ESE 531: Digital Signal Processing

Lec 23: April 20, 2017

Compressive Sensing

Previously

- Today
  - DTFT, DFT, FFT practice
  - Compressive Sampling/Sensing

Example 2:

A sequence $x = \{x[n], n = 0,1,\dots,N-1\}$ is given; let $X(e^{j\omega})$ be its DTFT.

(a) Suppose $N = 10$. You want to evaluate both $X(e^{j2\pi 7/12})$ and $X(e^{j2\pi 3/8})$. The only computation you can perform is one DFT, on any one input sequence of your choice. Can you find the desired DTFT values? *(Show your analysis and explain clearly.)*

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(b) Suppose N is large. You want to obtain $X(e^{j\omega})$ at the following $2M$ frequencies:

$$\omega = \frac{2\pi}{M}m,\quad m = 0,1,\ldots,M-1 \quad \text{and} \quad \omega = \frac{2\pi}{M}m + \frac{2\pi}{N},\quad m = 0,1,\ldots,M-1.$$

Here $M = 2^{\mu} \ll N = 2^{\nu}$

A standard radix-2 FFT algorithm is available. You may execute the FFT algorithm *once* or *more than once*, and *multiplications* and *additions* outside of the FFT are *allowed*, if necessary.

You want to get the $2M$ DTFT values with as few *total multiplications* as possible (*including those in the FFT*). Give explicitly the best method you can find for this, with an estimate of the *total number of multiplications* needed in terms of M and N.

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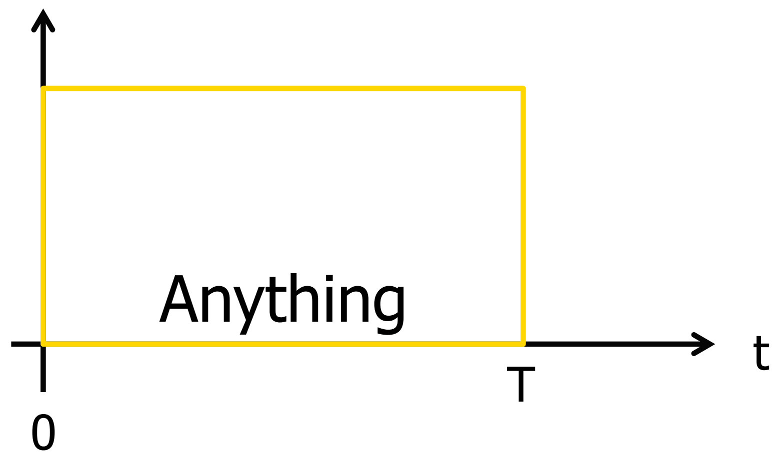
Does your result change if extra multiplications outside of FFTs are *not* allowed?

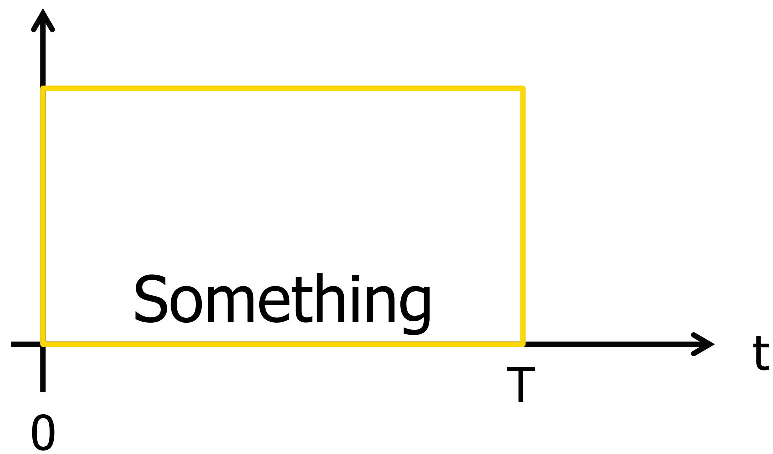
Compressive Sampling

Compressive Sampling



- What is the rate you need to sample at?
  - At least Nyquist

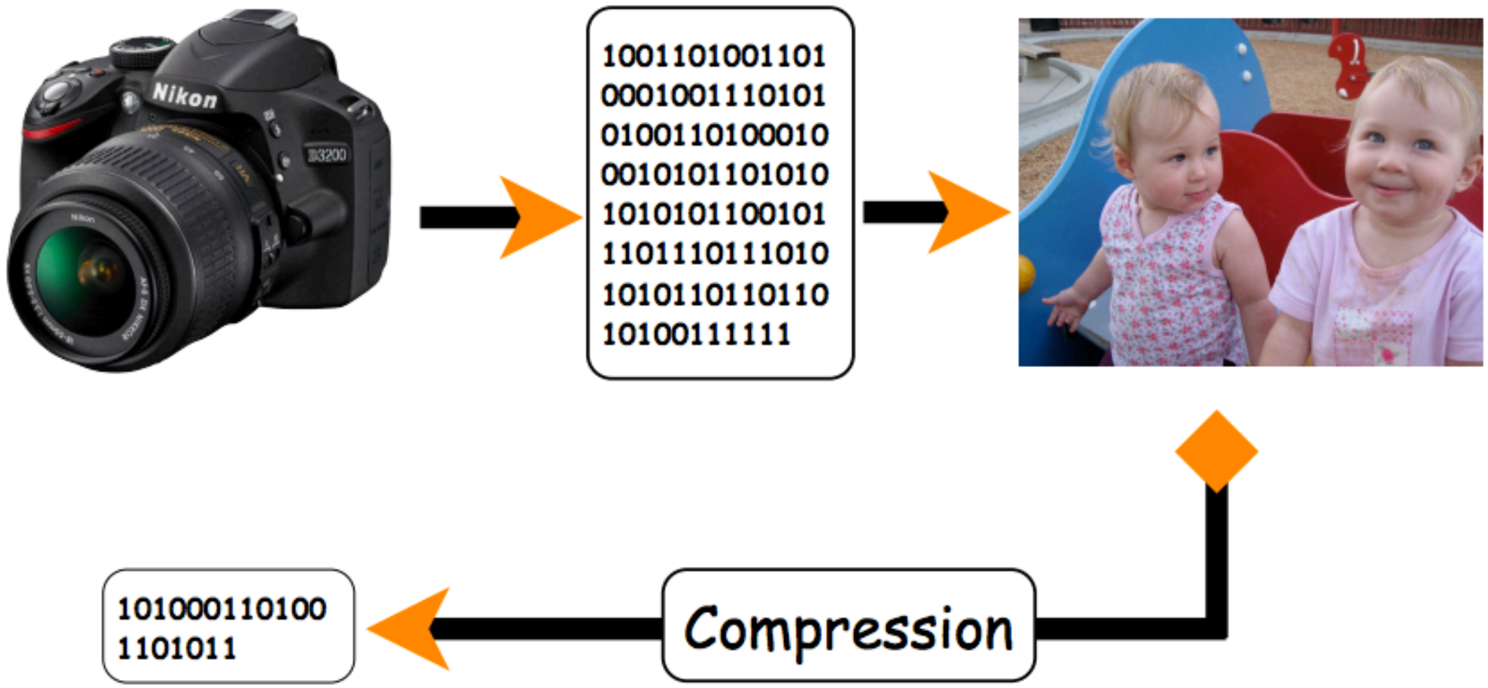
Compressive Sampling



- What is the rate you need to sample at?
  - Maybe less than Nyquist…

First: Compression

- Standard approach
  - First collect, then compress
    - Throw away unnecessary data



First: Compression

- Examples
  - Audio – 10x
    - Raw audio: 44.1kHz, 16bit, stereo = 1378 Kbit/sec
    - MP3: 44.1kHz, 16 bit, stereo = 128 Kbit/sec
  - Images – 22x
    - Raw image (RGB): 24bit/pixel
    - JPEG: 1280x960, normal = 1.09bit/pixel
  - Videos – 75x
    - Raw Video: (480x360)p/frame x 24b/p x 24frames/s + 44.1kHz x 16b x 2 = 98,578 Kbit/s
    - MPEG4: 1300 Kbit/s

First: Compression

- Almost all compression algorithm use transform coding
  - mp3: DCT
  - JPEG: DCT
  - JPEG2000: Wavelet
  - MPEG: DCT & time-difference

