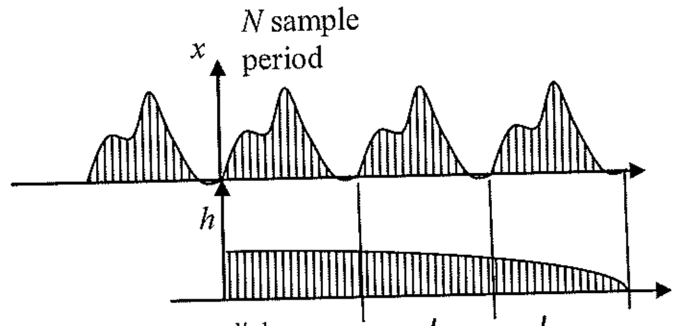
Example 1

(a) $y[n]=\sum_{k=0}^{K-1}x[n-k]h[k]$; since x is preiodic of period N, we anticipate the same for y

Consider $y[n+N]=\sum_{k=0}^{K-1}x[n+N-k]h[k]=\sum_{k=0}^{K-1}x[n-k]h[k]=y[n]$

Therfore y is periodic of period N



(b) $K=mN$

We see that the result of the convolution in each *output* periodic block of length N is a circular convolution of one period of

the input $\tilde{x}=\{x[0],...,x[N-1]\}$ *with* $\left\{\tilde{h}[n]\right\}_{n=0}^{N-1}=\left\{\sum_{p=0}^{m-1}h[n+pN]\right\}_{n=0}^{N-1}$

This can be implemented by multiplying the DFT's of $\tilde{x}$ and of

the $\left\{\sum_{p=0}^{m-1}h[n+pN]\right\}_{n=0}^{N-1}$, each of which requires $\frac{N}{2}\log N$ multiplications (using the

FFT), followed by an IDFT. Thus total number of multiplies is $\left(\frac{3N}{2}\log N\right)+N$ to

compute the output block of N samples; the rest of the output is just this block repeated.

- For the given numbers, this is a total of $1536\times 10+1024=$ $\boxed{16384 \text{ multiplications}}$

- Regular *convolution* requires mN multiplies for each output point, so for a total of N output points we need mN^2 multiplies.
- Even regular *circular* convolution (not using FFT) requires a total of N^2 multiplies which is $>10^6$ here.

Example 2

(a)

$e^{j2\pi 7/12} = e^{j2\pi 14/24}$, and $e^{j2\pi 3/8} = e^{j2\pi 9/24}$

We can compute the DTFT values on 24 equi-spaced points around unit circle, by computing the DFT of a sequence of length 24. Therefore in this case we can append 14 zeros to the given x sequence and obtain its DFT. Then the 10th and 15th output coefficients $X[9]$ and $X[14]$ will be the desired values.

With zeros appended, DFT is $X[k] = \sum_{n=0}^{23} x[n] e^{-j\frac{2\pi}{24}kn} = \sum_{n=-\infty}^{\infty} x[n]\left(e^{j\frac{2\pi}{24}k}\right)^{-n} = X(e^{j\omega})\big|_{\omega=\frac{2\pi}{24}k}$, $k = 0,1,\ldots,23$

(b) We are now computing the DTFT on (i) M uniformly spaced locations on the full unit circle (starting at $\omega = 0$), and also at (ii) a set of M locations offset from these by a small angle $2\pi / N$

-- The first set of M DTFT values can be obtained by taking the M-point DFT of an M-point sequence x_1 derived from x as follows:

$$x_1[m] = \sum_{r=0}^{\infty} x[m+Mr], \quad m = 0,1,\ldots,M-1, \text{ where } x[k] = 0 \text{ for } k > N-1.$$

No multiplications are needed in forming this finite sequence from x.

--For the DTFT at $\omega = \frac{2\pi}{M}m + \frac{2\pi}{N}$, $m = 0,1,\ldots,M-1$ we want

$$X\left(e^{j\left(\frac{2\pi}{M}m+\frac{2\pi}{N}\right)}\right) = \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi}{M}m+\frac{2\pi}{N}\right)n} = \sum_{n=0}^{N-1} \underbrace{x[n] e^{-j\frac{2\pi}{N}n}}_{y[n]} e^{-j\frac{2\pi}{M}mn}, \quad m = 0,1,\ldots,M-1$$

Thus with $y[n] \triangleq x[n] e^{-j\frac{2\pi}{N}n}$, $n = 0,1,\ldots,N-1$, we are now looking for the DTFT of y at M uniformly spaced locations on the full unit circle (starting at $\omega = 0$). This can again be found by taking the M-point DFT of an M-point sequence y_1 derived from y as follows:

$$y_1[n] = \sum_{r=0}^{\infty} y[n+Mr], \quad n = 0,1,\ldots,M-1, \text{ where } y[k] = 0 \text{ for } k > N-1.$$

No multiplications are needed in forming this finite sequence from y.

Therefore we need a total of 2 M-point FFT's, and N multiplications (to obtain sequence y from x). The total number of multiplications is then approximately

$$2\frac{M}{2}\log_2 M + N = \boxed{M\log_2 M + N}$$

Using a single FFT on the original sequence x, since N is divisible by M, we would need an N-point FFT with total number of multiplications $\frac{N}{2}\log_2 N$. This can be substantially larger than the result above; e.g. for $N=2^{14}=16{,}384$ and $M=2^8=256$, we need approximately 18,416 vs. 114,688