Q1

a)

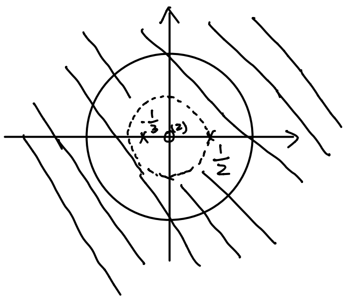
$$H(z) = \frac{A z^2}{(z-\frac{1}{2})(z+\frac{1}{3})} \qquad \frac{A}{\frac{1}{2} \times \frac{4}{3}} = \frac{3}{2}A = 6$$

$$A = \frac{12}{3} = 4$$

Average: 58.8
Stdev: 18.2

$$\boxed{H(z) = \frac{4z^2}{(z-\frac{1}{2})(z+\frac{1}{3})}}$$

$$\frac{z^2}{z^2 - \frac{1}{6}z - \frac{1}{6}}$$



b)

$$H(z) = 4\left(1 + \frac{\frac{1}{6}z + \frac{1}{6}}{(z-\frac{1}{2})(z+\frac{1}{3})}\right)$$

$$= 4 + \frac{2}{3}\left(\frac{\frac{9}{5}}{z-\frac{1}{2}} - \frac{\frac{4}{5}}{z+\frac{1}{3}}\right)$$

$$= 4 + \frac{\frac{6}{5}}{1-\frac{1}{2}z^{-1}} z^{-1} - \frac{\frac{8}{15}}{1+\frac{1}{3}z^{-1}} z^{-1}$$

$$\boxed{h[n] = 4\,\delta[n] + \frac{6}{5}\cdot\left(\frac{1}{2}\right)^{n-1} u[n-1] - \frac{8}{15}\cdot\left(-\frac{1}{3}\right)^{n-1} u[n-1]}$$

$$z^2 - \frac{1}{6}z - \frac{1}{6} \,\big)\, z^2 = 1, \qquad z^2 - \left(z^2 - \frac{1}{6}z - \frac{1}{6}\right) = \frac{1}{6}z + \frac{1}{6}$$

c) $\boxed{\text{Yes.}}$

d) $T = \frac{2\pi}{w} = \frac{1}{40}s$

$$x[n] = 50 + 30\cos[\pi n]$$

$$H(z) = \frac{4z^2}{(z-\frac{1}{2})(z+\frac{1}{3})} \Rightarrow H(e^{jw}) = \frac{4e^{2jw}}{(e^{jw}-\frac{1}{2})(e^{jw}+\frac{1}{3})}$$

$$H(e^{j\pi}) = \frac{4}{(-1-\frac{1}{2})(-1+\frac{1}{3})} = 4$$

$$H(e^{j0}) = \frac{4}{(1-\frac{1}{2})(1+\frac{1}{3})} = 6$$

$$\boxed{y[n] = 300 + 120\cos[\pi n]}$$

Points 100 Published

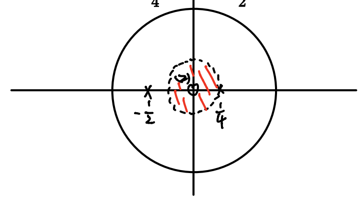
Details | Questions

Q1 Pick 1 questions, 35 pts per question

Q1

Consider an anti-causal LTI system with the transfer function $H(z)$. The transfer function has a second order zero at $z=0$ and two poles at $z=\frac{1}{4}$, $z=-\frac{1}{2}$. It is also knows that $H(z)=8$ when $z=1$. It might be helpful to draw the pole-zero diagram.

$$H(z) = \frac{9z^2}{(z-\frac{1}{4})(z+\frac{1}{2})} = \frac{9}{(1-\frac{1}{4}z^{-1})(1+\frac{1}{2}z^{-1})} = \frac{3}{1-\frac{1}{4}z^{-1}} + \frac{6}{1+\frac{1}{2}z^{-1}}$$



a) Determine $H(z)$. Include the ROC.

b) Determine the impulse response $h[n]$ of the system. $h[n] = -3(\frac{1}{4})^n u[-n-1] - 6(-\frac{1}{2})^n u[-n-1]$

c) Is the system stable? No.

d) Determine the output, $y[n]$, of the system when the input is the sequence $x[n]$, which is obtained from sampling the continuous-time signal $x(t) = 25 + 15\cos(40\pi t)$, at a sampling frequency $\Omega_S = 2\pi(40)$ rad/s.