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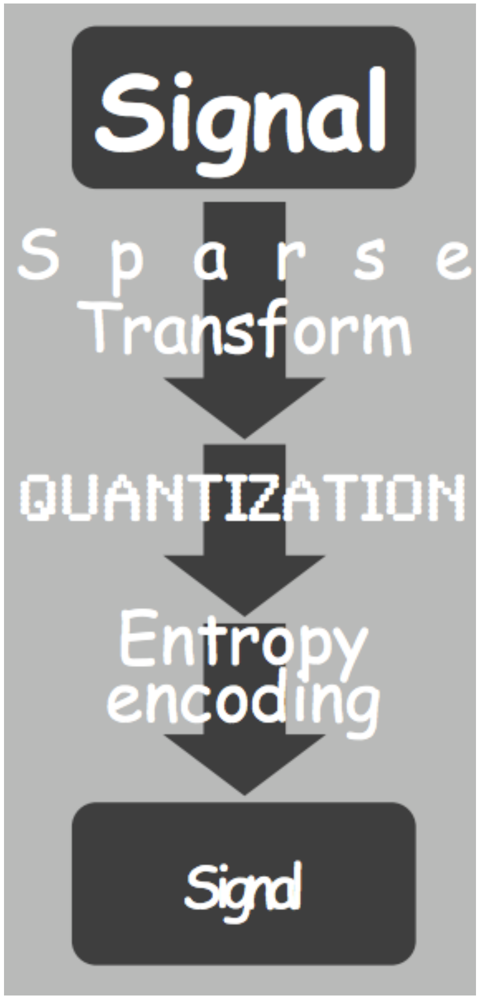
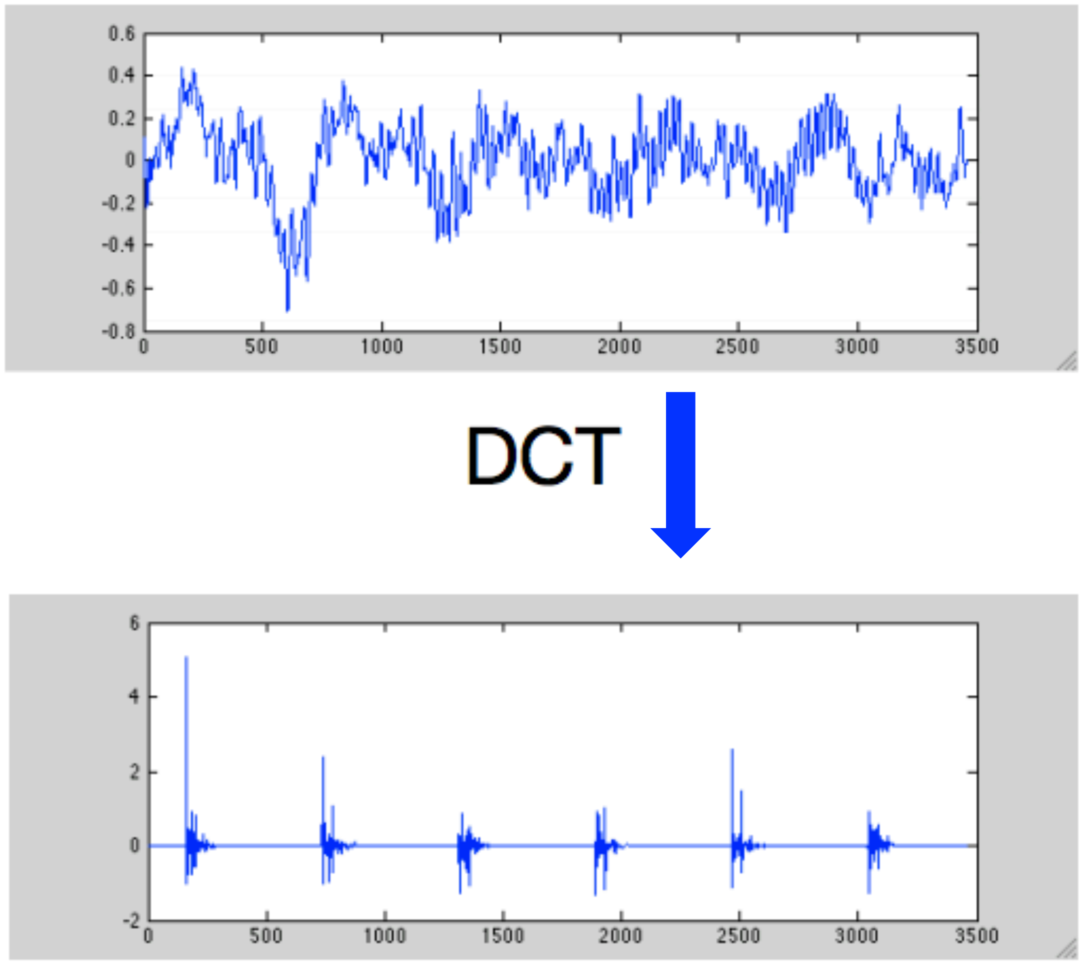


Sparse Transform



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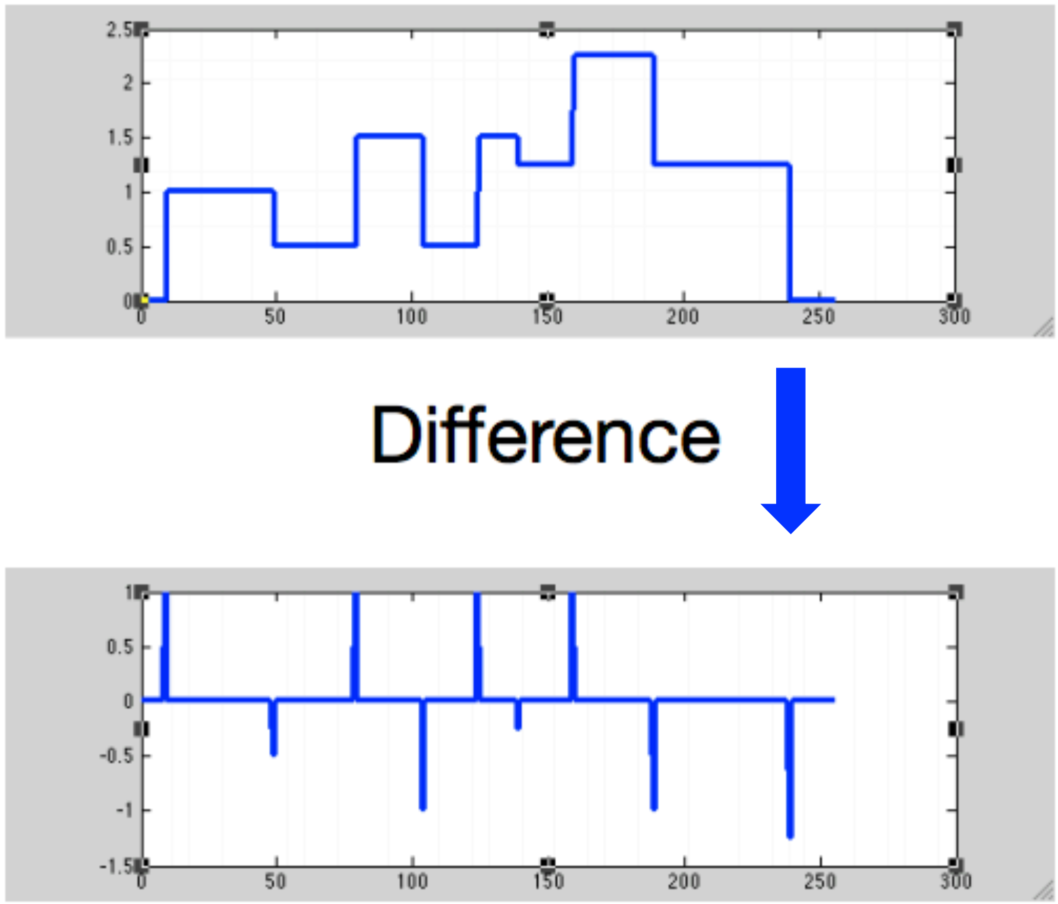

