ESE 531: Digital Signal Processing

Lec 15: March 21, 2017

Review, Generalized Linear Phase Systems

John A. Quinn Lecture

The 2017 John A. Quinn Lecture in Chemical Engineering

"Computation and Uncertainty: The Past, Present and Future of Control"

Manfred Morari
Distinguished Faculty Fellow
Department of Electrical and Systems Engineering
University of Pennsylvania

Thursday, March 23, 2017
3:00 pm, Wu and Chen Auditorium
Levine Hall

Reception to follow seminar - Levine Lobby

Abstract:

In reflecting on our work over the last 40 years, I found that it was dominated by two themes: computation and uncertainty. In this talk, I will describe how the rapidly increasing computational resources have affected our approaches to deal with uncertainty in feedback control. The lecture will be illustrated by examples from process control and other application areas like automotive and power systems.

Lecture Outline

- Review: All Pass Systems
- Review: Minimum Phase Systems
- General Linear Phase Systems

Frequency Response of LTI System

$$Y\left(e^{j\omega}\right) = H\left(e^{j\omega}\right)X\left(e^{j\omega}\right)$$

- We can define a magnitude response

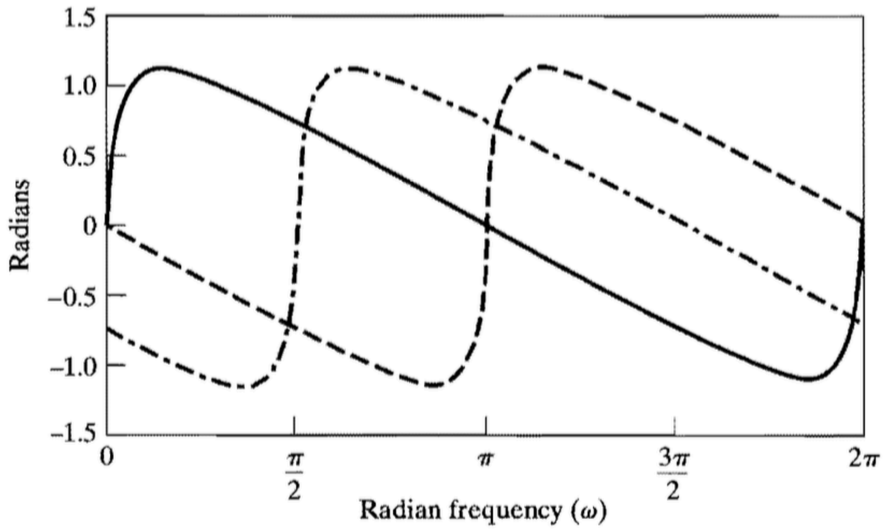
$$\left|Y\left(e^{j\omega}\right)\right| = \left|H\left(e^{j\omega}\right)\right|\left|X\left(e^{j\omega}\right)\right|$$

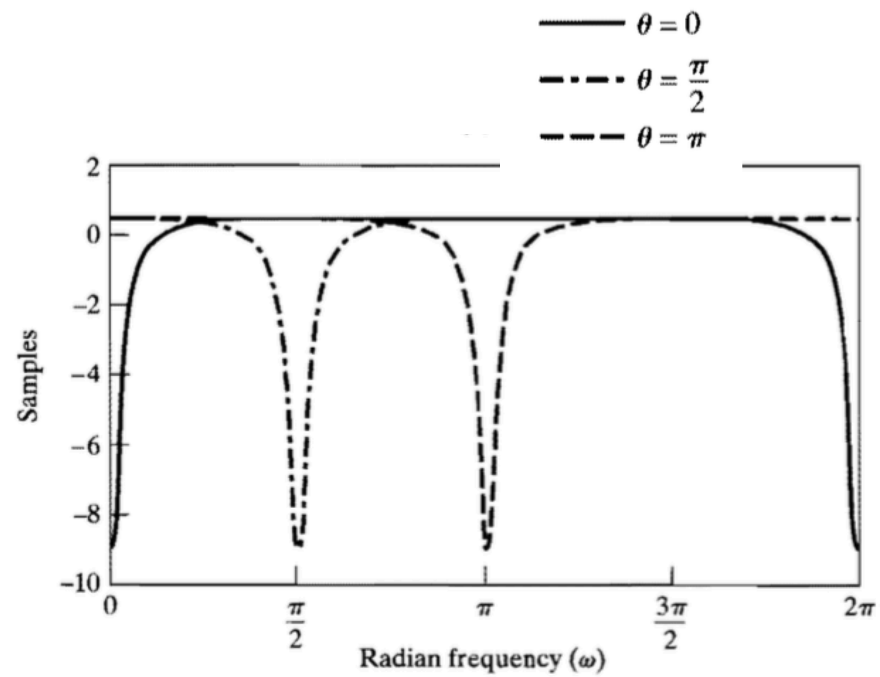
- And a phase response

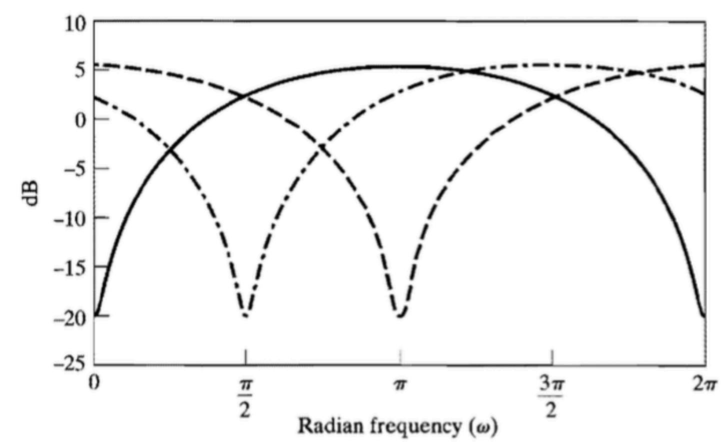
$$\angle Y\left(e^{j\omega}\right) = \angle H\left(e^{j\omega}\right) + \angle X\left(e^{j\omega}\right)$$

Example: Zero on Real Axis

- For $\theta \neq 0$





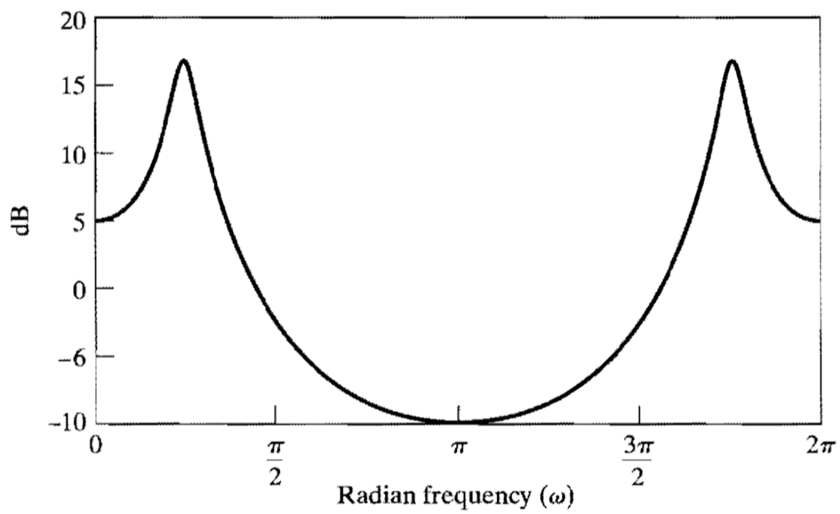


2nd Order IIR with Complex Poles

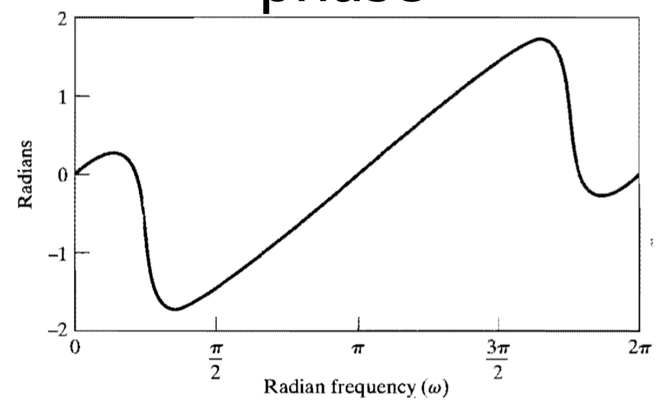
$$H(z) = \frac{1}{(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})}$$

r=0.9, $\theta = \pi / 4$

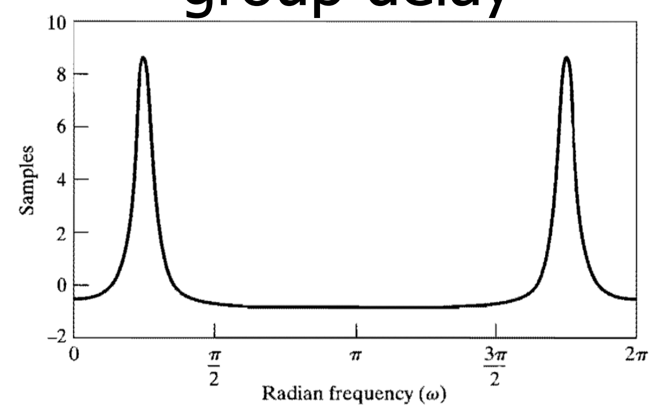
magnitude



phase

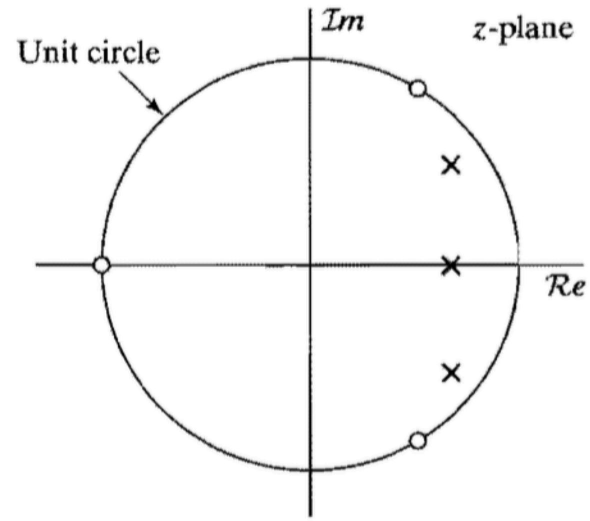


group delay



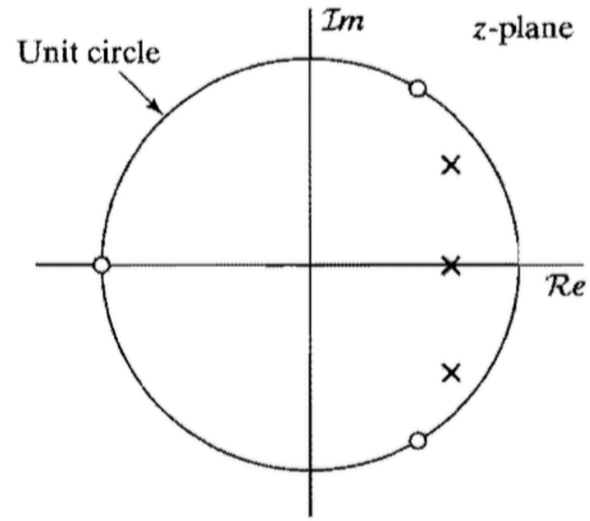
3rd Order IIR Example

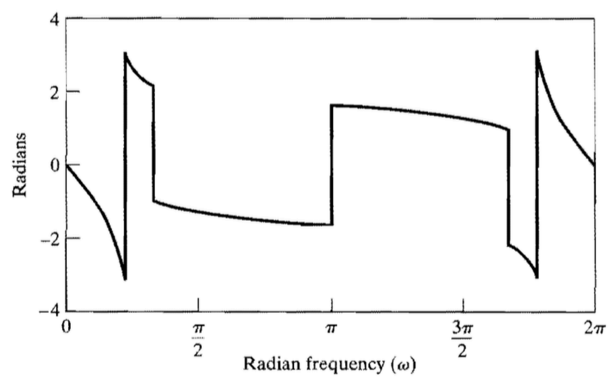
$$H(z) = \frac{0.05634(1 + z^{-1})(1 - 1.0166z^{-1} + z^{-2})}{(1 - 0.683z^{-1})(1 - 1.4461z^{-1} + 0.7957z^{-2})}$$

