ESE 531: Digital Signal Processing

Lec 15: March 15, 2018

Design of IIR Filters

Linear Filter Design

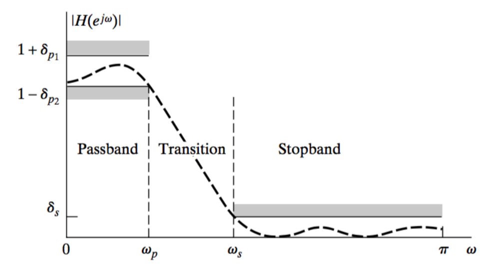
- Used to be an art
  - Now, lots of tools to design optimal filters
- For DSP there are two common classes
  - Infinite impulse response IIR
  - Finite impulse response FIR
- Both classes use finite order of parameters for design

What is a Linear Filter?

- Attenuates certain frequencies
- Passes certain frequencies
- Affects both phase and magnitude

- What does it mean to design a filter?
  - Determine the parameters of a transfer function or difference equation that approximates a desired impulse response (h[n]) or frequency response ($H(e^{j\omega})$).

Filter Specifications



What is a Linear Filter?

- Attenuates certain frequencies
- Passes certain frequencies
- Affects both phase and magnitude

- IIR
  - Mostly non-linear phase response
  - Could be linear over a range of frequencies
- FIR
  - Much easier to control the phase
  - Both non-linear and linear phase

Today

- IIR Filter Design
  - Impulse Invariance
  - Bilinear Transformation
- Transformation of DT Filters
- FIR Filters

IIR Filter Design

- Transform continuous-time filter into a discrete-time filter meeting specs
  - Pick suitable transformation from s (Laplace variable) to z (or t to n)
  - Pick suitable analog $H_c(s)$ allowing specs to be met, transform to $H(z)$
- We've seen this before… impulse invariance

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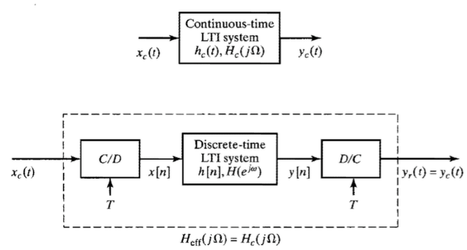
Impulse Invariance

- Want to implement continuous-time system in discrete-time



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Impulse Invariance

- With $H_c(j\Omega)$ bandlimited, choose

$$H(e^{j\omega}) = H_c(j\frac{\omega}{T}), \qquad |\omega| < \pi$$

- With the further requirement that T be chosen such that

$$H_c(j\Omega) = 0, \qquad |\Omega| \geq \pi / T$$

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$$\boxed{h[n] = Th_c(nT)}$$

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IIR by Impulse Invariance

- If $H_c(j\Omega) \approx 0$ for $|\Omega_d| > \pi/T_d$, there is no aliasing and $H(e^{j\omega}) = H(j\omega/T_d)$, $|\omega| < \pi$
- To get a particular $H(e^{j\omega})$, find corresponding H_c and T_d for which above is true (within specs)
- Note: T_d is not for aliasing control, used for frequency scaling.

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Example

Example: If $\boxed{H_c(s) = \frac{A_k}{s - p_k}}$ (e.g. one term in PF expansion)

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$h_c(t) = A_k e^{p_k t}, \quad t \geq 0;$

$$e^{at} \overset{L}{\longleftrightarrow} \frac{1}{s-a}$$

Example

Example: If $\boxed{H_c(s) = \frac{A_k}{s - p_k}}$ (e.g. one term in PF expansion)

$h_c(t) = A_k e^{p_k t}, \quad t \geq 0; \quad h[n] = T_d A_k e^{p_k T_d n} = T_d A_k \left(e^{p_k T_d}\right)^n$

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$\therefore \boxed{H(z) = T_d A_k \frac{1}{1 - e^{p_k T_d} z^{-1}}}$ Pole mapping is $z \leftarrow e^{sT_d}$