Compressive Sensing/Sampling

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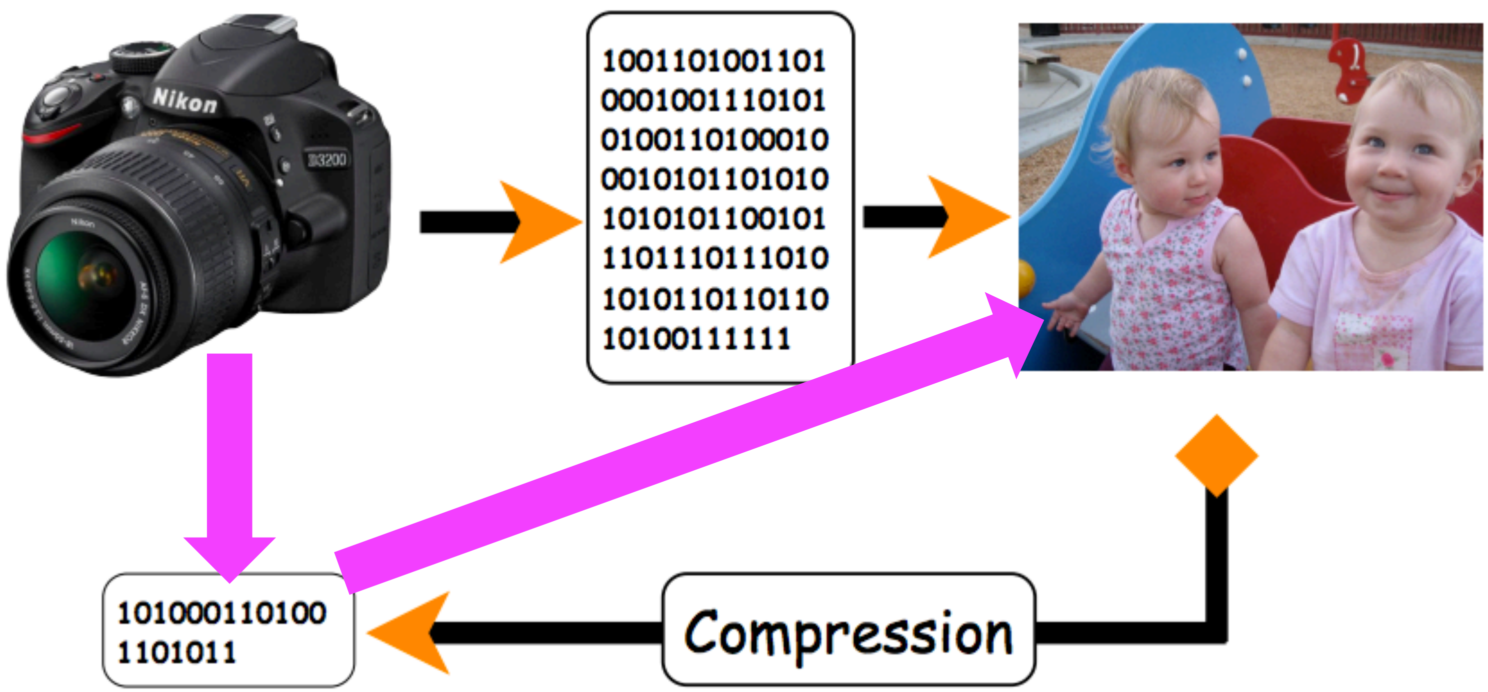

Compressive Sampling

- Sample at lower than the Nyquist rate and still accurately recover the signal, and in some cases exactly recover

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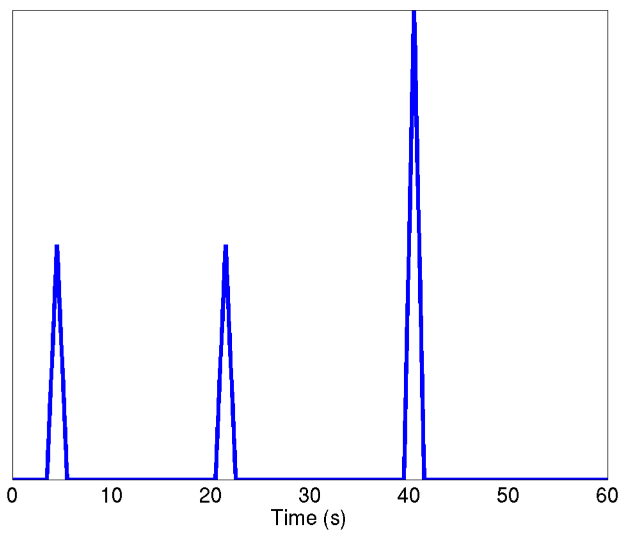
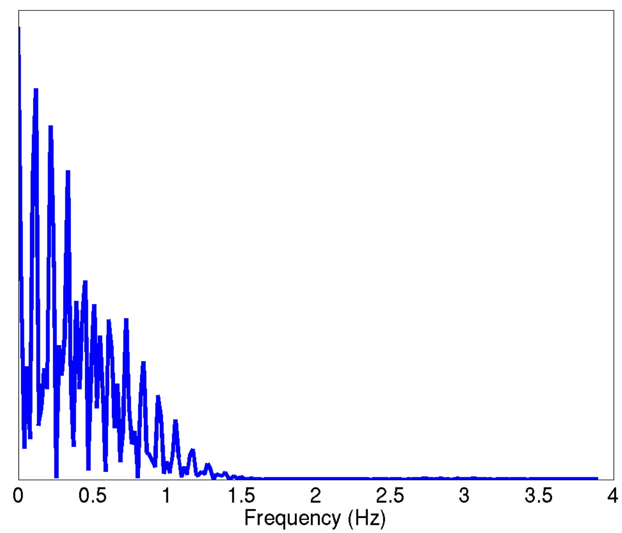
Sparse signal in time



Frequency spectrum



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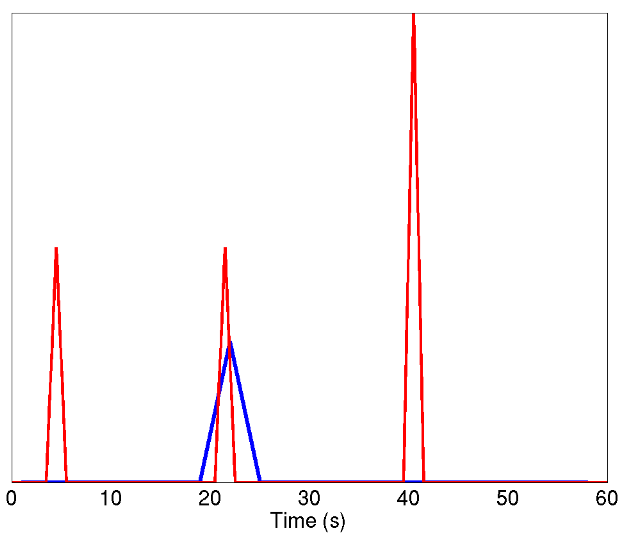
Undersampled in time



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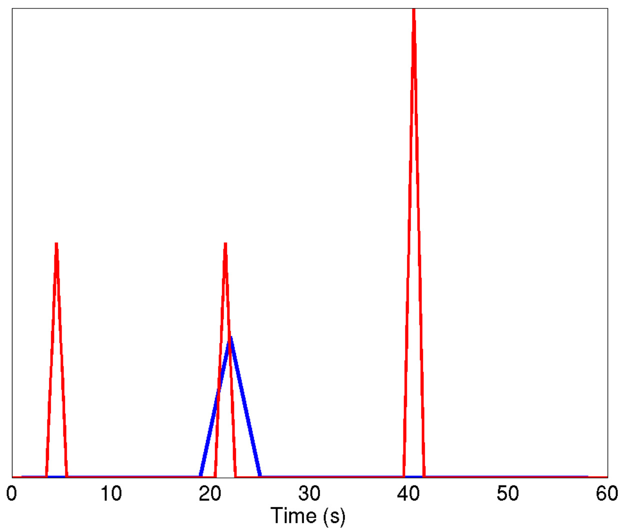

