University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

ESE531, Spring 2017 | Midterm | Thursday, March 16

- 3 Problems with point weightings shown. All 3 problems must be completed.
- Calculators allowed.
- Closed book = No text allowed. One two-sided 8.5x11 cheat sheet allowed.
- Final answers here.
- Additional workspace in "blue" book. Note where to find work in "blue" book if relevant.
- Sign Code of Academic Integrity statement at back of"blue" book.

Name:

Common DTFT pairs:

Sequence	DTFT
$\delta[n]$	1
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)$
1	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega-\omega_0+2\pi k)$
$\alpha^n u[n], \lvert\alpha\rvert<1$	$\frac{1}{1-\alpha e^{-j\omega}}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert \le \omega_c \\ 0, & \omega_c < \lvert\omega\rvert < \pi \end{cases}$

Common z-transform pairs:

Sequence	z-transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\delta[n-m]$	z^{-m}	All z except 0 (if m>0) or ∞ (if m<0)
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$

Trigonometric Identity:

$$e^{jA} = cos(A) + jsin(A)$$

Geometric Series:

$$\sum_{n=0}^{N} r^n = \frac{1-r^{N+1}}{1-r}$$
$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, |r| < 1$$

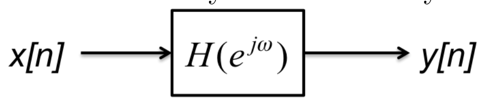
DTFT Equations:

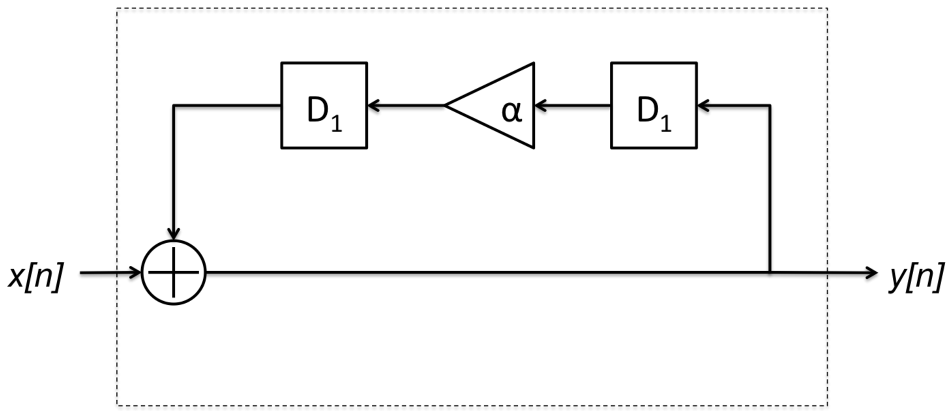
$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k]e^{-j\omega k}$$
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

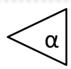
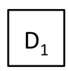
Upsampling/Downsampling:

Upsampling by L (↑L): $X_{up} = X(e^{j\omega L})$
Downsampling by M (↓M): $X_{down} = \frac{1}{M}\sum_{i=0}^{M-1} X(e^{j(\frac{\omega}{M}-\frac{2\pi}{M}i)})$

1. (40 pts) Consider a causal LTI system defined by the following block diagram:





Above, the symbol  denotes multiplication by a scalar α: if the input is $w[n]$, the output is $\alpha w[n]$. The symbol  denotes a delay by one sample: if the input is $w[n]$, the output is $w[n-1]$.

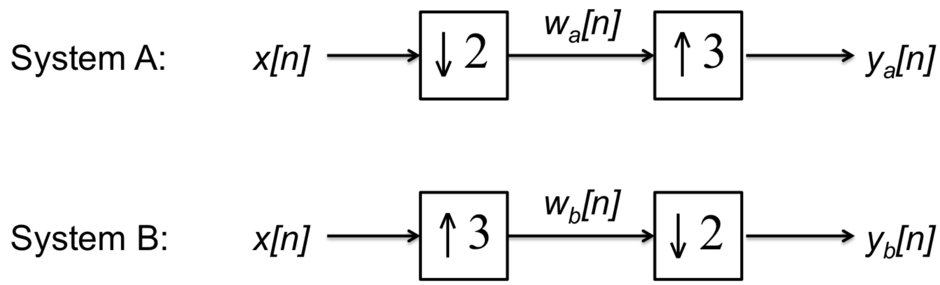
(a) What is the impulse response $h[n]$ of the system?