Zeros do *not* map the same way; *not* the general mapping of s to z

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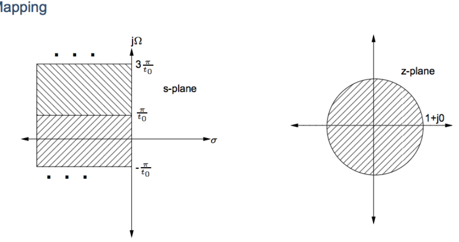
Zeros do *not* map the same way; *not* the general mapping of s to z

- Stability, causality, preserved.
- $j\Omega$ axis mapped linearly to unit-circle, with aliasing
- No control of zeros or of phase

Impulse Invariance Mapping

Mapping



Impulse Invariance

- Let, $\boxed{H_c(j\Omega) = 0, \quad |\Omega| \geq \pi / T}$

$$h[n] = T h_c(nT)$$

- If sampling at Nyquist Rate then

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c\left[j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right]$$

$$H(e^{j\omega}) = H_c\left(j\frac{\omega}{T}\right), \qquad |\omega| < \pi$$

Impulse Invariance

- Sampling the impulse response is equivalent to mapping the s-plane to the z-plane using:
  - $z = e^{st0} = r\,e^{j\omega}$
- The entire Ω axis of the s-plane wraps around the unit circle of the z-plane an infinite number of times

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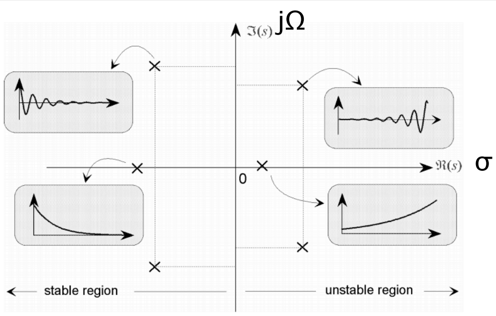
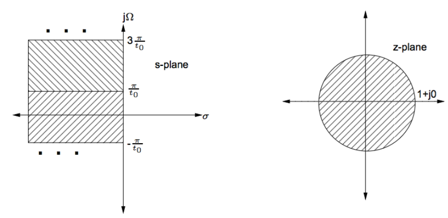
- The left half s-plane maps to the interior of the unit circle and the right half plane to the exterior

Review: S-plane and stability



Impulse Invariance Mapping

Mapping



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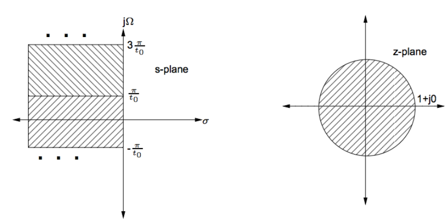
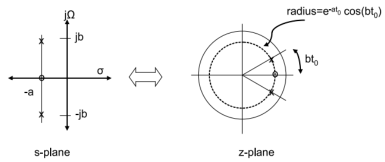
- Sampling the impulse response is equivalent to mapping the s-plane to the z-plane using:
  - $z = e^{st0} = r\,e^{j\omega}$
- The entire Ω axis of the s-plane wraps around the unit circle of the z-plane an infinite number of times
- The left half s-plane maps to the interior of the unit circle and the right half plane to the exterior
- This means stable analog filters (poles in LHP) will transform to stable digital filters (poles inside unit circle)
- This is a many-to-one mapping of strips of the s-plane to regions of the z-plane
  - Not a conformal mapping
  - The poles map according to $z = e^{st0}$, but the zeros do not always

Impulse Invariance Mapping

Mapping

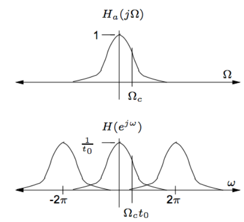


Impulse Invariance Mapping



Impulse Invariance

- Limitation of Impulse Invariance: overlap of images of the frequency response. This prevents it from being used for high-pass filter design



Bilinear Transformation

- The technique uses an algebraic transformation between the variables *s* and *z* that maps the entire jΩ-axis in the s-plane to one revolution of the unit circle in the z-plane.