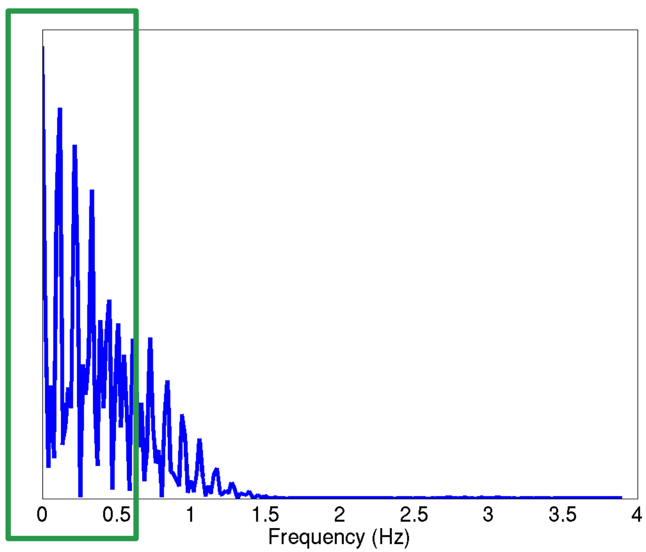

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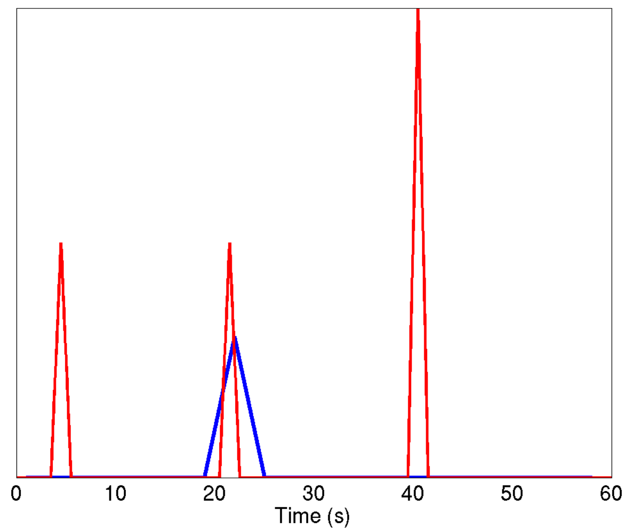
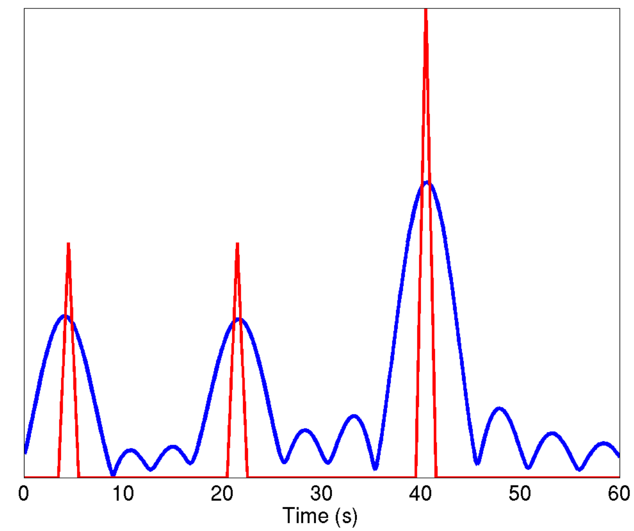


Undersampled in frequency
(reconstructed in time with IFFT)



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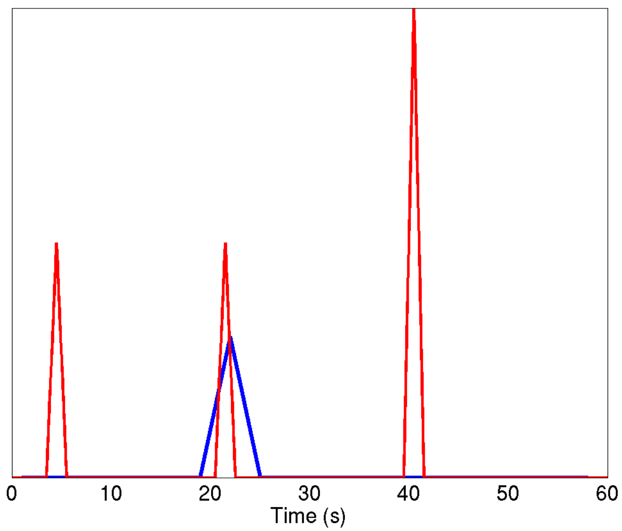

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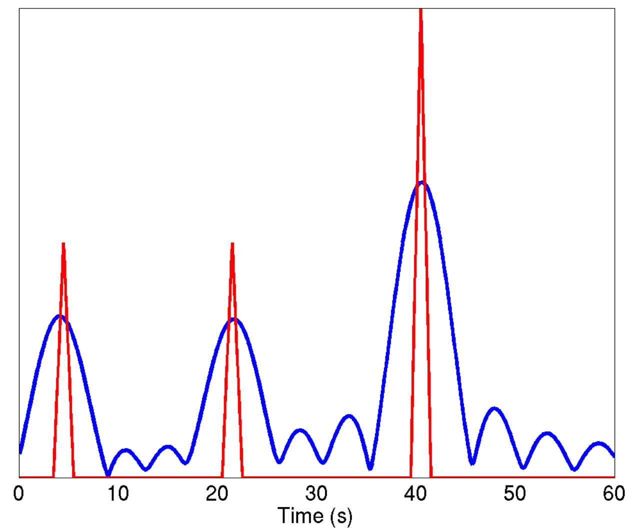
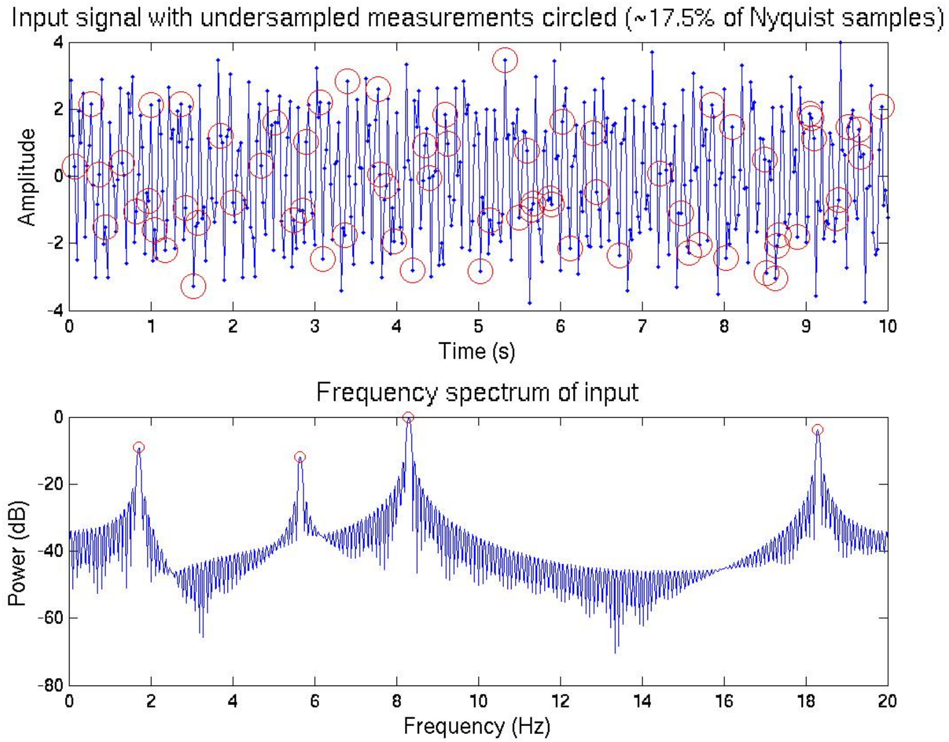


Requires sparsity and incoherent sampling

Compressive Sampling

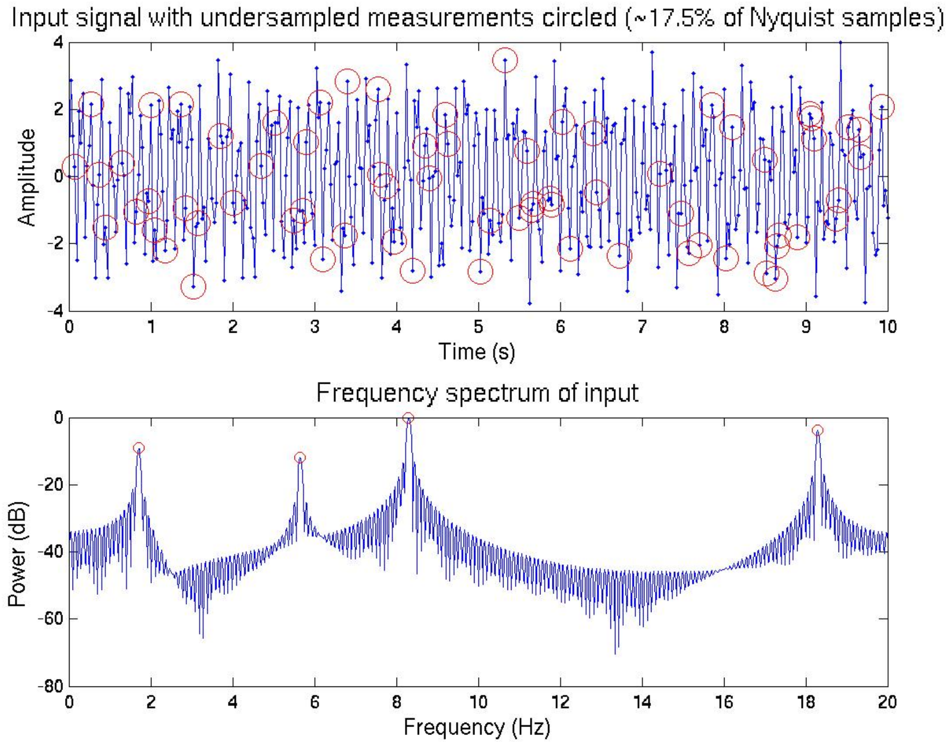


- Sense signal randomly M times
  - M > C· $\mu^2(\Phi, \Psi)$·S·log N
- Recover with linear program

$$\min \sum_{\omega} |\hat{g}(\omega)| \quad \text{subject to} \quad g(t_m) = f(t_m), \quad m = 1, \ldots, M$$