$H(e^{j0}) = 8$ $\quad y[n] = 200 + 216\cos[\pi n]$

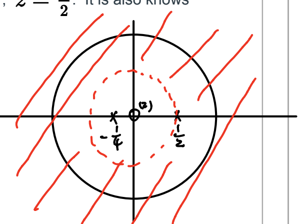
$H(e^{j\pi}) = \frac{72}{5}$

You can type your answers in the text box below or indicate your solution on your file upload.

Q1

Consider a causal LTI system with the transfer function $H(z)$. The transfer function has a second order zero at $z=0$ and two poles at $z=-\frac{1}{4}$, $z=\frac{1}{2}$. It is also knows that $H(z)=6$ when $z=1$. It might be helpful to draw the pole-zero diagram.

$$H(z) = \frac{\frac{15}{4}z^2}{(z+\frac{1}{4})(z-\frac{1}{2})} = \frac{\frac{15}{4}}{(1+\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{\frac{5}{4}}{1+\frac{1}{4}z^{-1}} + \frac{\frac{5}{2}}{1-\frac{1}{2}z^{-1}}$$



a) Determine $H(z)$. Include the ROC.

b) Determine the impulse response $h[n]$ of the system. $y[n] = \frac{5}{4}(-\frac{1}{4})^n u[n] + \frac{5}{2}(\frac{1}{2})^n u[n]$

c) Is the system stable? Yes.

d) Determine the output, $y[n]$, of the system when the input is the sequence $x[n]$, which is obtained from sampling the continuous-time signal $x(t) = 50 + 30\cos(40\pi t)$, at a sampling frequency $\Omega_S = 2\pi(20)$ rad/s.

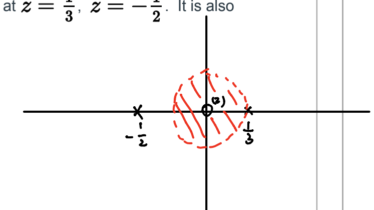
$H(e^{j0}) = 6$ $\quad y[n] = 480$

You can type your answers in the text box below or indicate your solution on your file upload.

Q1

Consider an anti-causal LTI system with the transfer function $H(z)$. The transfer function has a second order zero at $z=0$ and two poles at $z=\frac{1}{3}$, $z=-\frac{1}{2}$. It is also knows that $H(z)=5$ when $z=1$. It might be helpful to draw the pole-zero diagram.

$$H(z) = \frac{5z^2}{(z-\frac{1}{3})(z+\frac{1}{2})} = \frac{5}{(1-\frac{1}{3}z^{-1})(1+\frac{1}{2}z^{-1})} = \frac{2}{1-\frac{1}{3}z^{-1}} + \frac{3}{1+\frac{1}{2}z^{-1}}$$



a) Determine $H(z)$. Include the ROC.

b) Determine the impulse response $h[n]$ of the system. $h[n] = -2(\frac{1}{3})^n u[-n-1] - 3(-\frac{1}{2})^n u[-n-1]$

c) Is the system stable? No.

d) Determine the output, $y[n]$, of the system when the input is the sequence $x[n]$, which is obtained from sampling the continuous-time signal $x(t) = 25 + 15\cos(80\pi t)$, at a sampling frequency $\Omega_S = 2\pi(40)$ rad/s.

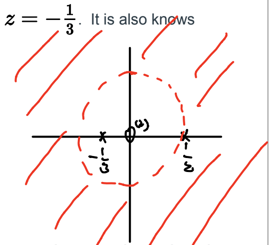
$H(e^{j0}) = 5$ $\quad y[n] = 200$

You can type your answers in the text box below or indicate your solution on your file upload.