$$s = \frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right);$$

$$H(z) = H_c\left(\frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right).$$

Bilinear Transformation

$$s = \frac{2}{T_d}\left(\frac{1-z^{-1}}{1+z^{-1}}\right);$$

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$$s = \sigma + j\Omega = \frac{2}{T_d}\left[\frac{2e^{-j\omega/2}(j\sin\omega/2)}{2e^{-j\omega/2}(\cos\omega/2)}\right] = \frac{2j}{T_d}\tan(\omega/2).$$

Bilinear Transformation

$$\Omega = \frac{2}{T_d}\tan(\omega/2),$$

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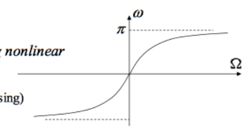
No aliasing, but mapping nonlinear

(Impulse invariance:

linear mapping, but with aliasing)



Example: Notch Filter

- The continuous time filter with:

$$H_a(s) = \frac{s^2 + \Omega_0^2}{s^2 + Bs + \Omega_0^2}$$

$$\Omega = \frac{2}{T_d}\tan(\omega/2),$$

$$\omega = 2\arctan(\Omega T_d/2).$$

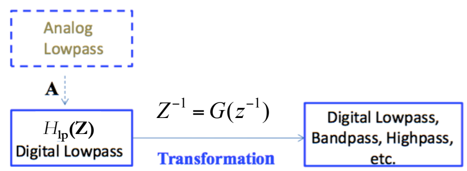
Simple Band-Stop (Notch) Filter

$$H_{BS}(z) = \frac{1+\alpha}{2}\,\frac{1-2\beta z^{-1}+z^{-2}}{1-\beta(1+\alpha)z^{-1}+\alpha z^{-2}} \qquad |\alpha|<1,\ |\beta|<1$$

Note: $1-2\beta z^{-1}+z^{-2} = (1-e^{j\omega_0}z^{-1})(1-e^{-j\omega_0}z^{-1})$

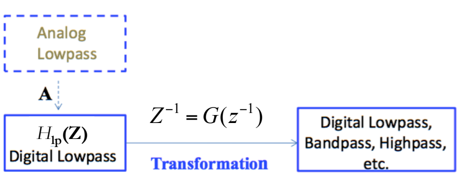
$\cos(\omega_0) = \beta$

Transformation of DT Filters



- Z – complex variable for the LP filter
- z – complex variable for the transformed filter
- Map Z-plane→z-plane with transformation G

Transformation of DT Filters



- Map Z-plane→z-plane with transformation G

$$H(z) = H_{lp}(Z)\big|_{Z^{-1}=G(z^{-1})}$$

Example 1:

- Lowpass→highpass
  - Shift frequency by π

so $\omega \to \omega - \pi$ (Lowpass to highpass)

$$Z^{-1} = -z^{-1} \quad \text{or} \quad e^{-j\omega} \to e^{-j(\omega-\pi)}$$

Example 1:

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so $\omega \rightarrow \omega - \pi$ (Lowpass to highpass)

$$Z^{-1} = -z^{-1} \text{ or } e^{-j\omega} \rightarrow e^{-j(\omega-\pi)}$$

$\boxed{y[n]=0.9y[n-1]+0.1x[n]}$ lowpass; $pole: z=0.9,\ H(z)=\frac{0.1}{1-0.9z^{-1}}$

$H(-z)=\frac{0.1}{1+0.9z^{-1}}$ highpass; $pole: z=-0.9$ $\boxed{y[n]=-0.9y[n-1]+0.1x[n]}$

Example 2:

- Lowpass→bandpass

$$Z^{-1} = -z^{-2}$$

$$H_{lp}(z) = \frac{1}{1-az^{-1}} \longrightarrow H_{bp}(z) = \frac{1}{1+az^{-2}}$$

Pole at z=a Pole at z=±j√a

Example 2:

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- Lowpass→bandstop

$$Z^{-1} = z^{-2}$$

$$H_{lp}(z) = \frac{1}{1-az^{-1}} \longrightarrow H_{bs}(z) = \frac{1}{1-az^{-2}}$$

Pole at z=±√a

Transformation Constraints on $G(z^{-1})$

- If $H_{lp}(Z)$ is the rational system function of a causal and stable system, we naturally require that the transformed system function *H(z)* be a rational function and that the system also be causal and stable.
  - $G(Z^{-1})$ must be a rational function of z^{-1}
  - The inside of the unit circle of the Z-plane must map to the inside of the unit circle of the z-plane
  - The unit circle of the Z-plane must map onto the unit circle of the z-plane.