Compressive Sampling

$$\hat{f}(\omega) = \sum_{i=1}^{K} \alpha_i \delta(\omega_i - \omega) \overset{\mathcal{F}}{\Leftrightarrow} f(t) = \sum_{i=1}^{K} \alpha_i e^{i\omega_i t}$$



- Sense signal randomly M times
  - $M > C \cdot \mu^2(\Phi, \Psi) \cdot S \cdot \log N$
- Recover with linear program

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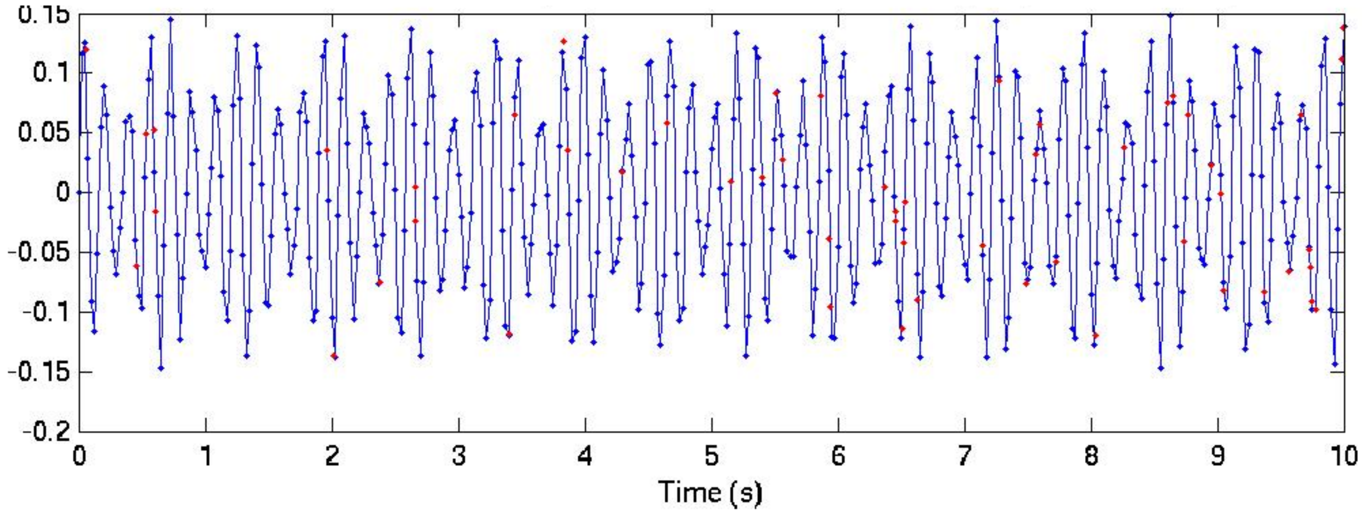
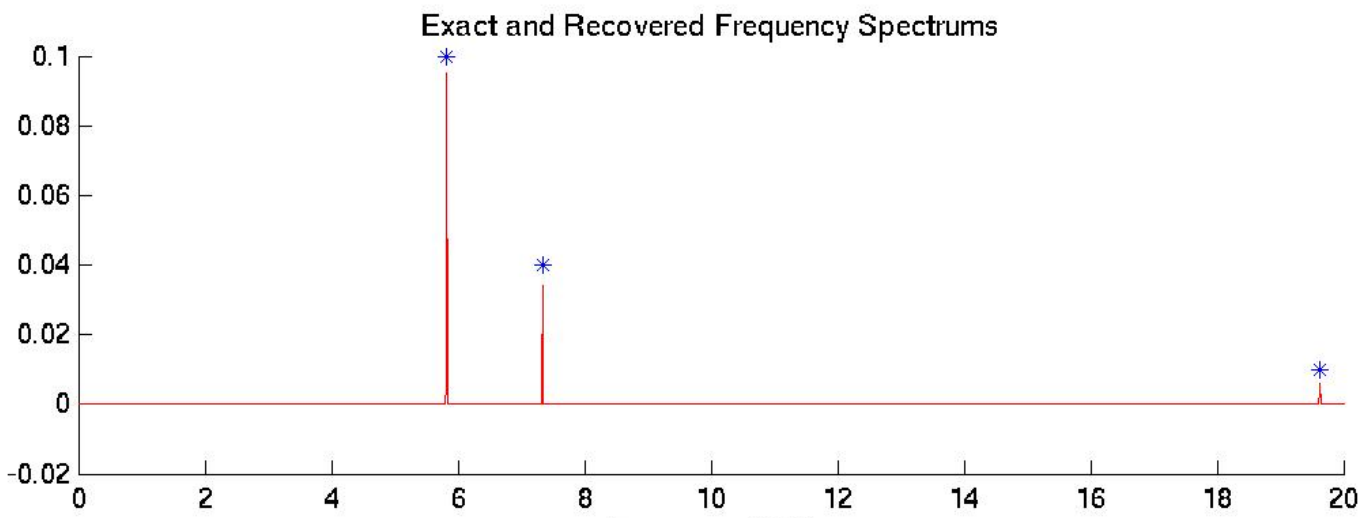
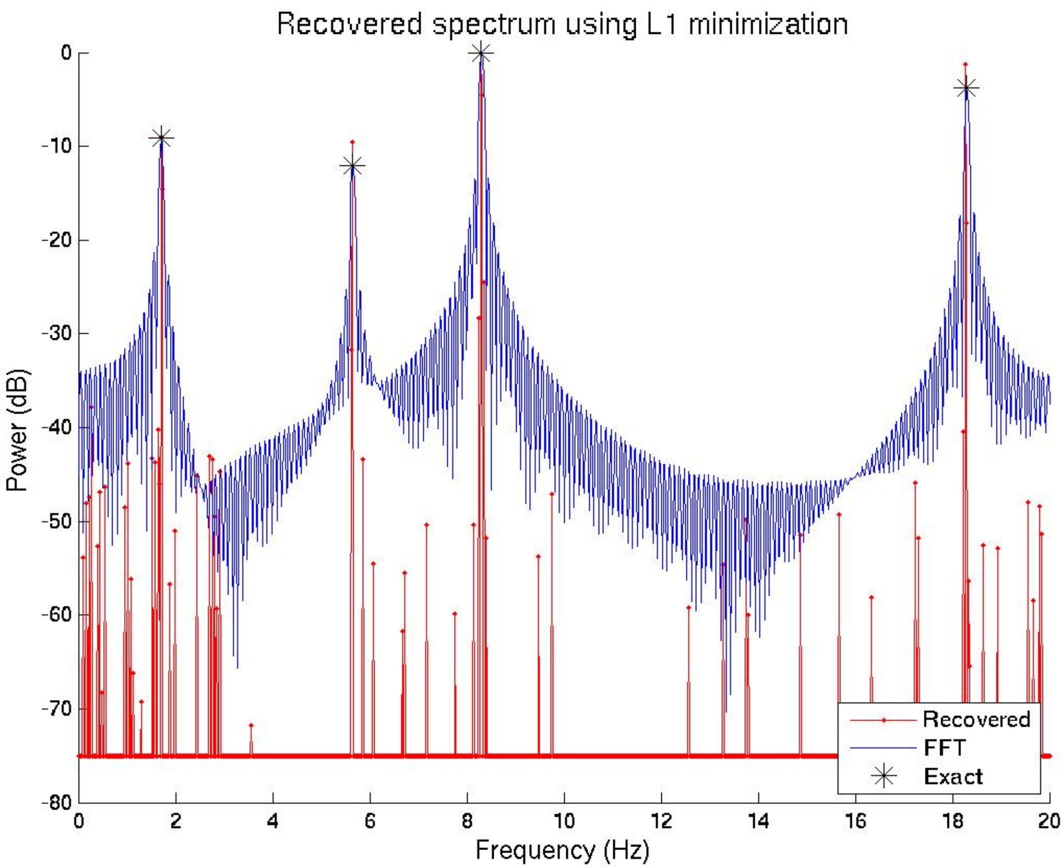
Compressive Sampling: Simple Example



