University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

ESE531, Spring 2018 — HW2: DTFT and z-transform — Thursday, January 25

Due: Friday, February 2nd, 11:59PM

- **Recommended Problems for Practice:** From the book: 3.10, 3.23, 3.45
- **Homework Problems:** All problems below must be turned in and are not optional for full credit

1. From the book: 3.31 (a), (b) and (d), 3.38, 3.42, 3.48
2. Matlab problem 1:
   The finite-length pulse is always used as a prime example of evaluating the DTFT. Suppose that the rectangular pulse $r[n]$ is defined by

   $$r[n] = \begin{cases} 1 & 0 \le n < L \\ 0 & \text{else} \end{cases}$$

   (a) Write an expression for the DTFT, $R(e^{j\omega})$, of $r[n]$.

   (b) Use the function `dtft` to evaluate the DTFT of a 12-point pulse. Make a plot of the DTFT versus ω over the range $-\pi \le \omega < \pi$. Plot the real and imaginary parts separately and submit your plots.

   (c) Plot the magnitude of the DTFT (see `abs` in MATLAB). To make the plot appear smooth, choose a number of frequency samples that is 5 to 10 times the length of the pulse length. Submit this plot. Note that the magnitude plot is more useful than the real and imaginary plots. Experiment plotting with different numbers of frequency samples. Label your axes.

   (d) Notice that the zero crossings of the magnitude of the DTFT are evenly spaced. How many zero crossings are there? What is the peak height of the magnitude?

   (e) Repeat the DTFT plot for L=15. Submit the plot. What is the peak height? How many zero crossings?

   (f) Write an expression for the location of the zero crossings as a function of L. Write an expression for the peak height as an function of L.

3. Matlab problem 2:
   Create an .m file that contains a MATLAB function `psinc(w,L)` that will evaluate the periodic sinc function on a frequency grid using the formula

   $$psinc(\omega, L) = \frac{sin(\frac{1}{2}\omega L)}{sin\frac{1}{2}\omega} \tag{1}$$

The function should have two inputs: a length `L` and a vector of frequencies `w`. It must check for division by zero as happens for $\omega = 0$. Directly evaluate the DTFT from MATLAB problem 1(a) using your function and plot the magnitude. Compare it with your plot using the `dtft` function. Submit your function code and the code you use to create your plot as well as the plot.

4. Matlab problem 3:
   For the signal $x[n] = (0.9)^n u[n]$, compute the DTFT $X(e^{j\omega})$ using `freqz`.
   (a) Make a plot of both the magnitude and the phase versus ω over the range $[-\pi, \pi)$. This will require a shift of the `[X,W]` vectors returned from `freqz`. Submit your plot. Explain why the magnitude is even and the phase is an odd function of ω.
   (b) Derive formulas for the magnitude and phase of the DTFT.
   (c) Compute and plot the magnitude and phase by a direct evaluation of the formulas derived in (b) and compare to plot in (a). How do they compare?