Transformation Constraints on $G(z^{-1})$

- Respective unit circles in both planes

$$Z = e^{j\theta} \text{ and } z = e^{j\omega}$$

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$$Z^{-1} = G(z^{-1})$$

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$$e^{-j\theta} = \left|G(e^{-j\omega})\right| e^{j\angle G(e^{-j\omega})}$$

Transformation Constraints on $G(z^{-1})$

- Respective unit circles in both planes

$$Z = e^{j\theta} \text{ and } z = e^{j\omega}$$

$$Z^{-1} = G(z^{-1})$$
$$e^{-j\theta} = G(e^{-j\omega})$$
$$e^{-j\theta} = \left|G(e^{-j\omega})\right| e^{j\angle G(e^{-j\omega})}$$

$$1 = \left|G(e^{-j\omega})\right| \qquad -\theta = \angle G(e^{-j\omega})$$

Transformation Constraints on $G(z^{-1})$

- General form that meets all constraints:
  - a_k real and $|a_k|<1$

$$G(z^{-1}) = \pm \prod_{k=1}^{N} \frac{z^{-1} - \alpha_k}{1 - \alpha_k z^{-1}}$$

General Transformation

- Lowpass→lowpass

$$G(z^{-1}) = \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$$

- Changes passband/stopband edge frequencies

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From $e^{-j\theta} = \dfrac{e^{-j\omega} - \alpha}{1 - \alpha e^{-j\omega}}$, get

$$\omega(\theta) = \tan^{-1}\left(\frac{(1-\alpha^2)\sin(\theta)}{2\alpha + (1+\alpha^2)\cos(\theta)}\right)$$

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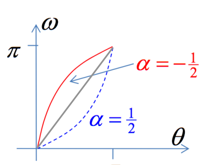


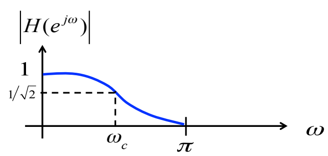
General Transformations

TABLE 7.1 TRANSFORMATIONS FROM A LOWPASS DIGITAL FILTER PROTOTYPE OF CUTOFF FREQUENCY θ_p TO HIGHPASS, BANDPASS, AND BANDSTOP FILTERS

Filter Type	Transformations	Associated Design Formulas
Lowpass	$Z^{-1} = \frac{z^{-1} - \alpha}{1 - az^{-1}}$	$\alpha = \frac{\sin\left(\frac{\theta_p - \omega_p}{2}\right)}{\sin\left(\frac{\theta_p + \omega_p}{2}\right)}$ ω_p = desired cutoff frequency
Highpass	$Z^{-1} = -\frac{z^{-1} + \alpha}{1 + \alpha z^{-1}}$	$\alpha = -\frac{\cos\left(\frac{\theta_p + \omega_p}{2}\right)}{\cos\left(\frac{\theta_p - \omega_p}{2}\right)}$ ω_p = desired cutoff frequency
Bandpass	$Z^{-1} = -\frac{z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + \frac{k-1}{k+1}}{\frac{k-1}{k+1} z^{-2} - \frac{2\alpha k}{k+1} z^{-1} + 1}$	$\alpha = \frac{\cos\left(\frac{\omega_{p2} + \omega_{p1}}{2}\right)}{\cos\left(\frac{\omega_{p2} - \omega_{p1}}{2}\right)}$ $k = \cot\left(\frac{\omega_{p2} - \omega_{p1}}{2}\right)\tan\left(\frac{\theta_p}{2}\right)$ ω_{p1} = desired lower cutoff frequency ω_{p2} = desired upper cutoff frequency
Bandstop	$Z^{-1} = \frac{z^{-2} - \frac{2\alpha}{1+k} z^{-1} + \frac{1-k}{1+k}}{\frac{1-k}{1+k} z^{-2} - \frac{2\alpha}{1+k} z^{-1} + 1}$	$\alpha = \frac{\cos\left(\frac{\omega_{p2} + \omega_{p1}}{2}\right)}{\cos\left(\frac{\omega_{p2} - \omega_{p1}}{2}\right)}$ $k = \tan\left(\frac{\omega_{p2} - \omega_{p1}}{2}\right)\tan\left(\frac{\theta_p}{2}\right)$ ω_{p1} = desired lower cutoff frequency ω_{p2} = desired upper cutoff frequency