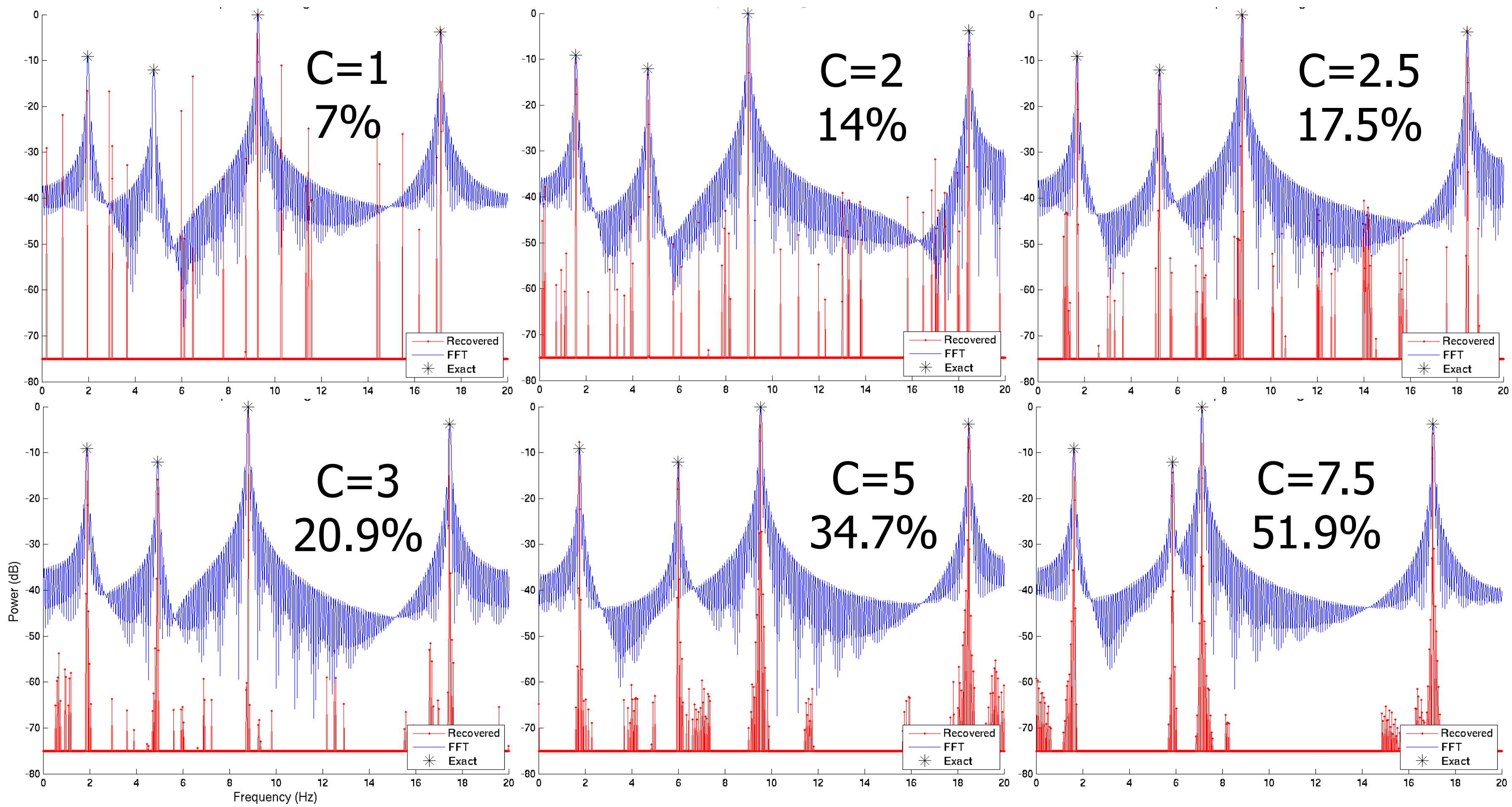


Example: Sum of Sinusoids



- Two relevant "knobs"
  - percentage of Nyquist samples as altered by adjusting optimization factor, *C*
  - input signal duration, *T*
    - Data block size

Example: Increasing C



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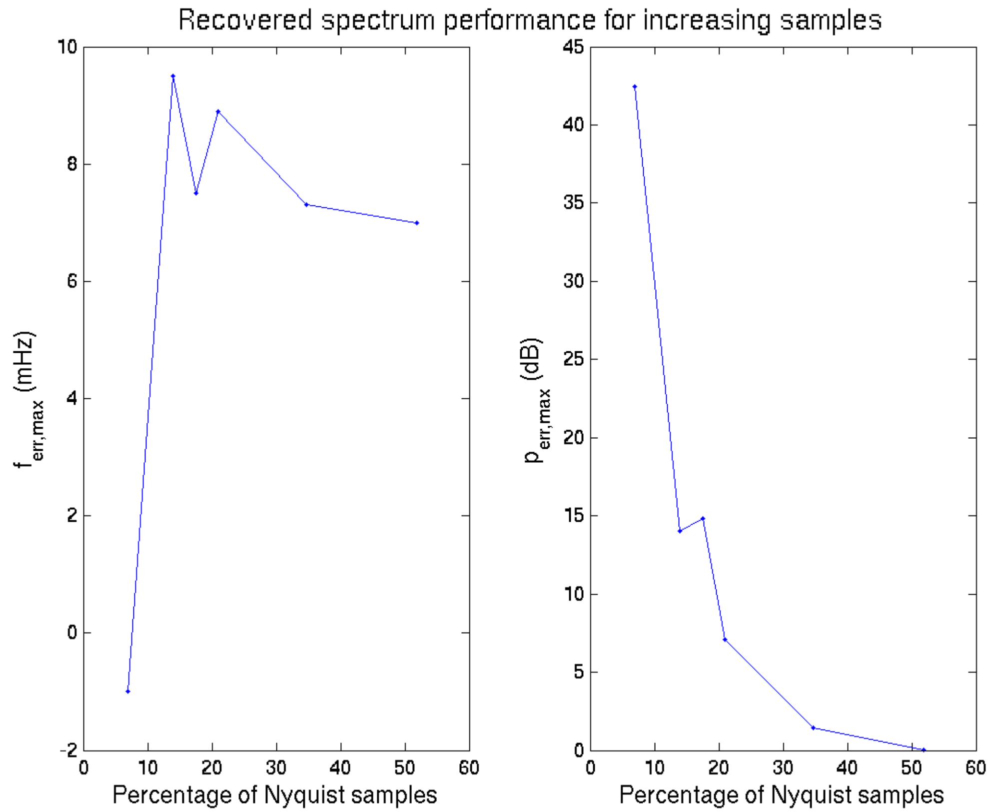
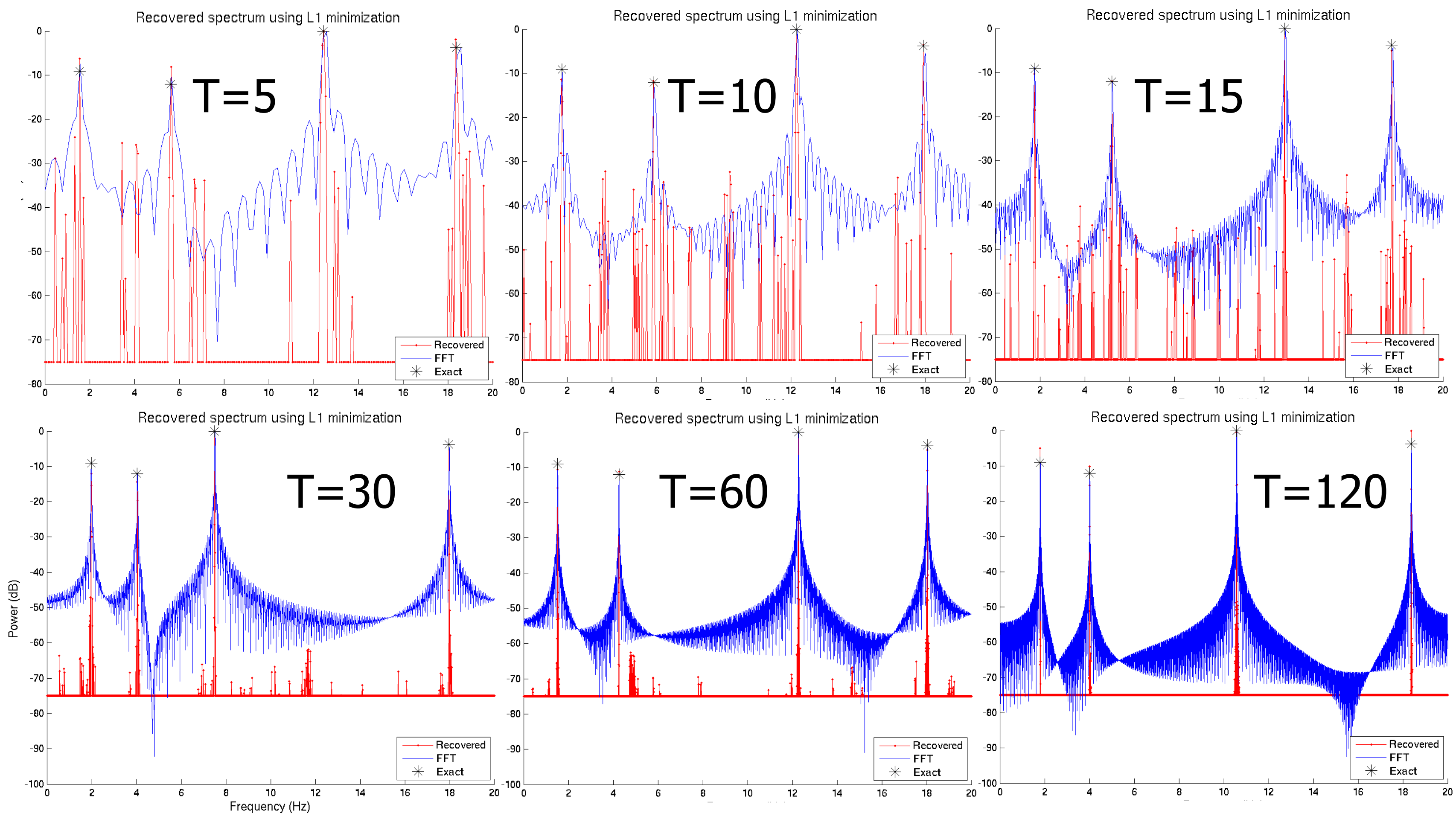