(b) For what choices of α is the system stable?

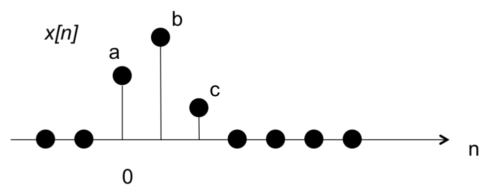
(c) For any α such that the system is stable, what is the frequency response $H(e^{j\omega})$ of the system?

(d) What is the output $y[n]$ when the input is the oscillatory signal $x[n] = (-1)^n$?

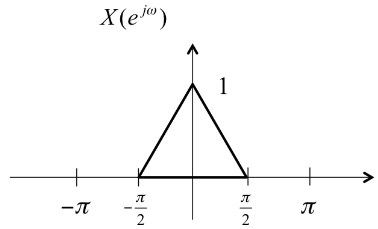
2. (30 pts)Below are two systems consisting of a compressor and expander.



(a) For $x[n]$ shown below, sketch $y_a[n]$ and $y_b[n]$ (assume $x[n] = 0$ outside the range shown).



(b) For $X(e^{j\omega})$ shown below, sketch $W_a(e^{j\omega})$ and $Y_a(e^{j\omega})$.



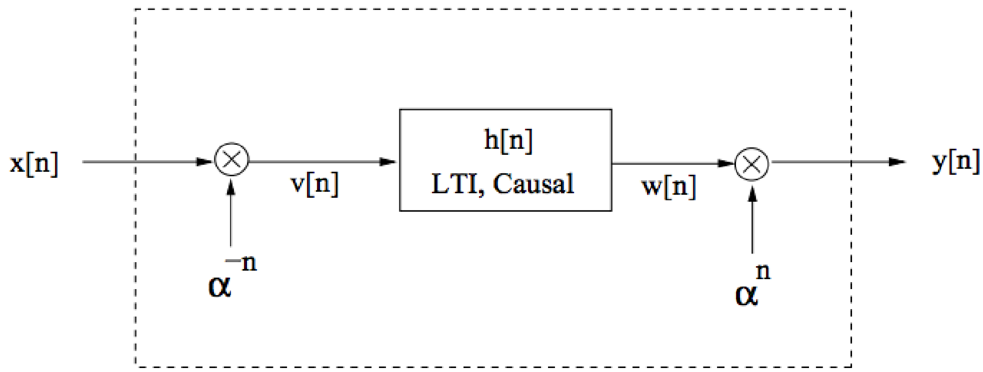
(c) $X(e^{j\omega})$ is now the Fourier transform of an arbitrary signal $x[n]$. Express $Y_b(e^{j\omega})$ in terms of $X(e^{j\omega})$. Your answer should be in the form of an equation, not a sketch.

(d) For any arbitrary $x[n]$, will $y_a[n] = y_b[n]$? If your answer is yes, algebraically justify your answer. If your answer is no, clearly explain or show a counterexample.

3. (30 pts)Consider a causal linear time-invariant system with impulse response $h[n]$. The z-transform of $h[n]$ is

$$H(z) = \frac{1 - z^{-1}}{1 - 0.5z^{-1}} \tag{1}$$

Consider the cascade configuration given in the figure below. Here we assume that α is a real number.



(a) Draw the pole-zero plot of $H(z)$. Also, sketch the region of convergence. Is this system stable?

(b) Assume that $\alpha = \frac{1}{3}$ and $x[n] = \left(\frac{1}{4}\right)^n u[n]$. Compute the output $y[n]$ for the given input $x[n]$.

(c) Find the response $g[n]$ of the overall system within the enclosed box to the input $x[n] = \delta[n]$.

(d) Plot the pole-zero plot of $G(z)$, the z-transform of $g[n]$. Also, sketch the region of convergence.