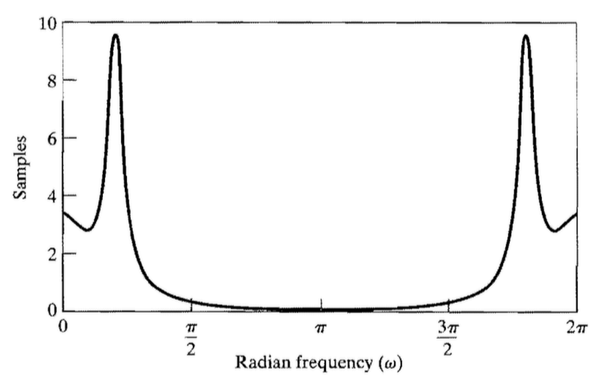


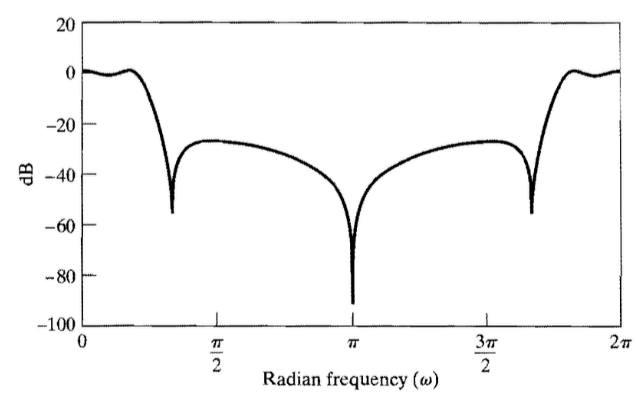
3rd Order IIR Example

$$H(z) = \frac{0.05634(1 + z^{-1})(1 - 1.0166z^{-1} + z^{-2})}{(1 - 0.683z^{-1})(1 - 1.4461z^{-1} + 0.7957z^{-2})}$$









All-Pass Systems

All-Pass Filters

- A system is an all-pass system if

$$\left|H(e^{j\omega})\right| = 1, \quad \text{all } \omega$$

- Its phase response $\theta(\omega)$ may be non-trivial

General All-Pass Filter

- d_k=real pole, e_k=complex poles paired w/ conjugate, e_k^*

$$H_{ap}(z) = A \prod_{k=1}^{M_r} \frac{z^{-1} - d_k}{1 - d_k z^{-1}} \prod_{k=1}^{M_c} \frac{(z^{-1} - e_k^*)(z^{-1} - e_k)}{(1 - e_k z^{-1})(1 - e_k^* z^{-1})}$$

General All-Pass Filter

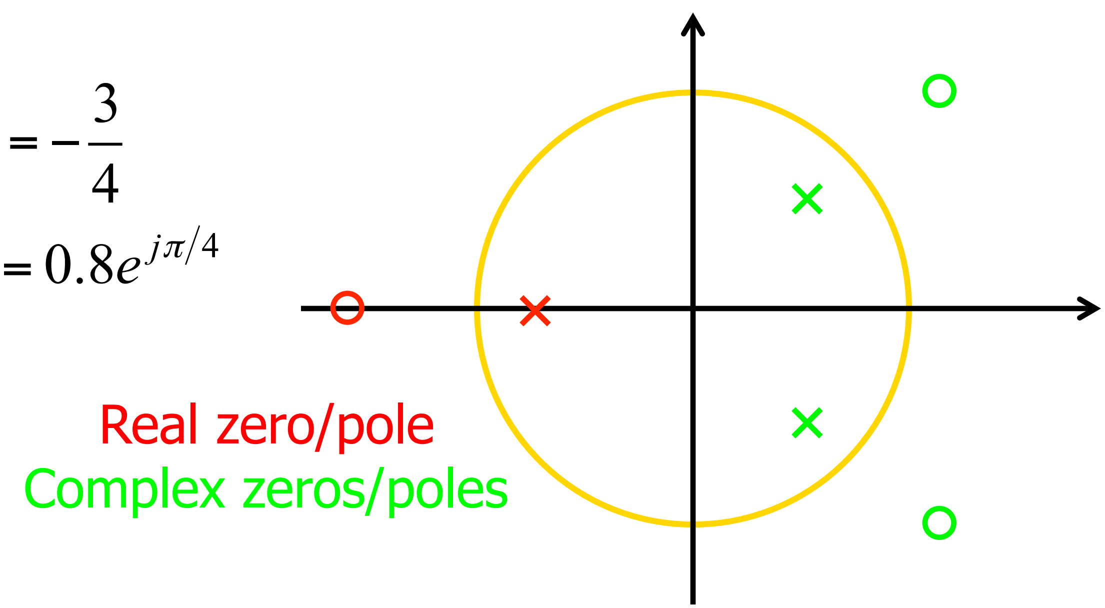
- d_k=real pole, e_k=complex poles paired w/ conjugate, e_k^*

$$H_{ap}(z) = A \prod_{k=1}^{M_r} \frac{z^{-1} - d_k}{1 - d_k z^{-1}} \prod_{k=1}^{M_c} \frac{(z^{-1} - e_k^*)(z^{-1} - e_k)}{(1 - e_k z^{-1})(1 - e_k^* z^{-1})}$$

- Example:

$$d_k = -\frac{3}{4}$$

$$e_k = 0.8e^{j\pi/4}$$



All Pass Filter Phase Response

- First order system

$$H(e^{j\omega}) = \frac{e^{-j\omega} - a^*}{1 - ae^{-j\omega}}$$

$$= \frac{e^{-j\omega} - re^{-j\theta}}{1 - re^{j\theta}e^{-j\omega}}$$

- phase

All Pass Filter Phase Response

- First order system $$H(e^{j\omega}) = \frac{e^{-j\omega} - a^*}{1 - ae^{-j\omega}} = \frac{e^{-j\omega} - re^{-j\theta}}{1 - re^{j\theta}e^{-j\omega}}$$

- phase $$\arg\left(\frac{e^{-j\omega} - re^{-j\theta}}{1 - re^{j\theta}e^{-j\omega}}\right)$$

$$= \arg\left(\frac{e^{-j\omega}(1 - re^{-j\theta}e^{j\omega})}{1 - re^{j\theta}e^{-j\omega}}\right)$$

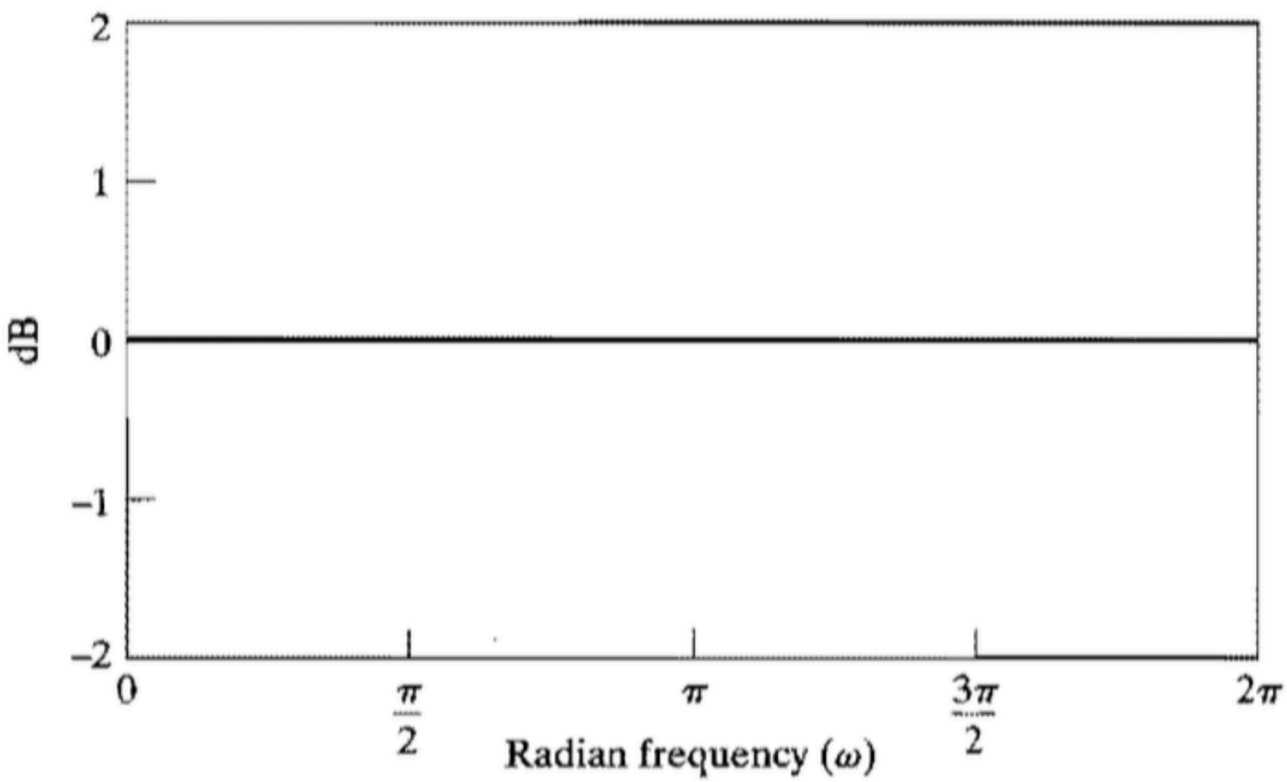
$$= \arg(e^{-j\omega}) + \arg(1 - re^{-j\theta}e^{j\omega}) - \arg(1 - re^{j\theta}e^{-j\omega})$$

$$= -\omega - \arg(1 - re^{j\theta}e^{-j\omega}) - \arg(1 - re^{j\theta}e^{-j\omega})$$

$$= -\omega - 2\arg(1 - re^{j\theta}e^{-j\omega})$$

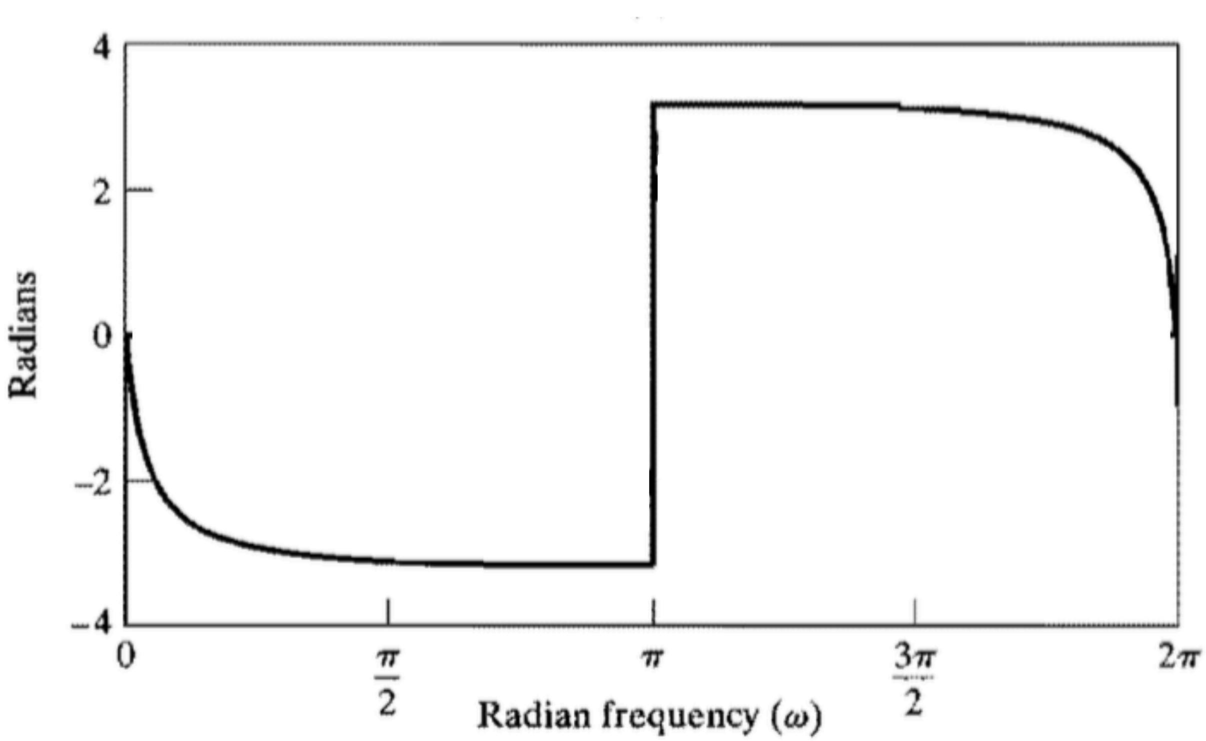
First Order Example

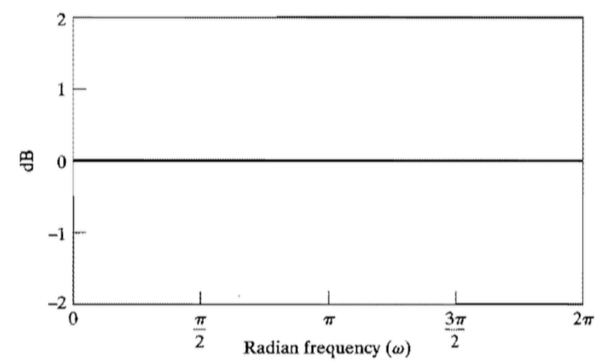
- Magnitude:



First Order Example

- Phase:



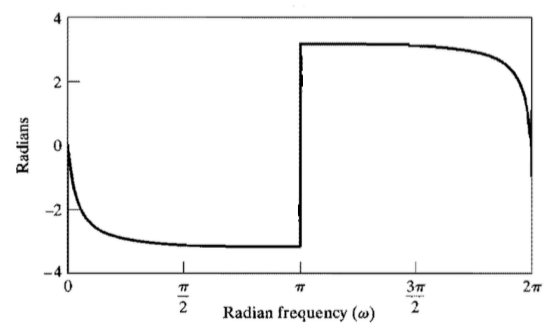


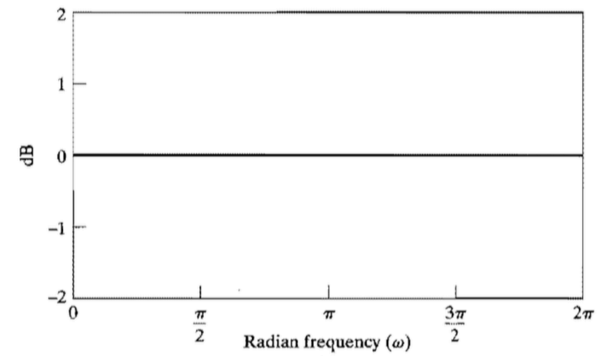
$z = 0.9$

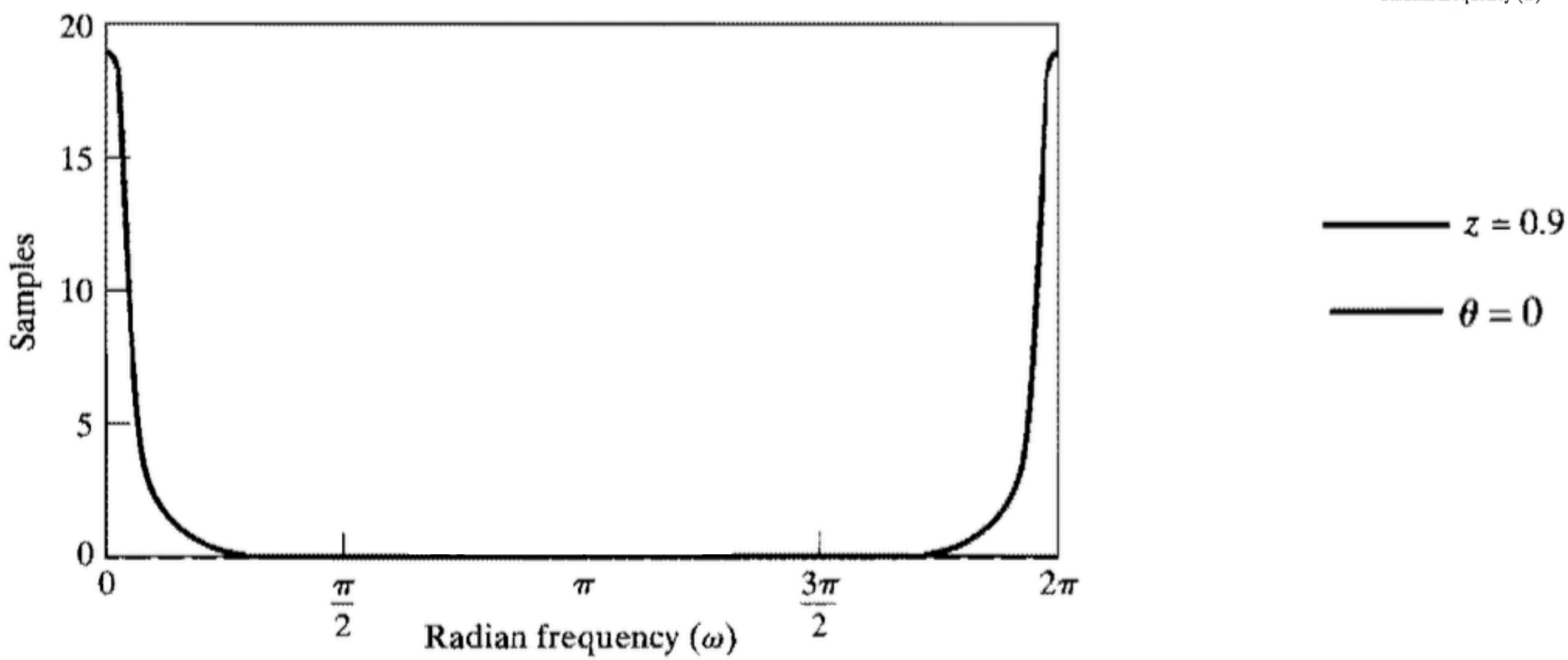
$\theta = 0$

First Order Example

- Group Delay:







All Pass Filter Phase Response

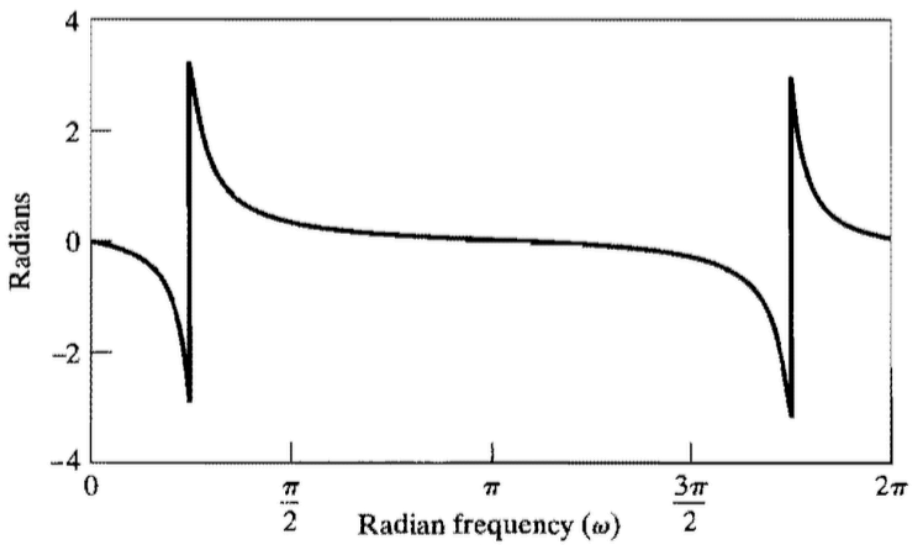
- Second order system with poles at $z = re^{j\theta}, re^{-j\theta}$

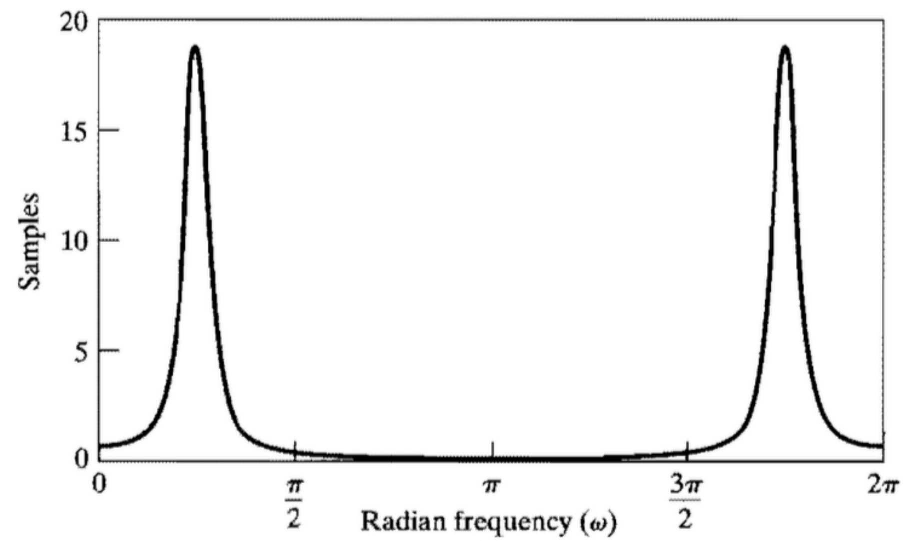
$$\angle\left[\frac{(e^{-j\omega}-re^{-j\theta})(e^{-j\omega}-re^{j\theta})}{(1-re^{j\theta}e^{-j\omega})(1-re^{-j\theta}e^{-j\omega})}\right] = -2\omega - 2\arctan\left[\frac{r\sin(\omega-\theta)}{1-r\cos(\omega-\theta)}\right]$$

$$-2\arctan\left[\frac{r\sin(\omega+\theta)}{1-r\cos(\omega+\theta)}\right].$$

Second Order Example

- Poles at $z = 0.9e^{\pm j\pi/4}$ (zeros at conjugates)





All-Pass Properties

- Claim: For a stable all-pass system:
  - $\arg[H_{ap}(e^{j\omega})] \leq 0$
    - Unwrapped phase always non-positive and decreasing
  - $\mathrm{grd}[H_{ap}(e^{j\omega})] > 0$
    - Group delay always positive
  - Intuition
    - delay is positive → system is causal
    - Phase negative → phase lag

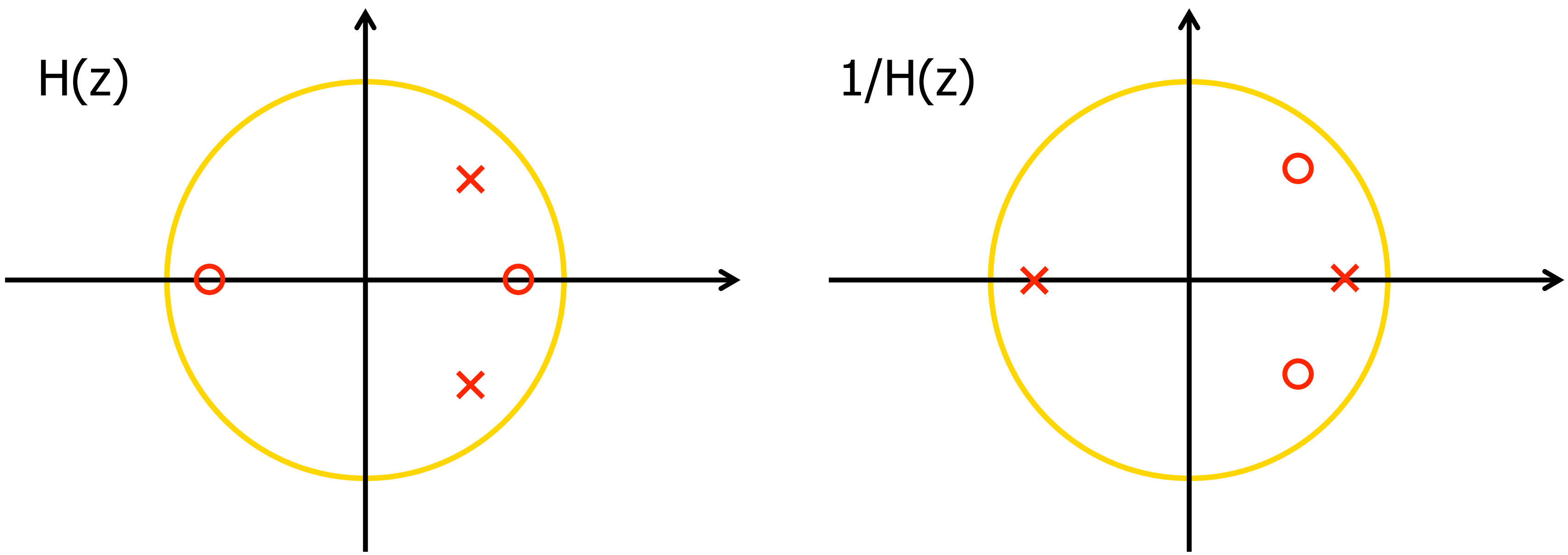
Minimum-Phase Systems

Minimum-Phase Systems

- Definition: A stable and causal system H(z) (i.e. poles inside unit circle) whose inverse 1/H(z) is also stable and causal (i.e. zeros inside unit circle)
  - All poles and zeros inside unit circle

Minimum-Phase Systems

- Definition: A stable and causal system H(z) (i.e. poles inside unit circle) whose inverse 1/H(z) is also stable and causal (i.e. zeros inside unit circle)
  - All poles and zeros inside unit circle



All-Pass Min-Phase Decomposition

- Any stable, causal system can be decomposed to:

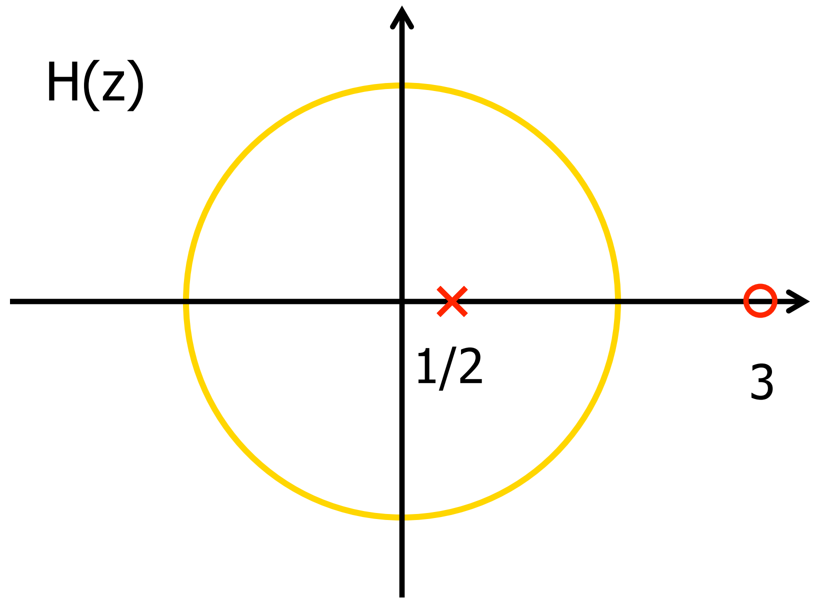
$$H(z) = H_{\min}(z) \cdot H_{ap}(z)$$

- Approach:
  - (1) First construct H_{ap} with all zeros outside unit circle
  - (2) Compute

$$H_{\min}(z) = \frac{H(z)}{H_{ap}(z)}$$

Min-Phase Decomposition Example

$$H(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}}$$

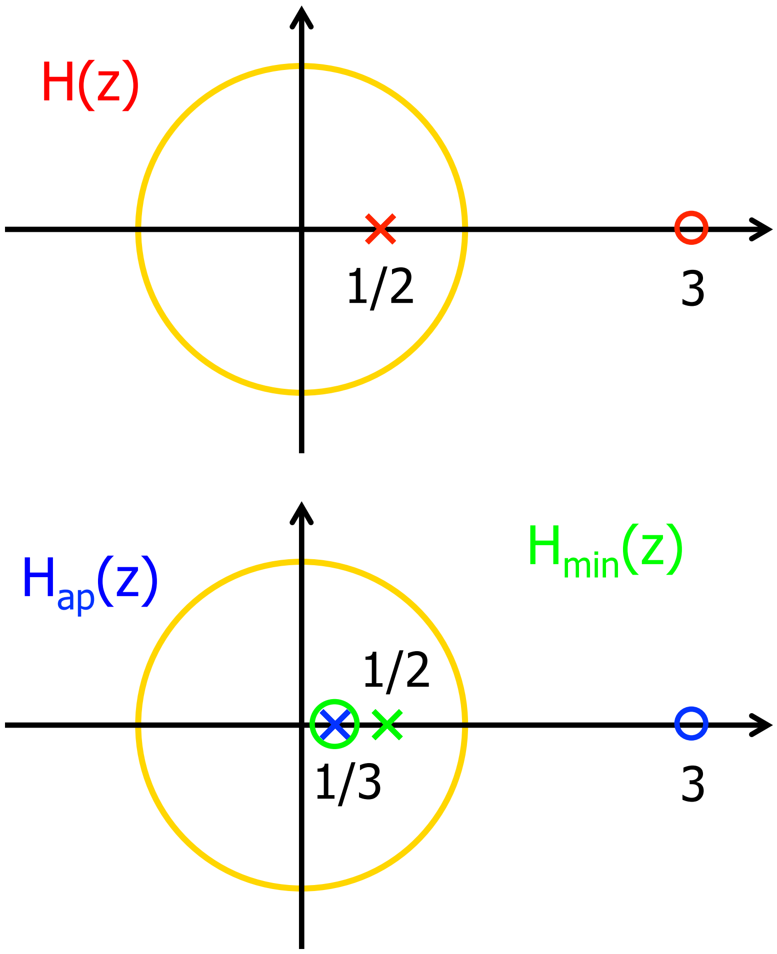


Min-Phase Decomposition Example

$$H(z) = \frac{1-3z^{-1}}{1-\frac{1}{2}z^{-1}}$$

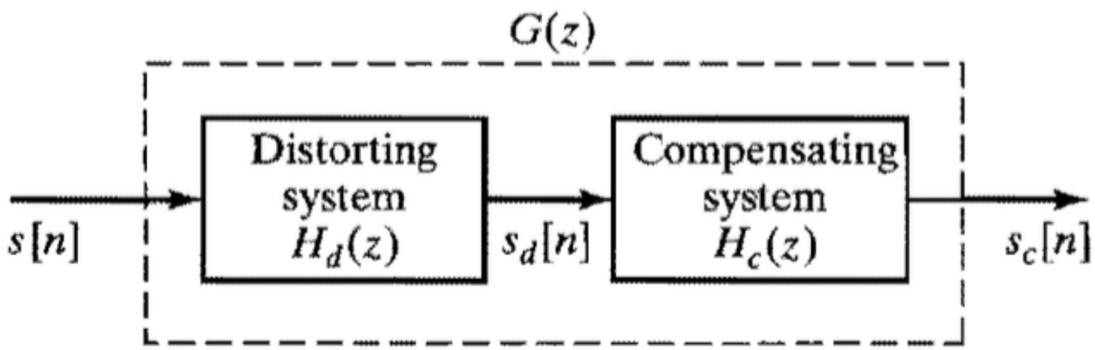
- Set $H_{ap}(z) = \dfrac{z^{-1}-\frac{1}{3}}{1-\frac{1}{3}z^{-1}}$

$$H_{\min}(z) = -3\frac{1-\frac{1}{3}z^{-1}}{1-\frac{1}{2}z^{-1}}$$



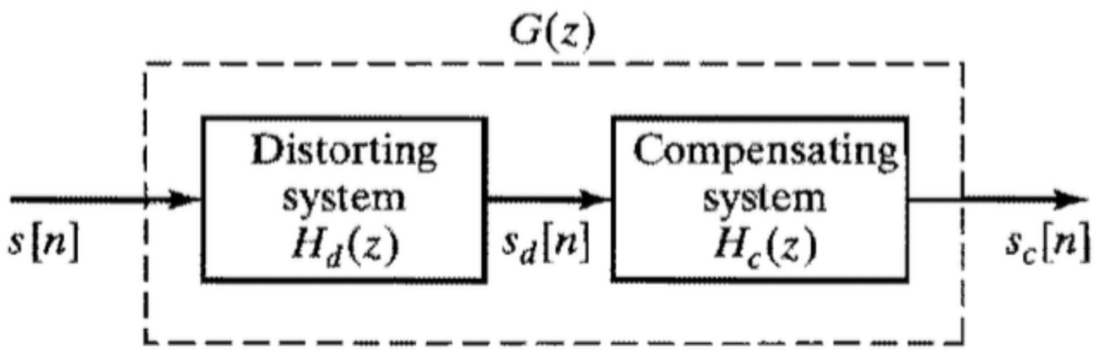
Min-Phase Decomposition Purpose

- Have some distortion that we want to compensate for:



Min-Phase Decomposition Purpose

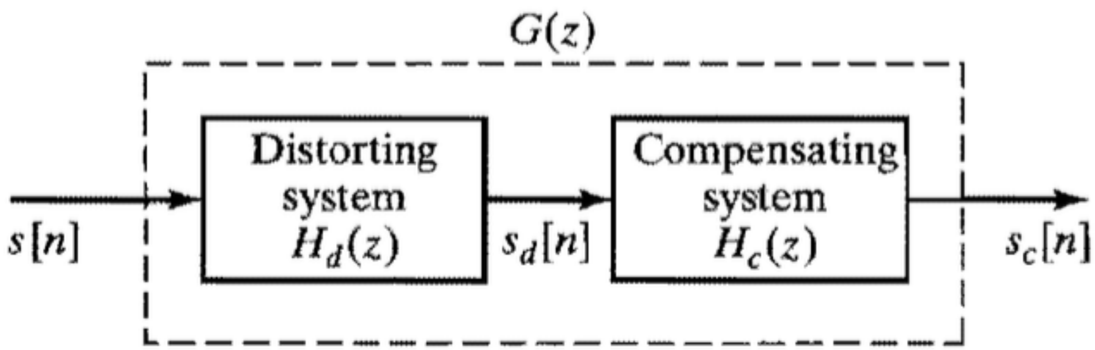
- Have some distortion that we want to compensate for:



- If $H_d(z)$ is min phase, easy:
  - $H_c(z)=1/H_d(z)$ ← also stable and causal

Min-Phase Decomposition Purpose

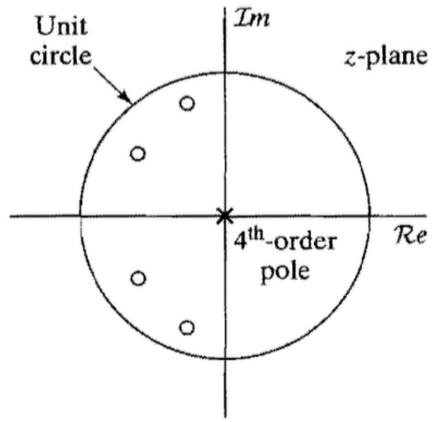
- Have some distortion that we want to compensate for:



- If $H_d(z)$ is min phase, easy:
  - $H_c(z)=1/H_d(z)$ ← also stable and causal
- Else, decompose $H_d(z)=H_{d,min}(z)\ H_{d,ap}(z)$
  - $H_c(z)=1/H_{d,min}(z) \rightarrow H_d(z)H_c(z)=H_{d,ap}(z)$
    - Compensate for magnitude distortion

Minimum Energy-Delay Property

Min phase

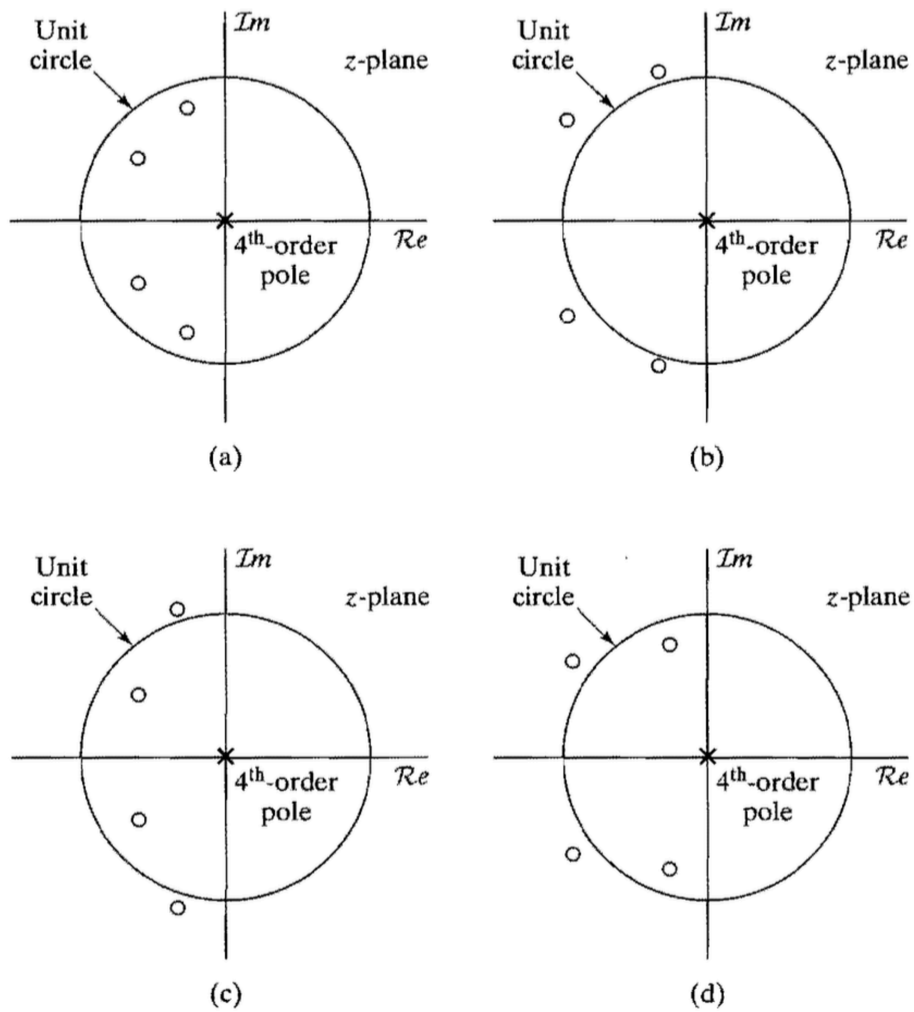


(a)

Minimum Energy-Delay Property

Min phase

Max phase



Generalized Linear Phase Systems

Generalized Linear Phase

- An LTI system has generalized linear phase if frequency response $H(e^{j\omega})$ can be expressed as:

$$H(e^{j\omega}) = A(\omega)e^{-j\omega\alpha + j\beta}, \quad |\omega| < \pi$$

- Where A(ω) is a real function.

Generalized Linear Phase

- An LTI system has generalized linear phase if frequency response $H(e^{j\omega})$ can be expressed as:

$$H(e^{j\omega}) = A(\omega)e^{-j\omega\alpha + j\beta}, \quad |\omega| < \pi$$

- Where A(ω) is a real function.

- What is the group delay?