Q1

Consider a causal LTI system with the transfer function $H(z)$. The transfer function has a second order zero at $z = 0$ and two poles at $z = \frac{1}{2}$, $z = -\frac{1}{3}$. It is also knows that $H(z) = 6$ when $z = 1$. It might be helpful to draw the pole-zero diagram.

$$H(z) = \frac{4z^2}{(z-\frac{1}{2})(z+\frac{1}{3})} = \frac{4}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})} = \frac{\frac{12}{5}}{1-\frac{1}{2}z^{-1}} + \frac{\frac{8}{5}}{1+\frac{1}{3}z^{-1}}$$



a) Determine $H(z)$. Include the ROC.

b) Determine the impulse response $h[n]$ of the system. $h[n] = \frac{12}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{8}{5}\left(-\frac{1}{3}\right)^n u[n]$

c) Is the system stable? Yes.

d) Determine the output, $y[n]$, of the system when the input is the sequence $x[n]$, which is obtained from sampling the continuous-time signal $x(t) = 50 + 30\cos(40\pi t)$, at a sampling frequency $\Omega_S = 2\pi(40)$ rad/s.

$H(e^{j0}) = 6$ $\quad y[n] = 300 + 120\cos[\pi n]$

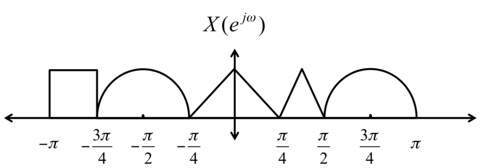
$H(e^{j\pi}) = 4$

You can type your answers in the text box below or indicate your solution on your file upload.

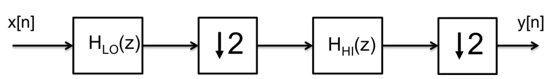
Q2 Pick 1 questions, 20 pts per question

Q2

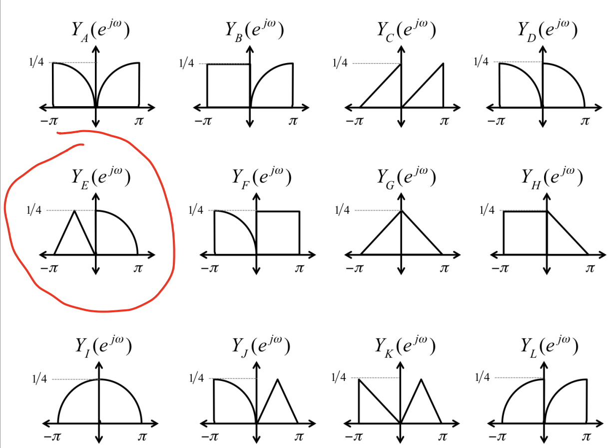
Consider an input signal $x[n]$ with the spectrum $X(e^{j\omega})$.



You pass $x[n]$ through the below filter-bank where $H_{LO}(z)$ and $H_{HI}(z)$ are ideal low-pass and high-pass filters with cutoff frequency of $\omega_c = \frac{\pi}{2}$.



a) Below are possible output spectrums, $Y(e^{j\omega})$ labelled A-L. Indicate which is the output spectrum of the above filter bank or say none if the correct spectrum is not present.



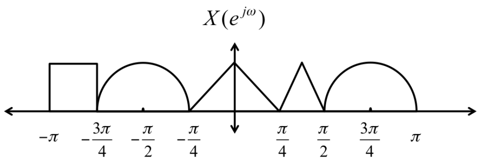
b) Design a new filter bank which results in the output spectrum $Y_L(e^{j\omega})$.

$\boxed{H_{HI}(z)} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{H_{HI}(z)} \rightarrow \boxed{\downarrow 2}$

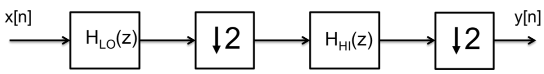
Write your answers in the text block below or indicate your solution on your file upload.

