a) LTI System B has system function $H_B(z) = H_A(z)H_A(z^{-1})$.

Sketch its *unit-sample response* h_B <u>for L=4</u>. *(Exact values not required for sketch).*

Does System B have *linear phase*? If so, what is its phase response?

(b) LTI System C is defined by $H_C(z) = H_A(-z) - H_A(-z^{-1})$

Find its *unit-sample response* h_C in terms of a, <u>for L=4</u>.

Determine if System C is (for arbitrary L) a *generalized linear phase* system.

Problem 2

A real, causal, length-7, generalized linear phase FIR filter is known to have a zero at $z = j\sqrt{2}$, and its frequency response $H(e^{j\omega})$ has value 9 at $\omega = 0$ and at $\omega = \pi$. The filter is of one of the four types (Type I, II, III, or IV).

Can you find the unit-sample response $h = \{h[0], h[1], \ldots, h[6]\}$ of the filter?

(Either *find* h *explicitly*, or *explain* why it is not possible to find it using only the given information.)

Problem 3

A long *periodic* sequence x of period $N = 2^r$ (r is an integer) is to be convolved with a finite-length sequence h of length K.

(a) *Show* that the output y of this convolution (filtering) is *periodic*; what is its period?

(b) Let $K = mN$ where m is an integer; N is large. How would you implement this convolution *efficiently*? Explain your analysis clearly.

Compare the *total number of multiplications* required in your scheme to that in the direct implementation of FIR filtering. (Consider the case $r = 10$, $m = 10$).

Problem 4

A sequence $x = \{x[n],\ n = 0,1,\ldots,N-1\}$ is given; let $X(e^{j\omega})$ be its DTFT.

(a) Suppose $N = 10$. You want to evaluate both $X(e^{j2\pi 7/12})$ and $X(e^{j2\pi 3/8})$. The only computation you can perform is one DFT, on any one input sequence of your choice. Can you find the desired DTFT values? *(Show your analysis and explain clearly.)*

(b) Suppose N is large. You want to obtain $X(e^{j\omega})$ at the following $2M$ frequencies:

$$\omega = \frac{2\pi}{M}m,\quad m = 0,1,\ldots,M-1 \quad \text{and} \quad \omega = \frac{2\pi}{M}m + \frac{2\pi}{N},\quad m = 0,1,\ldots,M-1.$$

Here $M = 2^{\mu} \ll N = 2^{\nu}$

A standard radix-2 FFT algorithm is available. You may execute the FFT algorithm *once* or *more than once*, and *multiplications* and *additions* outside of the FFT are *allowed*, if necessary.

You want to get the $2M$ DTFT values with as few *total multiplications* as possible (*including those in the FFT*). Give explicitly the best method you can find for this, with an estimate of the *total number of multiplications* needed in terms of M and N.

Does your result change if extra multiplications outside of FFTs are *not* allowed?

Common DTFT pairs:

Sequence	DTFT
$\delta[n]$	1
$u[n]$	$\dfrac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)$
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega-\omega_0+2\pi k)$
$\alpha^n u[n], \lvert\alpha\rvert < 1$	$\dfrac{1}{1-\alpha e^{-j\omega}}$

Definition of the DFT:

DFT of $\{x[n],\, n = 0,1,\ldots, N-1\}$ is $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi k n/N}, \qquad k = 0,1,\ldots, N-1$

- ***N*-point FFT requires approximately $\dfrac{N}{2}\log_2 N$ multiplications.**

Common z-transform pairs:

Sequence	z-transform	ROC
$\delta[n]$	1	All z
$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$

Trigonometric Identities:

$$\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\cos(A) + \cos(B) = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$
$$\cos(A) = \frac{e^{jA} + e^{-jA}}{2}$$

Geometric Series: $\sum_{n=0}^{N} r^n = \dfrac{1-r^{N+1}}{1-r}$