Causal FIR Systems

$$y[n] = b_0 x[n] + b_1 x[n-1] + \ldots + b_M x[n-M], \quad \text{all } n$$

$$H(z) = b_0 + b_1 z^{-1} + \ldots + b_M z^{-M} = b_0 \prod_{k=1}^{M} (1 - c_k z^{-1})$$

$$h[n] = \begin{cases} b_n, & n = 0,1,\ldots,M \\ 0, & otherwise \end{cases}$$

Causal FIR Systems

- Causal FIR systems have generalized linear phase if they have impulse response length (M+1)
- It can be shown if

$$h[n] = \begin{cases} h[M-n], & 0 \le n \le M, \\ 0, & \text{otherwise,} \end{cases}$$

- then

$$H(e^{j\omega}) = A_e(e^{j\omega})e^{-j\omega M/2},$$

Example: Moving Average

- Moving Average Filter
  - Causal: $M_1=0$, $M_2=M$

$$y[n] = \frac{x[n-M] + \ldots + x[n]}{M+1}$$

Impulse response

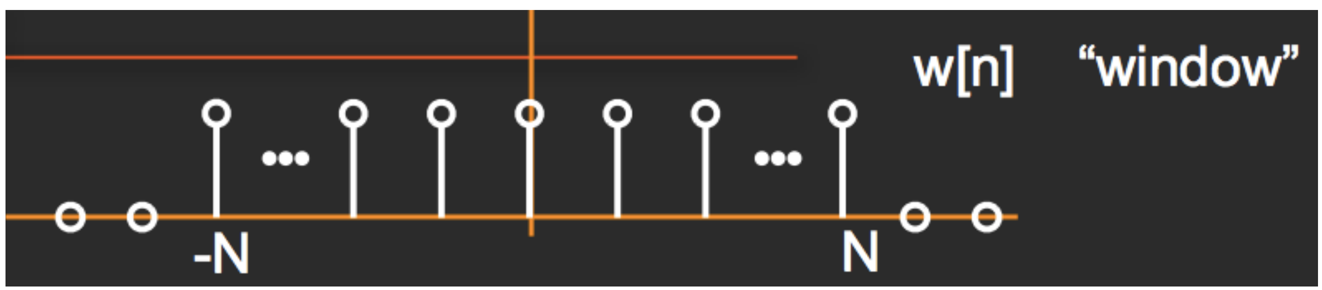
Example: Moving Average

- Moving Average Filter
  - Causal: $M_1=0$, $M_2=M$

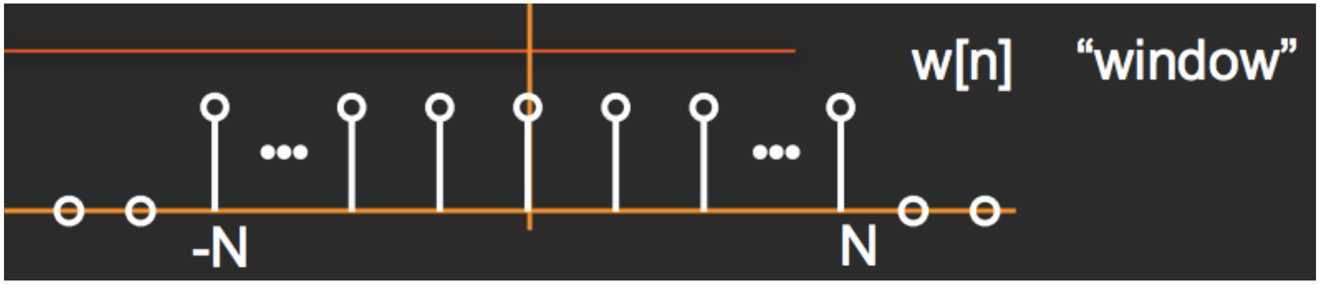
$$y[n] = \frac{x[n-M] + \ldots + x[n]}{M+1}$$

Impulse response

Scaled &Time Shifted window



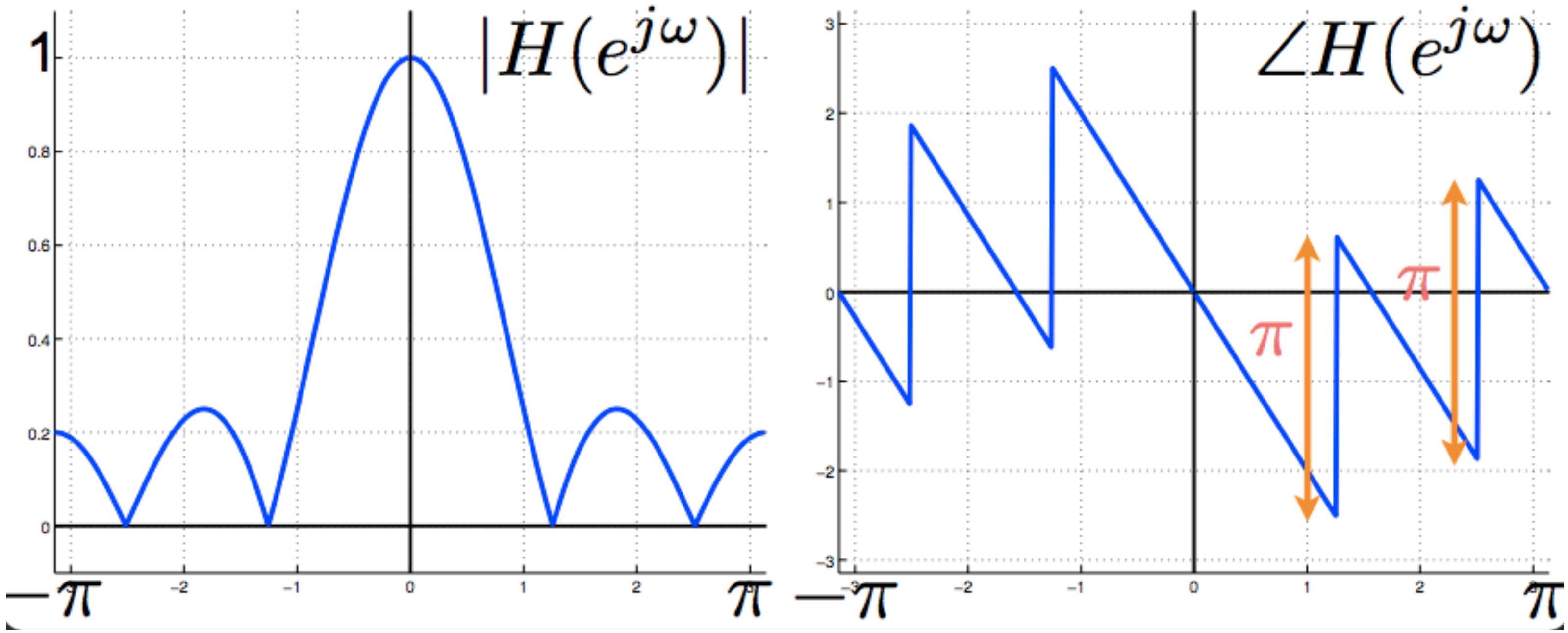
Example: Moving Average



$$w[n] \leftrightarrow W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

$$\frac{1}{M+1} w[n-M/2] \leftrightarrow W(e^{j\omega}) = \frac{e^{-j\omega M/2}}{M+1} \frac{\sin\left((M/2+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

Example: Moving Average



Causal FIR Systems

- Causal FIR systems have generalized linear phase if they have impulse response length (M+1)
- It can be shown if

$$h[n] = \begin{cases} h[M-n], & 0 \le n \le M, \\ 0, & \text{otherwise,} \end{cases}$$

- Then

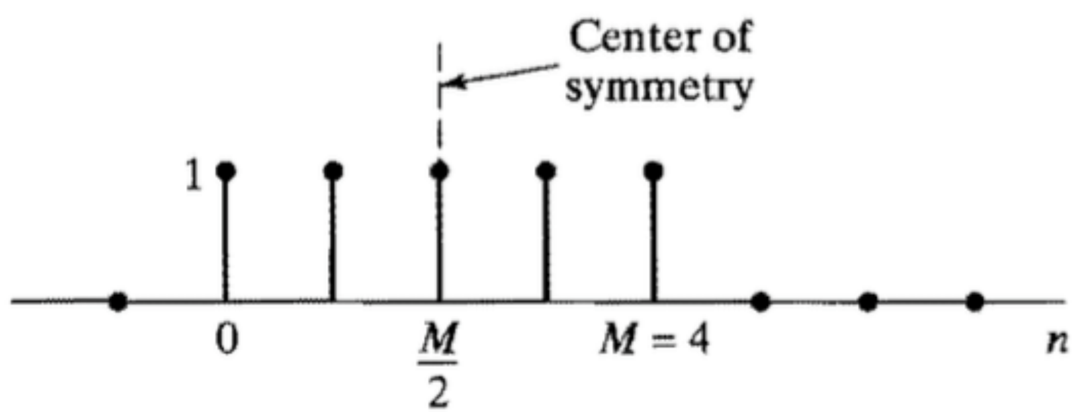
$$H(e^{j\omega}) = A_e(e^{j\omega})e^{-j\omega M/2},$$

- Sufficient conditions to guarantee GLP, not necessary
  - Causal IIR can also have GLP

FIR GLP: Type I

Type I Even Symmetry, M even

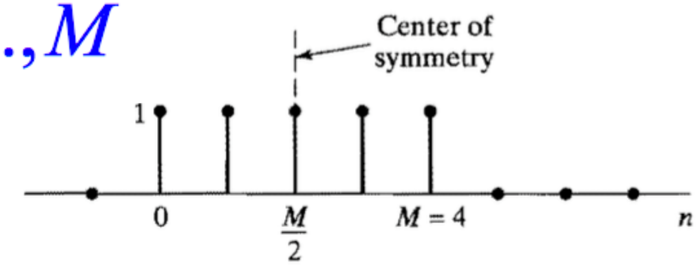
$$h[n] = h[M-n], \quad n = 0,1,\ldots,M$$



FIR GLP: Type I

Type I Even Symmetry, M even

$$h[n] = h[M-n], \quad n = 0,1,\ldots,M$$

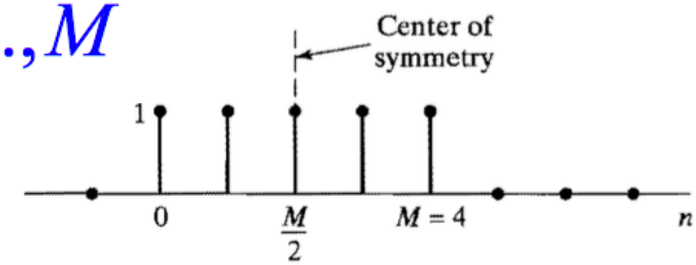


Then $H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real, Even}} \; e^{-j\omega M/2}$ ← integer delay

FIR GLP: Type I

Type I Even Symmetry, M even

$$h[n] = h[M-n], \quad n = 0,1,\ldots,M$$



Then $H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real, Even}} \; e^{-j\omega M/2}$ ← integer delay

$$H(e^{j\omega}) = e^{-j\omega M/2} \left(\sum_{k=0}^{M/2} a[k] \cos \omega k \right)$$

$$a[0] = h[M/2],$$

$$a[k] = 2h[(M/2) - k], \qquad k = 1, 2, \ldots, M/2.$$

FIR GLP: Type I – Example, M=4

Type I Even Symmetry, M even

$$h[n] = h[M-n], \quad n = 0,1,\ldots,M$$

Then $H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real, Even}} \; e^{-j\omega M/2}$ ← integer delay

$$H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} + h[4]e^{-j4\omega}$$

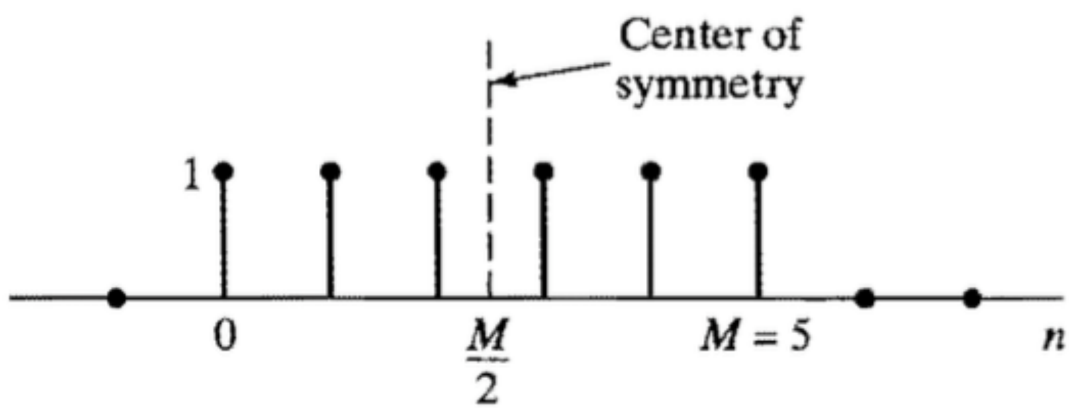
$$= e^{-j2\omega}\left[h[0]e^{j2\omega} + h[1]e^{j\omega} + h[2] + h[1]e^{-j\omega} + h[0]e^{-j2\omega}\right]$$

$$= \underbrace{\left[2h[0]\cos(2\omega) + 2h[1]\cos(\omega) + h[2]\right]}_{A(\omega)\ (even)} \; e^{-j2\omega}$$