Reminder: Simple Low Pass Filter

$$H_{LP}(z) = \frac{1-\alpha}{2}\frac{1+z^{-1}}{1-\alpha z^{-1}} \qquad |\alpha| < 1$$



w_c is the 3dB cutoff frequency $\qquad \alpha = \frac{1-\sin(\omega_c)}{\cos(\omega_c)}$

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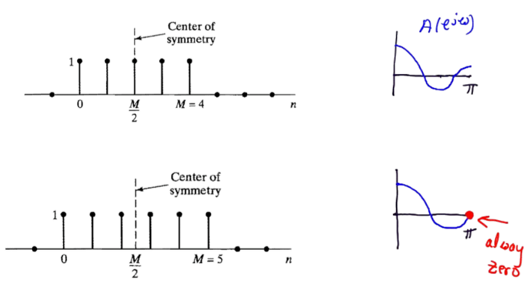
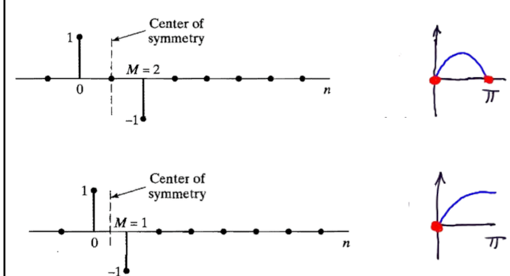
FIR GLP: Type I and II



FIR GLP: Type III and IV



FIR Design by Windowing

- Given desired frequency response, $H_d(e^{j\omega})$, find an impulse response

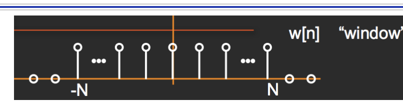
$$h_d[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}d\omega$$

ideal

- Obtain the M^th order causal FIR filter by truncating/windowing it

$$h[n] = \begin{cases} h_d[n]w[n] & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

Example: Moving Average



$$w[n] \leftrightarrow W(e^{j\omega}) = \frac{\sin\left((N+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

$$\frac{1}{M+1}w[n-M/2] \leftrightarrow W(e^{j\omega}) = \frac{e^{-j\omega M/2}}{M+1}\frac{\sin\left((M/2+1/2)\omega\right)}{\sin\left(\omega/2\right)}$$

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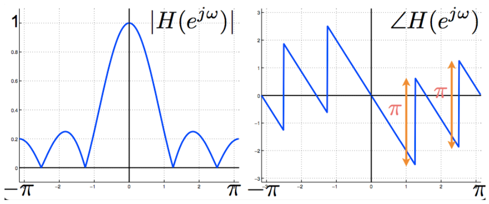


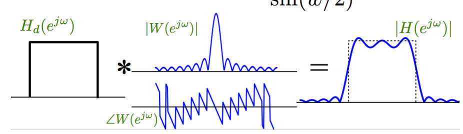
FIR Design by Windowing

- We already saw this before,

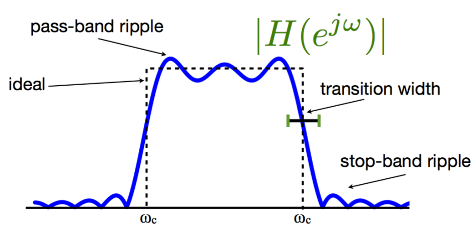
$$H(e^{j\omega}) = H_d(e^{j\omega}) * W(e^{j\omega})$$