- **$f_{err,max}$ within 10 mHz**
- **$p_{err,max}$ decreasing**

Example: Increasing T



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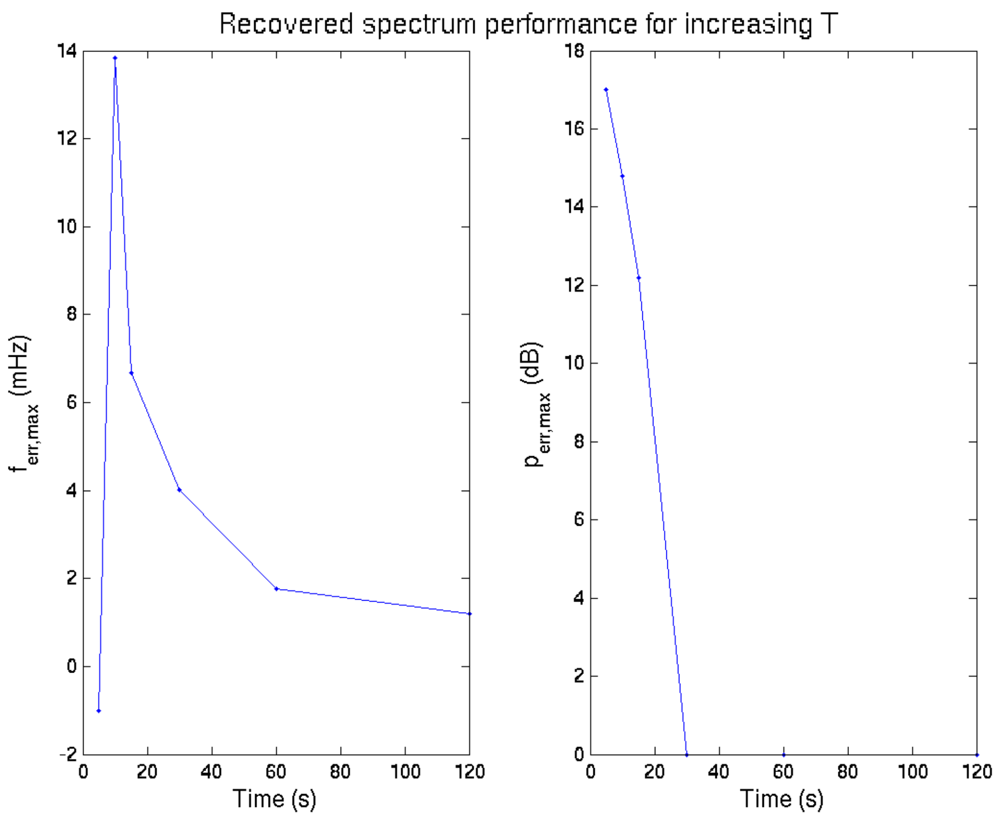
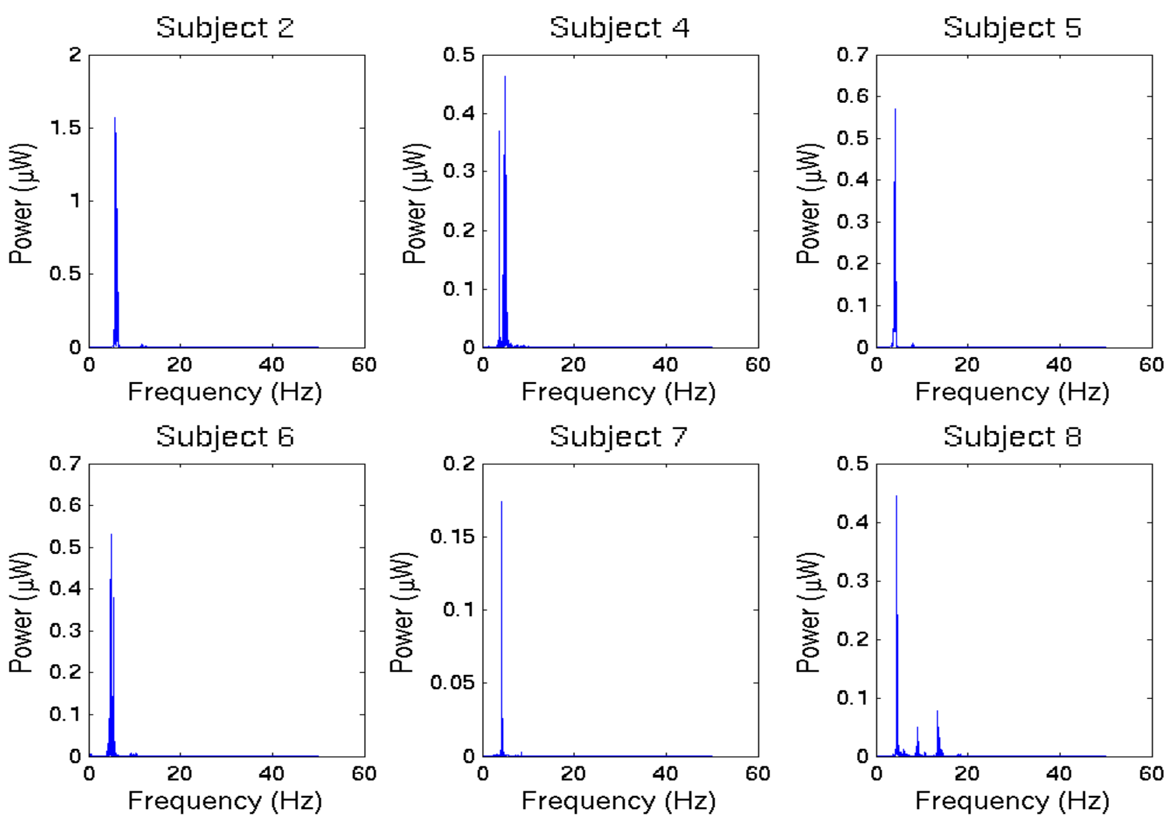


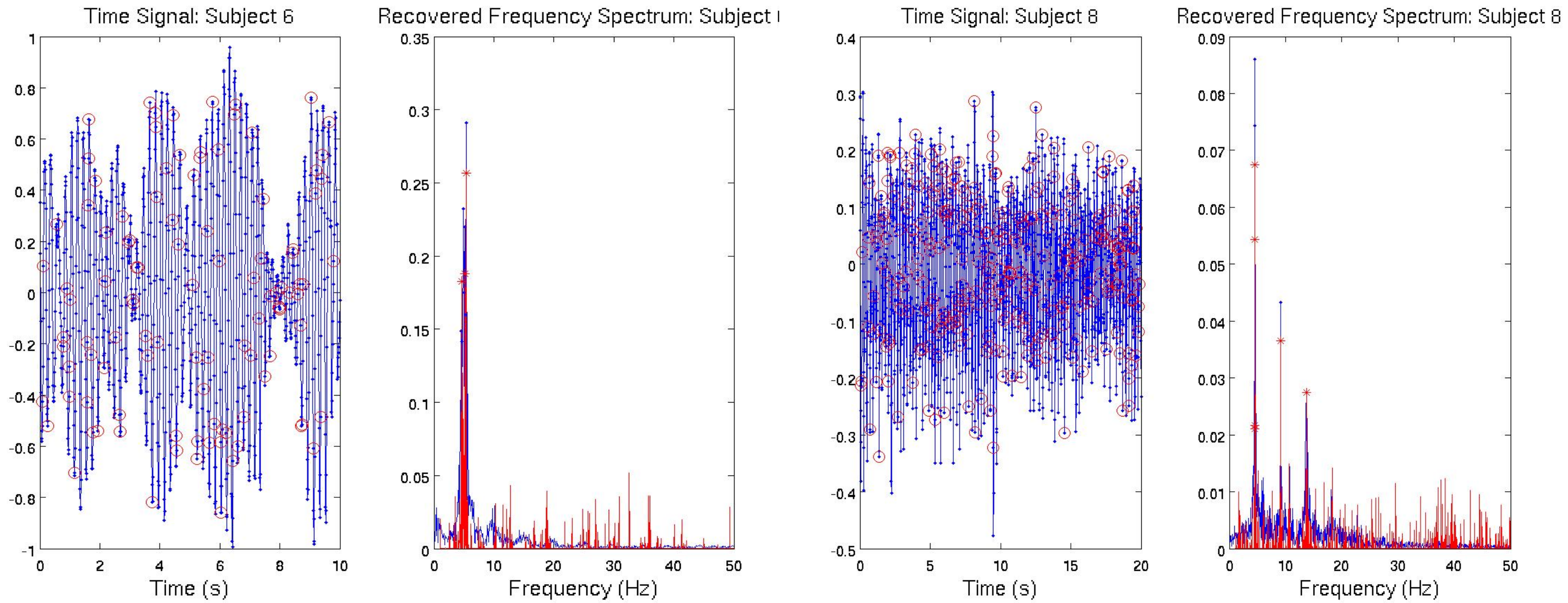
- $f_{err,max}$ **decreasing**
- $p_{err,max}$ **decreasing**