FIR GLP: Type II

Type II Even Symmetry, M odd

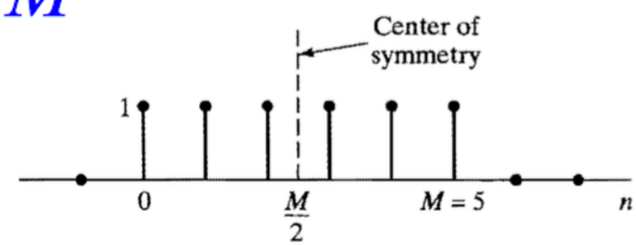
$$h[n] = h[M-n], \quad n = 0,1,\ldots,M$$



FIR GLP: Type II

Type II Even Symmetry, M odd

$$h[n] = h[M-n], \quad n = 0,1,\ldots,M$$

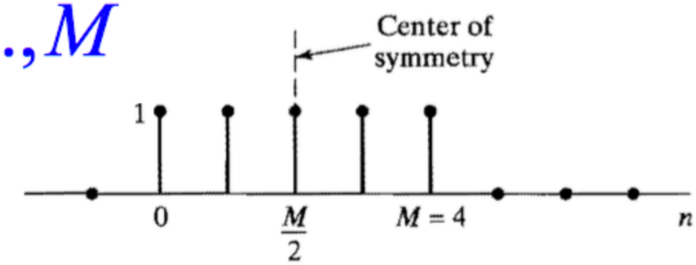


Then $H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real, Even}} \; e^{-j\omega M/2}$ ← integer delay

FIR GLP: Type II

Type I Even Symmetry, M even

$$h[n] = h[M-n], \quad n = 0,1,\ldots,M$$



Then $H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real, Even}} \; e^{-j\omega M/2}$ ← integer delay

$$H(e^{j\omega}) = e^{-j\omega M/2}\left\{\sum_{k=1}^{(M+1)/2} b[k]\cos\left[\omega\left(k-\tfrac{1}{2}\right)\right]\right\}$$

$$b[k] = 2h[(M+1)/2 - k], \qquad k = 1, 2, \ldots, (M+1)/2.$$

FIR GLP: Type II – Example, M=3

Type I Even Symmetry, M even

$$h[n] = h[M-n], \quad n = 0,1,\ldots,M$$

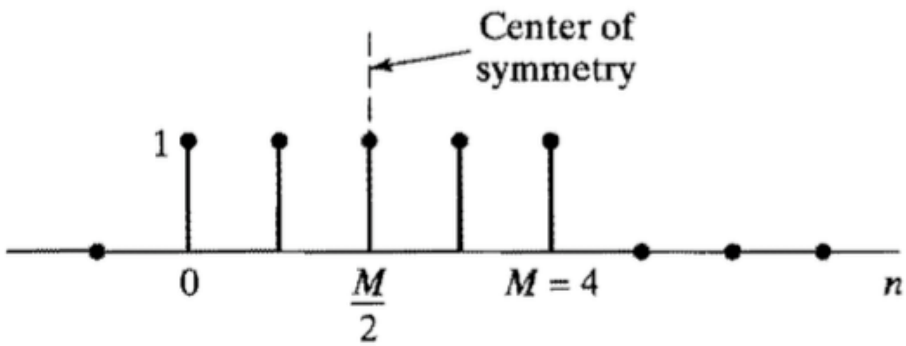
$$\text{Then } H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real, Even}} \; e^{-j\omega M/2}$$

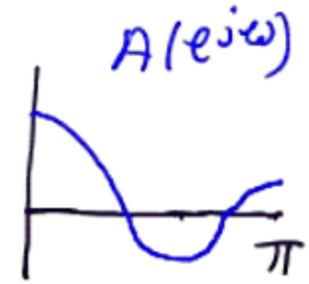
← integer delay

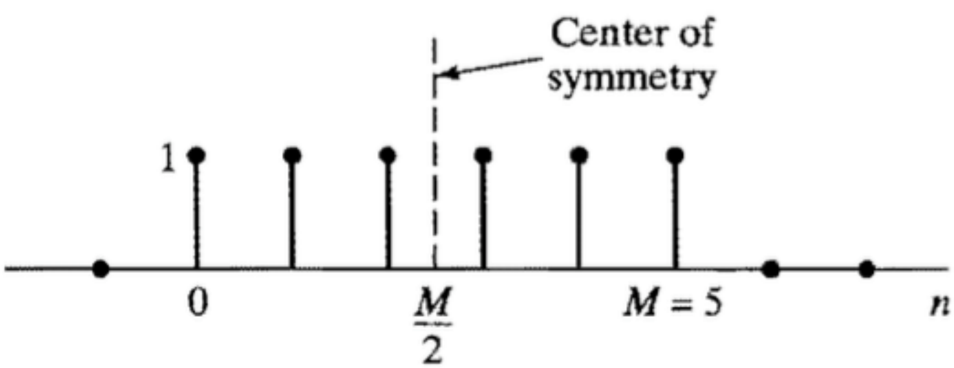
$$H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega}$$

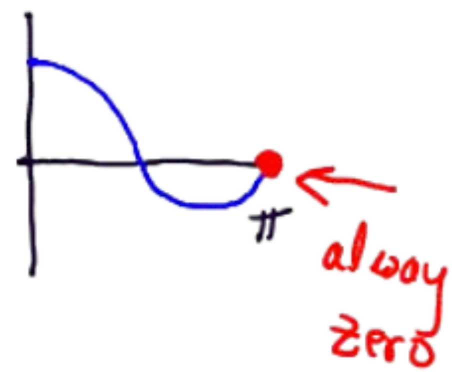
$$H(e^{j\omega}) = \underbrace{\left[2h[0]\cos(3\omega/2) + 2h[1]\cos(\omega/2)\right]}_{A(\omega)} \; e^{-j3\omega/2}$$

FIR GLP: Type I and II





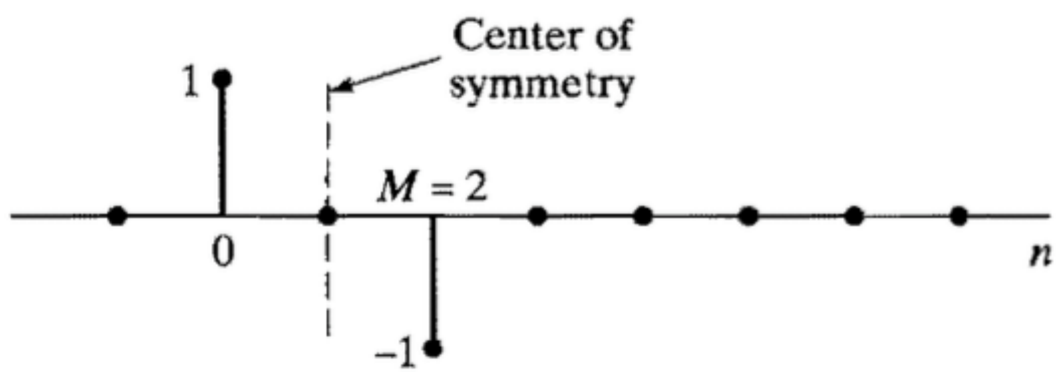




FIR GLP: Type III

Type III Odd Symmetry, M even

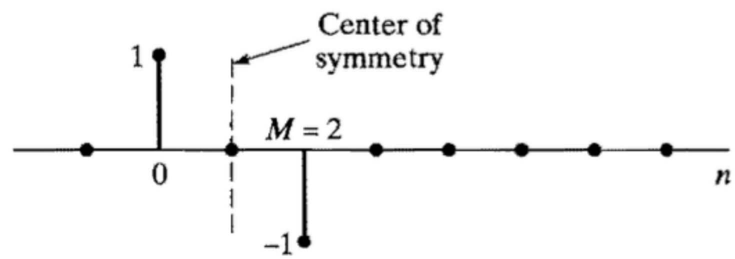
$$h[n] = -h[M-n], \quad n = 0,1,\ldots,M \qquad (note\ \ h[M/2] = 0)$$



FIR GLP: Type III

Type III Odd Symmetry, M even

$$h[n] = -h[M-n], \quad n = 0,1,\ldots,M \qquad (note\ \ h[M/2] = 0)$$



$$H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real,Odd}}\ e^{-j\omega M/2 + j\pi/2}$$

FIR GLP: Type III

Type III Odd Symmetry, M even

$$h[n] = -h[M-n], \quad n = 0,1,\ldots,M \qquad (note\ \ h[M/2] = 0)$$

$$H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real,Odd}}\ e^{-j\omega M/2 + j\pi/2}$$

$$H(e^{j\omega}) = je^{-j\omega M/2}\left[\sum_{k=1}^{M/2} c[k]\sin\omega k\right]$$

$$c[k] = 2h[(M/2)-k], \qquad k = 1, 2, \ldots, M/2.$$

FIR GLP: Type III – Example, M=4

Type III Odd Symmetry, M even

$$h[n] = -h[M-n], \quad n = 0,1,\ldots,M \qquad (note\ \ h[M/2]=0)$$

$$H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real,Odd}}\ e^{-j\omega M/2 + j\pi/2}$$

$$H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} + h[4]e^{-j4\omega}$$

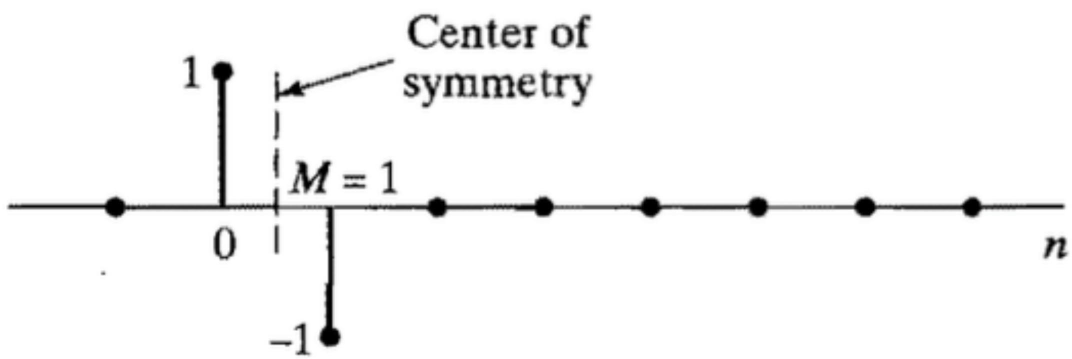
$$= e^{-j2\omega}\left[h[0]e^{j2\omega} + h[1]e^{j\omega} - h[1]e^{-j\omega} - h[0]e^{-j2\omega}\right]$$

$$= \underbrace{\left[2h[0]\sin(2\omega) + 2h[1]\sin(\omega)\right]}_{A(\omega)\ (odd)}\ j\,e^{-j2\omega}$$

FIR GLP: Type IV

Type IV Odd Symmetry, M Odd

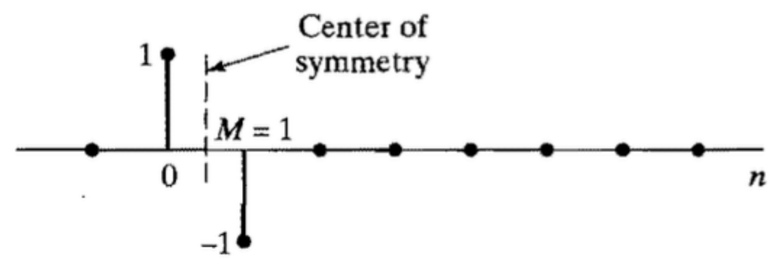
$$h[n] = -h[M-n], \quad n = 0,1,\ldots,M \qquad (M/2 \text{ not an integer})$$



FIR GLP: Type IV

Type IV Odd Symmetry, *M* Odd

$$h[n] = -h[M-n], \quad n = 0,1,\ldots,M \qquad (M/2 \text{ not an integer})$$



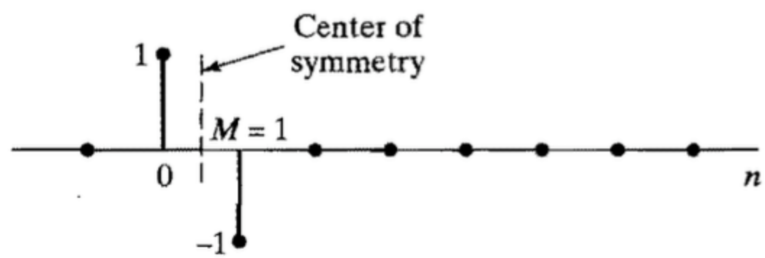
$$H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real, Odd}} \; e^{-j\omega M/2 + j\pi/2}$$

fractional delay

FIR GLP: Type IV

Type IV Odd Symmetry, M Odd

$$h[n] = -h[M-n], \quad n = 0,1,\ldots,M \qquad (M/2 \text{ not an integer})$$



$$H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real,Odd}} \; e^{-j\omega M/2 + j\pi/2}$$

fractional delay

$$H(e^{j\omega}) = je^{-j\omega M/2}\left[\sum_{k=1}^{(M+1)/2} d[k]\sin\left[\omega\left(k-\tfrac{1}{2}\right)\right]\right]$$

$$d[k] = 2h[(M+1)/2 - k], \qquad k = 1, 2, \ldots, (M+1)/2.$$

FIR GLP: Type IV – Example, M=3

Type IV Odd Symmetry, M Odd

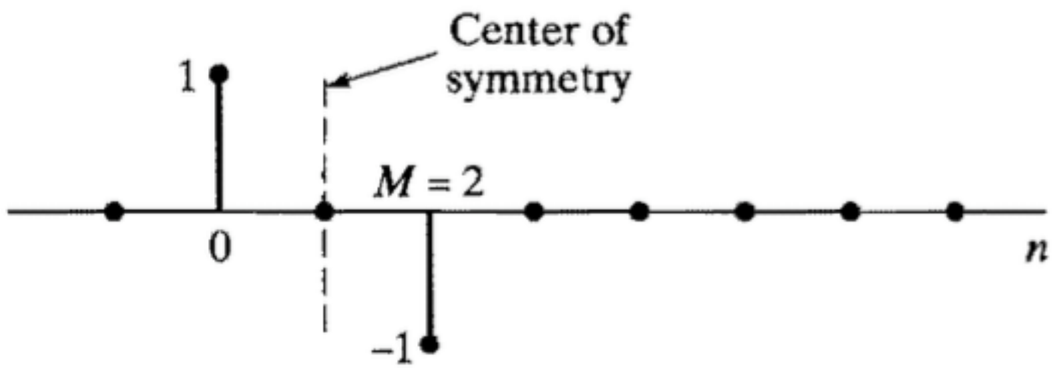
$$h[n] = -h[M-n], \quad n = 0,1,\ldots,M \qquad (M/2 \text{ not an integer})$$