Q2

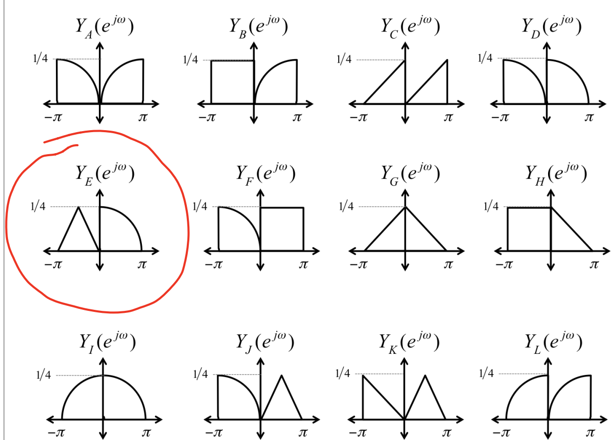
Consider an input signal $x[n]$ with the spectrum $X(e^{j\omega})$.



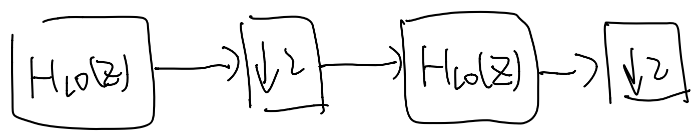
You pass $x[n]$ through the below filter-bank where $H_{LO}(z)$ and $H_{HI}(z)$ are ideal low-pass and high-pass filters with cutoff frequency of $\omega_c = \frac{\pi}{2}$.



a) Below are possible output spectrums, $Y(e^{j\omega})$ labelled A-L. Indicate which is the output spectrum of the above filter bank or say none if the correct spectrum is not present.



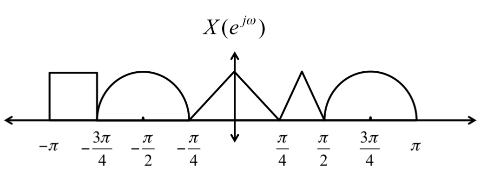
b) Design a new filter bank which results in the output spectrum $Y_G(e^{j\omega})$.



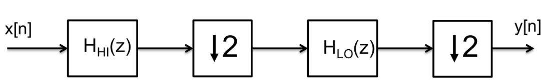
Write your answers in the text block below or indicate your solutions on your file upload.

Q2

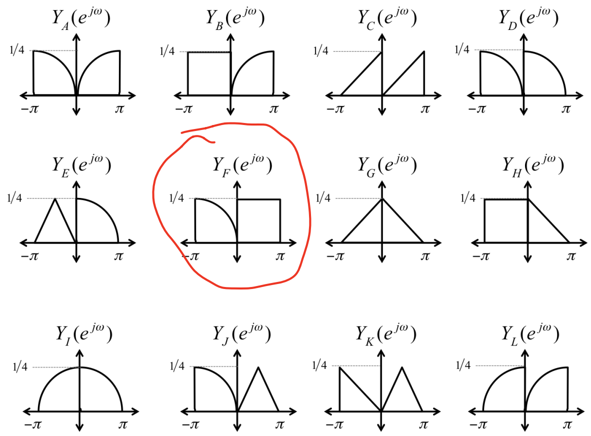
Consider an input signal $x[n]$ with the spectrum $X(e^{j\omega})$.



You pass $x[n]$ through the below filter-bank where $H_{LO}(z)$ and $H_{HI}(z)$ are ideal low-pass and high-pass filters with cutoff frequency of $\omega_c = \frac{\pi}{2}$.



a) Below are possible output spectrums, $Y(e^{j\omega})$ labelled A-L. Indicate which is the output spectrum of the above filter bank or say none if the correct spectrum is not present.



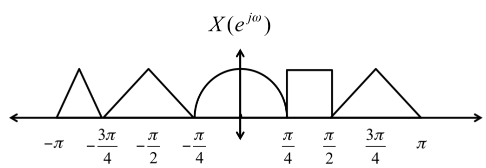
b) Design a new filter bank which results in the output spectrum $Y_E\left(e^{j\omega}\right)$.