Biometric Example: Parkinson's Tremors



- 6 Subjects of real tremor data
  - collected using low intensity velocity-transducing laser recording aimed at reflective tape attached to the subjects' finger recording the finger velocity
  - All show Parkinson's tremor in the 4-6 Hz range.
  - Subject 8 shows activity at two higher frequencies
  - Subject 4 appears to have two tremors very close to each other in frequency

Compressive Sampling: Real Data



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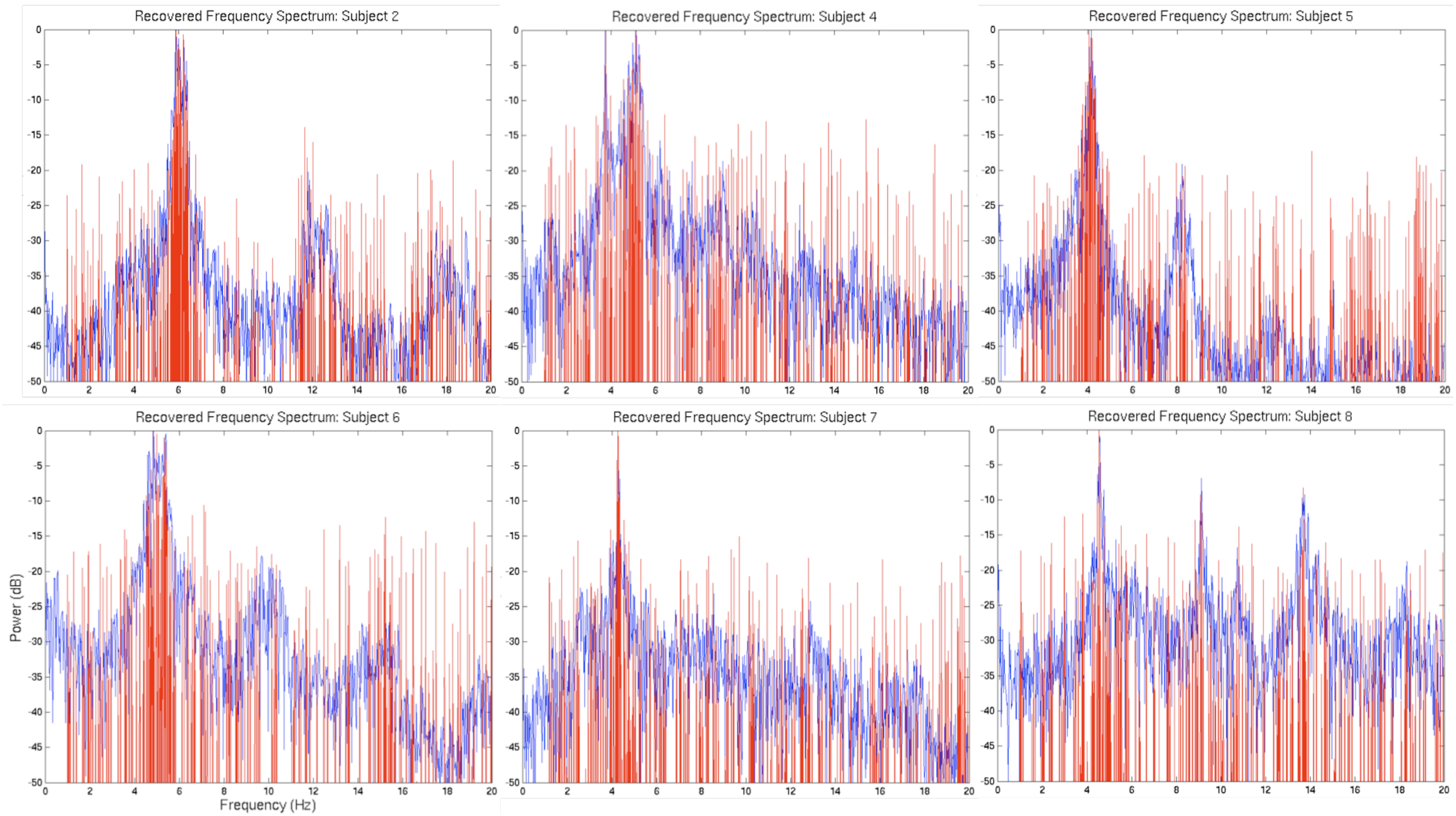
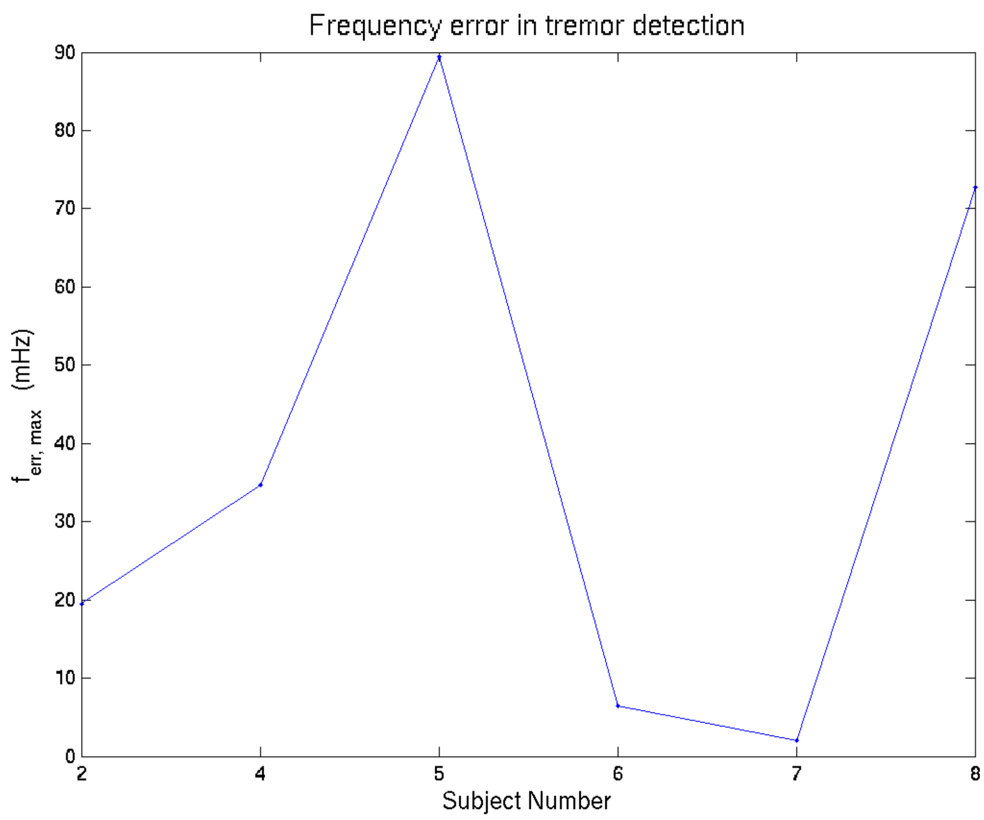


- **C=10.5, T=30**
  - 20% Nyquist required samples

Biometric Example: Parkinson's Tremors



- **Tremors detected within 100 mHz**

Implementing Compressive Sampling

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- Implement hardware on chip to "choose" samples in real time
  - Only write to memory the "chosen" samples
    - Design random-like sequence generator
  - Only convert the "chosen" samples
    - Design low energy ADC

Big Ideas

- Compressive Sampling
  - Integrated sensing/sampling, compression and processing
  - Based on sparsity and incoherency

Admin

- Tania extra office hours
  - M 4-6pm
- Tuesday lecture
  - Review
  - Final exam details
    - Old exam posted
- Project
  - Due 4/25