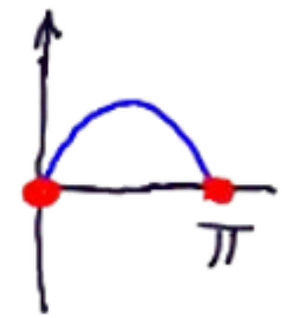
$$H(e^{j\omega}) = \sum_{n=0}^{M} h[n]e^{-j\omega n} = \underbrace{A(w)}_{\text{Real,Odd}} \; e^{-j\omega M/2 + j\pi/2}$$

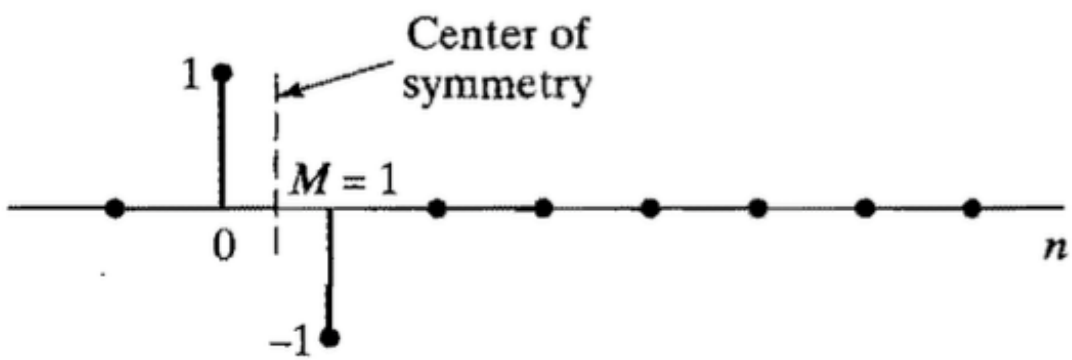
fractional delay

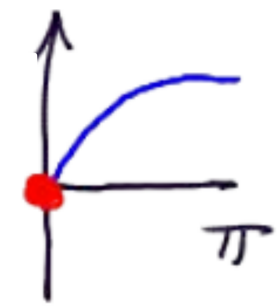
$$H(e^{j\omega}) = \underbrace{\left[2h[0]\sin(3\omega/2) + 2h[1]\sin(\omega/2)\right]}_{A(\omega)} \; je^{-j3\omega/2}$$

FIR GLP: Type III and IV









Zeros of GLP System – Type I and II

- FIR GLP System Function

$$H(z) = \sum_{n=0}^{M} h[n]z^{-n}$$

Zeros of GLP System – Type I and II

- FIR GLP System Function

$$H(z) = \sum_{n=0}^{M} h[n]z^{-n}$$

$$H(z) = \sum_{n=0}^{M} h[M-n]z^{-n} = \sum_{k=M}^{0} h[k]z^{k}z^{-M}$$
$$= z^{-M}H(z^{-1}).$$

If z_0 is a zero then z_0^{-1} is also a zero.

Zeros of GLP System – Type I and II

- FIR GLP System Function

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n}$$

If z_0 is a zero then z_0^{-1} is also a zero.

- If h[n] is real,

If z_0 is a zero then z_0^{*} is also a zero.

Zeros of GLP System – Type I and II

- FIR GLP System Function

$$H(z) = \sum_{n=0}^{M} h[n]z^{-n}$$

Real system → zeros occur in conjugate-reciprocal groups of 4

$$(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})(1 - r^{-1}e^{j\theta}z^{-1})(1 - r^{-1}e^{-j\theta}z^{-1})$$

Zeros of GLP System – Type I and II

- FIR GLP System Function

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n}$$

Real system → zeros occur in conjugate-reciprocal groups of 4

$$(1 - re^{j\theta} z^{-1})(1 - re^{-j\theta} z^{-1})(1 - r^{-1} e^{j\theta} z^{-1})(1 - r^{-1} e^{-j\theta} z^{-1})$$

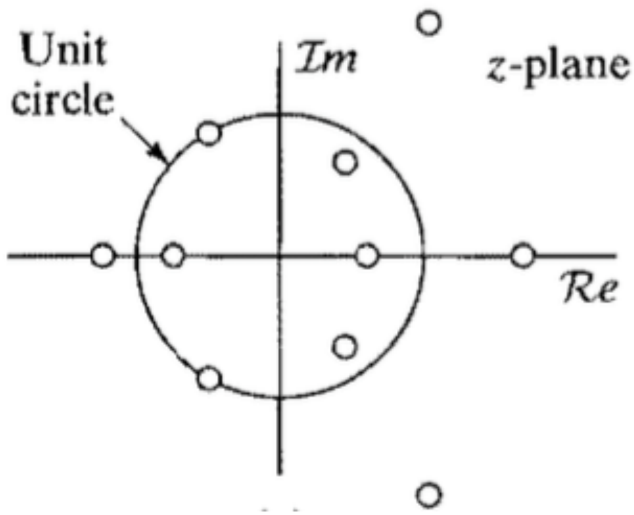
- If zero is on unit circle (r=1)

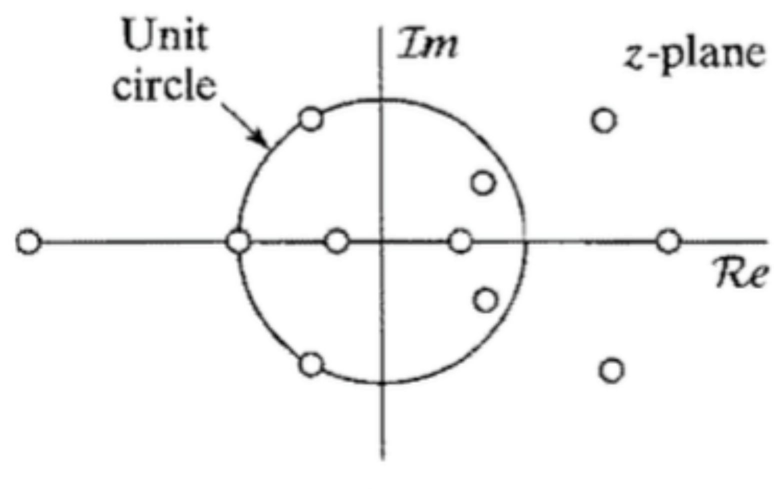
$$(1 - e^{j\theta} z^{-1})(1 - e^{-j\theta} z^{-1}).$$

- If zero is real and not on unit circle (θ =0)

$$(1 \pm r z^{-1})(1 \pm r^{-1} z^{-1}).$$

Zeros of GLP System – Type I and II





Zeros of GLP System – Type II

- FIR GLP System Function

$$H(z) = \sum_{n=0}^{M} h[M-n]z^{-n} = \sum_{k=M}^{0} h[k]z^{k}z^{-M}$$

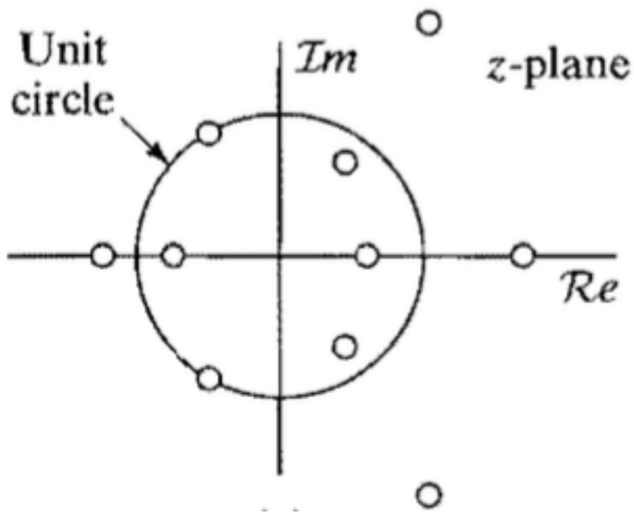
$$= z^{-M}H(z^{-1}).$$

Consider $z = -1$: $H(-1) = (-1)^{-M} H(-1)$

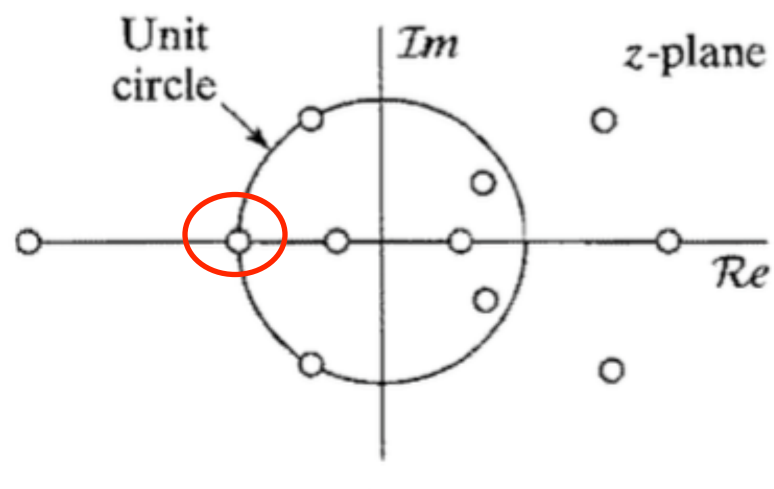
$\Rightarrow$ for M odd, $z = -1$ must be a zero (Type II)

Zeros of GLP System – Type I and II

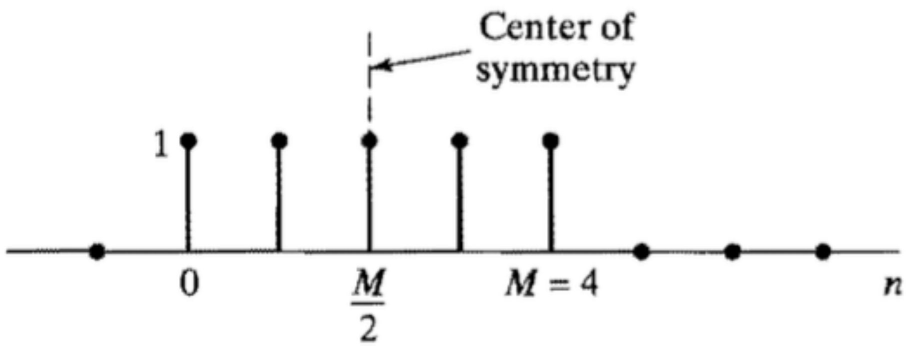
Type I

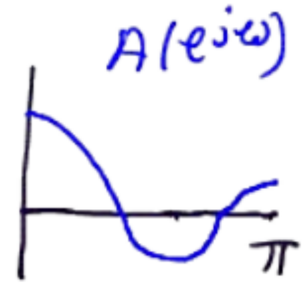


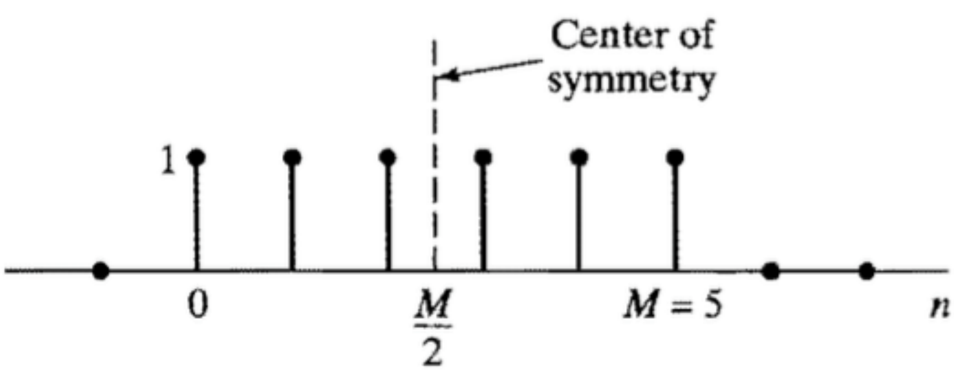
Type II

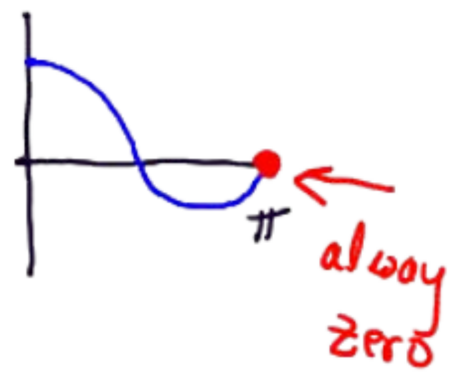


FIR GLP: Type I and II









Zeros of GLP System – Type III and IV

- FIR GLP System Function

$$H(z) = \sum_{n=0}^{M} h[n]z^{-n}$$

$$H(z) = -z^{-M} H(z^{-1}).$$

Zeros of GLP System – Type III and IV

- FIR GLP System Function

$$H(z) = \sum_{n=0}^{M} h[n]z^{-n}$$

$$H(z) = -z^{-M} H(z^{-1}).$$

If z_0 is a zero then z_0^{-1} is also a zero.