$H_{LO} \to \boxed{\downarrow 2} \to H_{HI} \to \boxed{\downarrow 2}$

Write your answers in the text block below or indicate your solutions on your file upload.

Q2

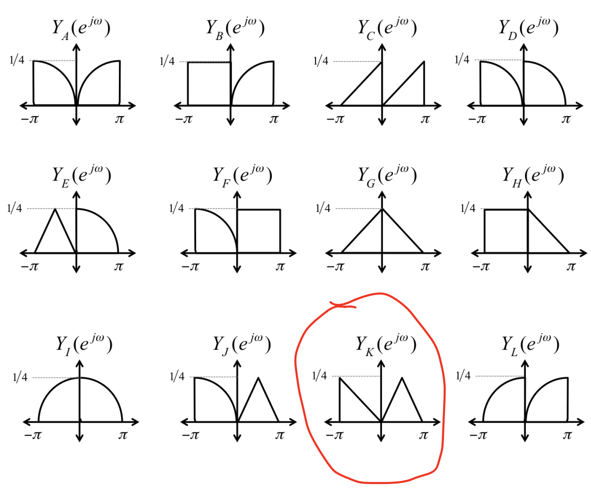
Consider an input signal $x\,[n]$ with the spectrum $X\left(e^{j\omega}\right)$.



You pass $x\,[n]$ through the below filter-bank where $H_{LO}\,(z)$ and $H_{HI}\,(z)$ are ideal low-pass and high-pass filters with cutoff frequency of $\omega_c = \frac{\pi}{2}$.

$x[n] \to \boxed{H_{HI}(z)} \to \boxed{\downarrow 2} \to \boxed{H_{LO}(z)} \to \boxed{\downarrow 2} \to y[n]$

a) Below are possible output spectrums, $Y\left(e^{j\omega}\right)$ labelled A-L. Indicate which is the output spectrum of the above filter bank or say none if the correct spectrum is not present.



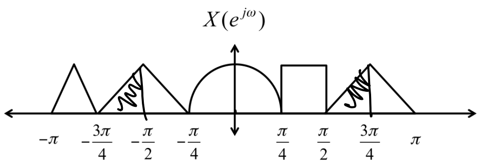
b) Design a new filter bank which results in the output spectrum $Y_I\left(e^{j\omega}\right)$.

$\boxed{H_{LO}} \to \boxed{\downarrow 2} \to \boxed{H_{LO}} \to \boxed{\downarrow 2}$

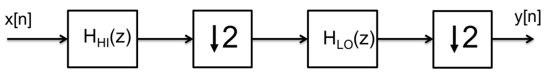
Write your answers in the text block below or indicate your solutions on your file upload.

Q2

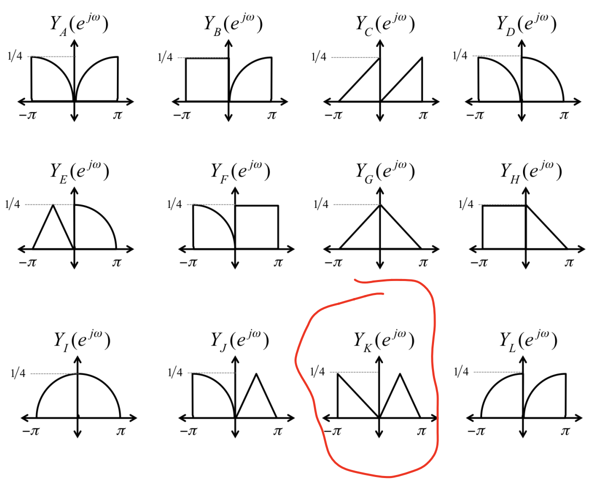
Consider an input signal $x[n]$ with the spectrum $X(e^{j\omega})$.