- For Boxcar (rectangular) window

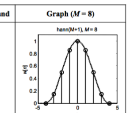
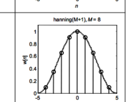
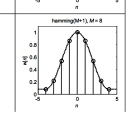
$$W(e^{j\omega}) = e^{-j\omega\frac{M}{2}}\frac{\sin(w(M+1)/2)}{\sin(w/2)}$$



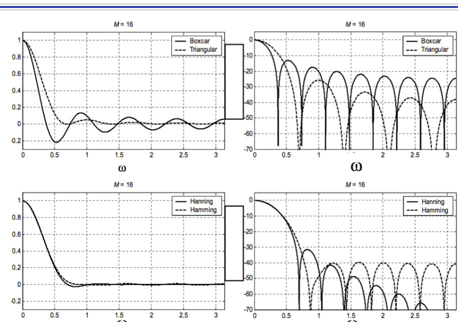
FIR Design by Windowing



Tapered Windows

Name(s)	Definition	MATLAB Command	Graph ($M = 8$)
Hann	$w[n] = \begin{cases} \frac{1}{2}\left[1+\cos\left(\frac{\pi n}{M/2}\right)\right] & \lvert n\rvert \le M/2 \\ 0 & \lvert n\rvert > M/2 \end{cases}$	`hann(M+1)`	
Hanning	$w[n] = \begin{cases} \frac{1}{2}\left[1+\cos\left(\frac{\pi n}{M/2+1}\right)\right] & \lvert n\rvert \le M/2 \\ 0 & \lvert n\rvert > M/2 \end{cases}$	`hanning(M+1)`	
Hamming	$w[n] = \begin{cases} 0.54+0.46\cos\left(\frac{\pi n}{M/2}\right) & \lvert n\rvert \le M/2 \\ 0 & \lvert n\rvert > M/2 \end{cases}$	`hamming(M+1)`	

Tradeoff – Ripple vs. Transition Width



FIR Filter Design

- Choose a desired frequency response $H_d(e^{j\omega})$
  - non causal (zero-delay), and infinite imp. response
  - If derived from C.T, choose T and use:

$$H_d(e^{j\omega}) = H_c(j\frac{\Omega}{T})$$

- Window:
  - Length M+1 ⇔ affects transition width
  - Type of window ⇔ transition-width/ ripple

FIR Filter Design

- Choose a desired frequency response $H_d(e^{j\omega})$
  - non causal (zero-delay), and infinite imp. response
  - If derived from C.T, choose T and use:

$$H_d(e^{j\omega}) = H_c(j\frac{\Omega}{T})$$

- Window:
  - Length M+1 ⇔ affects transition width
  - Type of window ⇔ transition-width/ ripple
  - Modulate to shift impulse response
    - Force causality

$$H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}$$

FIR Filter Design

- Determine truncated impulse response $h_1[n]$

$$h_1[n] = \begin{cases} \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}e^{j\omega n} & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

- Apply window

$$h_w[n] = w[n]h_1[n]$$

- Check:
  - Compute $H_w(e^{j\omega})$, if does not meet specs increase M or change window

Example: FIR Low-Pass Filter Design

$$H_d(e^{j\omega}) = \begin{cases} 1 & |\omega| \le \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Choose M ⇒ Window length and set

$$H_1(e^{j\omega}) = H_d(e^{j\omega})e^{-j\omega\frac{M}{2}}$$

$$h_1[n] = \begin{cases} \frac{\sin(\omega_c(n-M/2))}{\pi(n-M/2)} & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\omega_c}{\pi}\text{sinc}(\frac{\omega_c}{\pi}(n-M/2))$$