Zeros of GLP System – Type III and IV

- FIR GLP System Function

$$H(z) = \sum_{n=0}^{M} h[n]z^{-n}$$

Real system → zeros occur in conjugate-reciprocal groups of 4

$$(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})(1 - r^{-1}e^{j\theta}z^{-1})(1 - r^{-1}e^{-j\theta}z^{-1})$$

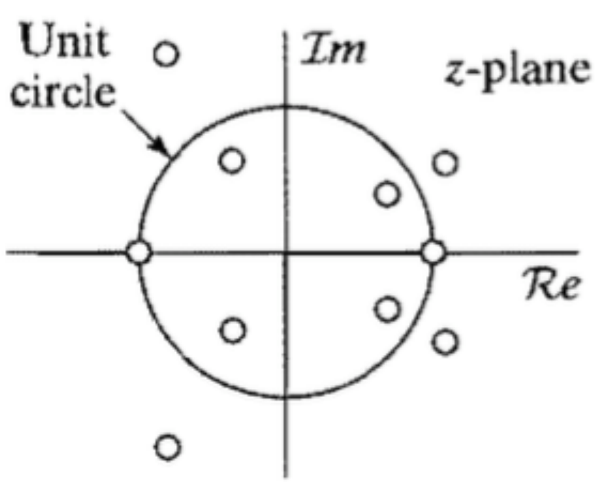
- If zero is on unit circle (r=1)

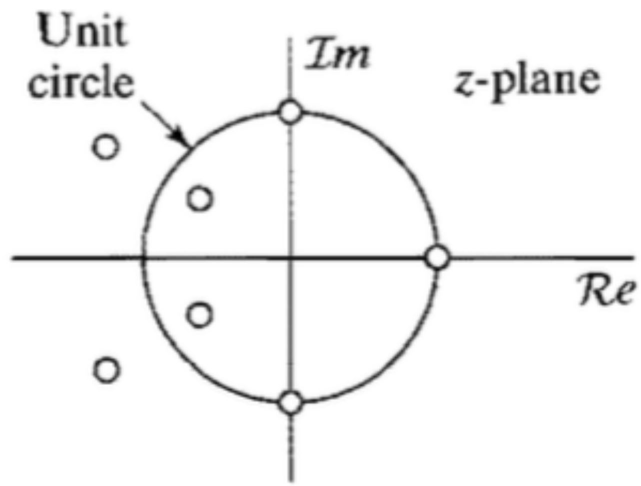
$$(1 - e^{j\theta}z^{-1})(1 - e^{-j\theta}z^{-1}).$$

- If zero is real and not on unit circle (θ =0)

$$(1 \pm rz^{-1})(1 \pm r^{-1}z^{-1}).$$

Zeros of GLP System – Type III and IV





Zeros of GLP System – Type III and IV

- FIR GLP System Function

$$H(z) = -z^{-M} H(z^{-1}).$$

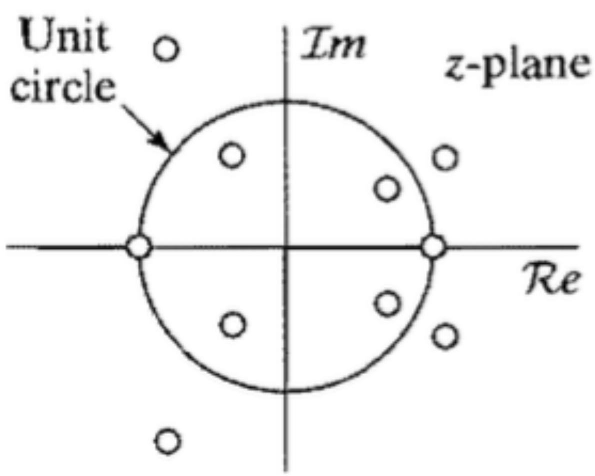
$$H(1) = -H(1) \;\Rightarrow\; z = 1 \text{ must be a zero}$$

$$H(-1) = (-1)^{-M+1} H(-1)$$

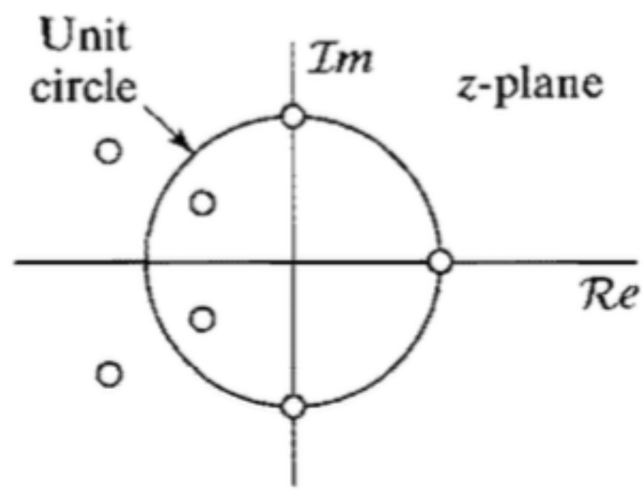
$$\boxed{\Rightarrow \;\text{for } M \text{ even, } z = -1 \text{ must be a zero (Type III)}}$$

Zeros of GLP System – Type III and IV

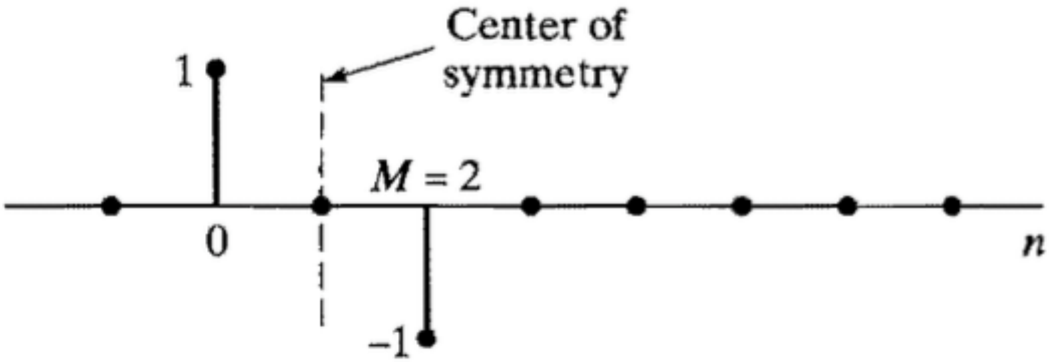
Type III

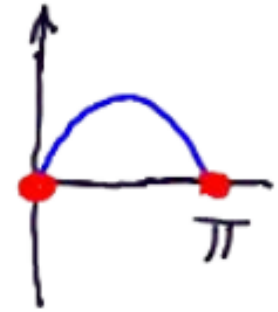


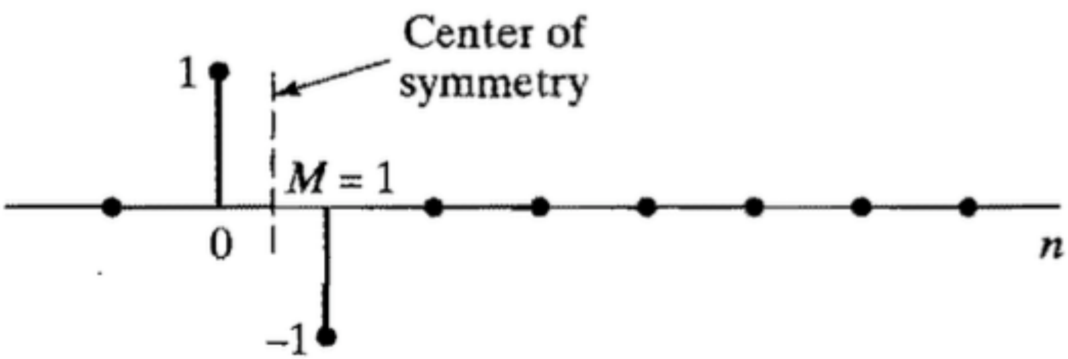
Type IV

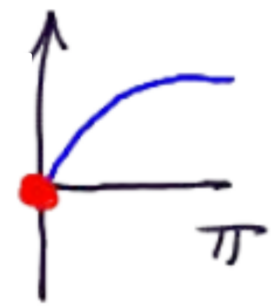


FIR GLP: Type III and IV









GLP and Min Phase Systems

- Any FIR linear-phase system can be decomposed into:

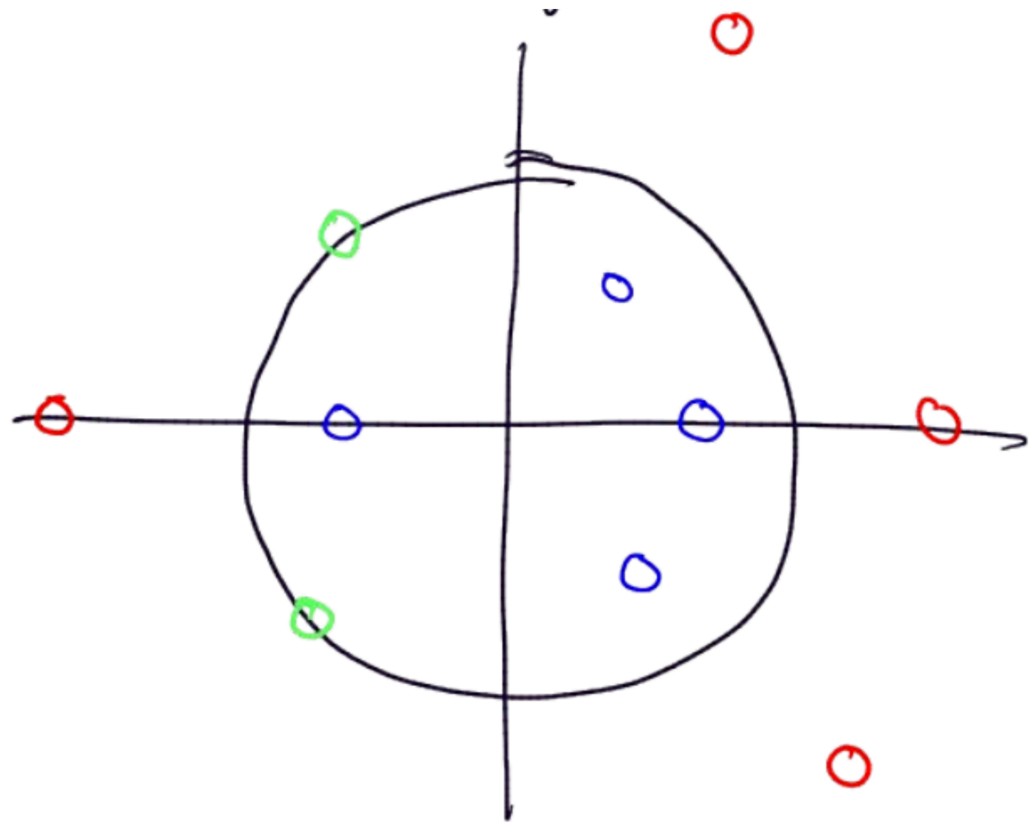
$$H(z) = H_{\min}(z) H_{uc}(z) H_{\max}(z)$$

- A min phase system, system containing only zeros on unit circle, and max phase system

GLP and Min Phase Systems

- Any FIR linear-phase system can be decomposed into:

$H(z) =$



- A min phase sy
on unit circle,

Big Ideas

- Frequency Response of LTI Systems
  - Magnitude Response, Phase Response, Group Delay
- All Pass Systems
  - Used for delay compensation
- Minimum Phase Systems
  - Can compensate for magnitude distortion
  - Minimum energy-delay property
- Generalized Linear Phase Systems
  - Useful for design of causal FIR filters

Admin

- HW 6
  - Out now
  - Due Tuesday 3/28
  - No MATLAB problem

- Midterm returned on Thursday
  - Exam and solutions will be posted