ESE 531 Digital Signal Processing — Mid-Term Exam

October 15, 2015 | **Closed Book/Notes** | **1 Hr. 20 Mins**

Given Information is on Page 3   ***Explain the reasoning leading to your answers.***

Problem 1

In Figure 1a the LTI system H in the inner box has unit-sample response h. a and b are complex constants.

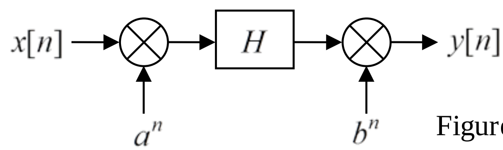


Figure 1a

(a) Suppose $b = \frac{1}{a} = e^{-j\pi}$, and H is an ideal, zero-phase LPF with cut-off frequency $\omega_c = \pi/2$.

Let x have the DTFT $X(e^{j\omega})$ shown in Figure 1b.

Sketch the DTFT of the output y.

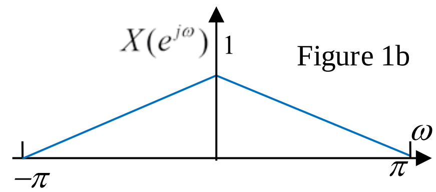


Figure 1b

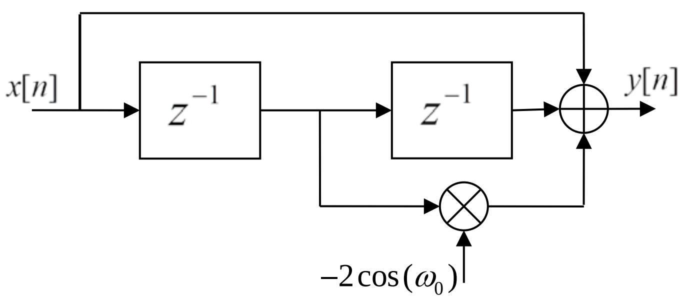
(b) Suppose more generally that $b = \frac{1}{a}$ in Figure 1a.

Is the overall system of Figure 1a *linear and time-invariant* (LTI)? *Prove or disprove*.

(Different approaches are possible for this).

Problem 2

Consider the causal system below:



(a) Find the transfer function $H(z)$ of the system and determine its poles and zeros.

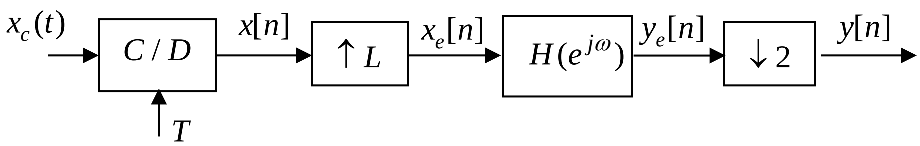
(b) Let $\omega_0 = 0$.

(i) Does the system have a *causal inverse*? If so, find the *unit-sample response* of the causal inverse.

(ii) Can you find a *causal, stable* inverse? If not, suggest an *approximate* inverse that is causal and stable. *(Brief answer, no detailed analysis)*

Problem 3

In the system below $H(e^{j\omega}) = \begin{cases} 1, & \text{for } \omega_1 < |\omega| < \omega_2 \\ 0, & \text{elsewhere} \end{cases}$, on $[-\pi, \pi]$



(a) Let $\boldsymbol{L} = \mathbf{2}$, $\omega_1 = \pi/2, \ \ \omega_2 = \pi$, and $x_c(t) = \cos\left(\frac{\pi}{3T} t\right)$.

Sketch $X_e(e^{j\omega})$. Find $\{y_e[n]\}$ and the output $\{y[n]\}$.

(b) Consider the case $\boldsymbol{L} = \mathbf{4}$ and $\omega_1 = \pi/4$, $\omega_2 = 3\pi/4$ for the filter. Assume

$$X_c(j\omega_a) = \begin{cases} 1 - |\omega_a| \frac{T}{\pi}, & |\omega_a| < \frac{\pi}{T} \\ 0, & \textit{otherwise} \end{cases}$$

Sketch $Y(e^{j\omega})$, labeling key values.

How does the answer change if a *unit-delay* is inserted before the compressor $\downarrow 2$?

Given: For $\uparrow L$ $\quad X_e(e^{j\omega}) = X(e^{j\omega L})$; $\quad$ for $\downarrow 2$ $\quad Y(e^{j\omega}) = \frac{1}{2} Y_e\left(e^{j\omega/2}\right) + \frac{1}{2} Y_e\left(e^{j[\omega - 2\pi]/2}\right)$

Common DTFT pairs:

Sequence	DTFT
$\delta[n]$	1
$u[n]$	$\frac{1}{1-e^{-j\omega}}+\sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k)$
1	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega+2\pi k)$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty}2\pi\delta(\omega-\omega_0+2\pi k)$
$\alpha^n u[n], \lvert\alpha\rvert<1$	$\frac{1}{1-\alpha e^{-j\omega}}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega})=\begin{cases}1, & \lvert\omega\rvert\le\omega_c\\ 0, & \omega_c<\lvert\omega\rvert<\pi\end{cases}$

Common z-transform pairs:

Sequence	z-transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert>1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert<1$
$\delta[n-m]$	z^{-m}	All z except 0 (if m>0) or ∞ (if m<0)
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert>\lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert<\lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert>\lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert<\lvert a\rvert$

Trigonometric Identities:

$$\sin(A-B)=\sin(A)\cos(B)-\cos(A)\sin(B)$$

$$\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B)$$

$$\cos(A)+\cos(B)=2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

Geometric Series: $\sum_{n=0}^{N}r^n=\frac{1-r^{N+1}}{1-r}$

$$\cos(A)=\frac{e^{jA}+e^{-jA}}{2}$$