Example: FIR Low-Pass Filter Design

- The result is a windowed sinc function

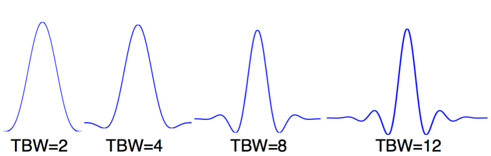
$$h_w[n] = w[n]h_1[n]$$

$$\frac{\omega_c}{\pi}\text{sinc}(\frac{\omega_c}{\pi}(n-M/2))$$

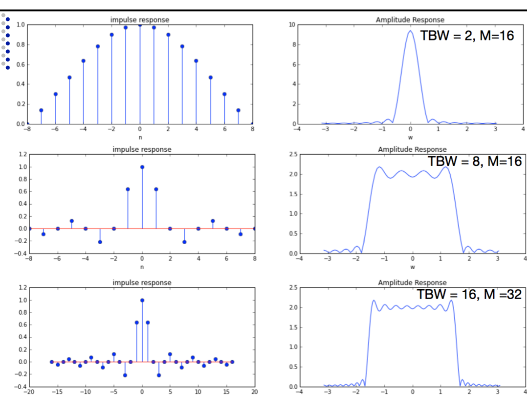
- High Pass Design:
  - Design low pass
  - Transform to $h_w[n](-1)^n$
- General bandpass
  - Transform to $2h_w[n]\cos(\omega_0 n)$ or $2h_w[n]\sin(\omega_0 n)$

Characterization of Filter Shape

Time-Bandwidth Product, a unitless measure
T(BW) = (M+1)ω/2π ⇒ also, total # of zero crossings

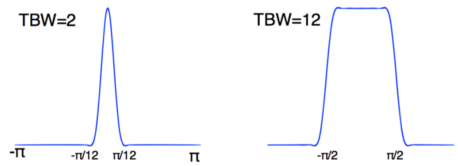


Larger TBW ⇒ More of the “sinc” function
hence, frequency response looks more like a rect function



Frequency Response Profile

Q: What are the lengths of these filters in samples?



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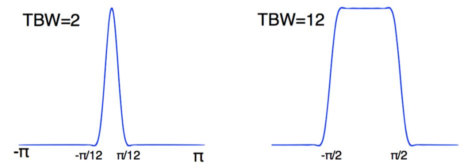
Frequency Response Profile

Q: What are the lengths of these filters in samples?



2 = (M+1)*(π/6) / (2π) ⇒ M=23   12= (M+1)*(π) / (2π) ⇒ M=23

Note that transition is the same!

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Alternative Design through FFT

- To design order M filter:
- Over-Sample/discretize the frequency response at P points where P >> M (P=15M is good)

$$H_1(e^{j\omega_k}) = H_d(e^{j\omega_k})e^{-j\omega_k \frac{M}{2}}$$

- Sampled at: $\omega_k = k\frac{2\pi}{P}$ $\quad |k = [0, \cdots, P-1]$
- Compute $h_1[n] = \mathrm{IDFT}_P(H_1[k])$
- Apply M+1 length window:

$$h_w[n] = w[n]h_1[n]$$

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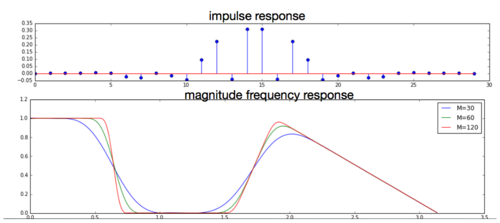
Example

- signal.firwin2(M+1,omega_vec/pi, amp_vec)
- taps1 = signal.firwin2(30, [0.0,0.2,0.21,0.5, 0.6, 1.0], [1.0, 1.0, 0.0,0.0,1.0,0.0])



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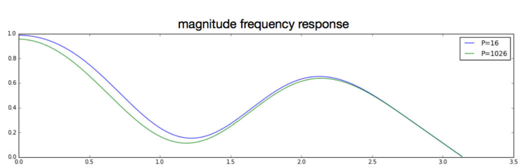
Example

- For M+1=14
  - P = 16 and P = 1026



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Admin

- HW 7
  - Out now
  - Due Friday 3/15
- Midterms back on Tuesday

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