You pass $x[n]$ through the below filter-bank where $H_{LO}(z)$ and $H_{HI}(z)$ are ideal low-pass and high-pass filters with cutoff frequency of $\omega_c = \frac{\pi}{2}$.



a) Below are possible output spectrums, $Y(e^{j\omega})$ labelled A-L. Indicate which is the output spectrum of the above filter bank or say none if the correct spectrum is not present.



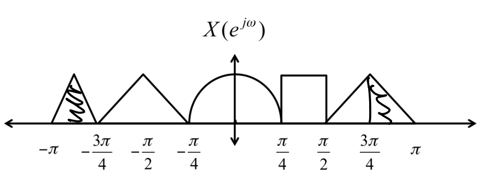
b) Design a new filter bank which results in the output spectrum $Y_C(e^{j\omega})$.

$\boxed{H_{HI}} \to \boxed{\downarrow 2} \to \boxed{H_{HI}} \to \boxed{\downarrow 2}$

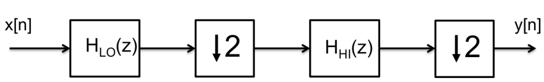
Write your answers in the text block below or indicate your solutions on your file upload.

Q2

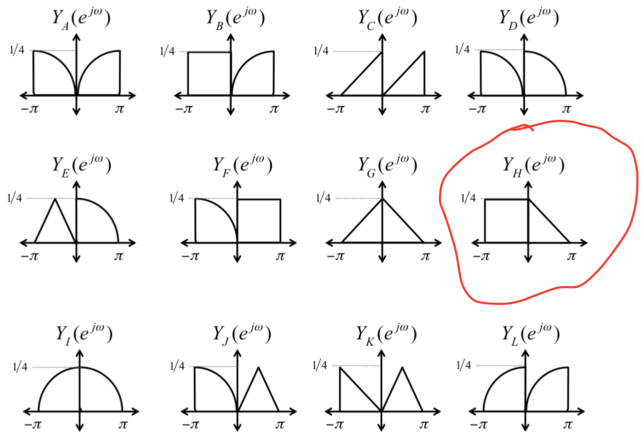
Consider an input signal $x[n]$ with the spectrum $X(e^{j\omega})$.



You pass $x[n]$ through the below filter-bank where $H_{LO}(z)$ and $H_{HI}(z)$ are ideal low-pass and high-pass filters with cutoff frequency of $\omega_c = \frac{\pi}{2}$.



a) Below are possible output spectrums, $Y(e^{j\omega})$ labelled A-L. Indicate which is the output spectrum of the above filter bank or say none if the correct spectrum is not present.



b) Design a new filter bank which results in the output spectrum $Y_K\left(e^{j\omega}\right)$.

$\boxed{H_{HI}} \to \boxed{\downarrow 2} \to \boxed{H_{Lo}} \to \boxed{\downarrow 2}$

Write your answers in the text block below or indicate your solutions on your file upload.

Q3 Pick 1 questions, 21 pts per question

Q3

A system has been built for computing the 8-point DFT $Y[0], Y[1], \ldots, Y[7]$ of a discrete-time sequence $y[0], y[1], \ldots, y[7]$. However, the system is not working properly and only the even DFT samples $Y[0], Y[2], Y[4], Y[6]$ are being computed correctly. To help you solve the problem, the data you can access are:

- the (correct) even DFT samples, $Y[0], Y[2], Y[4], Y[6]$;
- the first 4 input values $y[0], y[1], y[2], y[3]$ (the other inputs are unavailable).

a) If $y[0] = 1$, and $y[1] = y[2] = y[3] = 0$, and $Y[0] = Y[2] = Y[4] = Y[6] = 2$, what are the missing values $Y[1], Y[3], Y[5], Y[7]$? Explain how you got them.

b) You need to build an efficient system that computes the odd samples $Y[1], Y[3], Y[5], Y[7]$ for any set of inputs. The computational modules you have available are one 4-point DFT and one 4-point IDFT. Both are free. You can purchase adders, subtracters, or multipliers for \$10 each. Multiplying by 1 or 0 is also free. Design a system of the lowest possible cost that takes as input

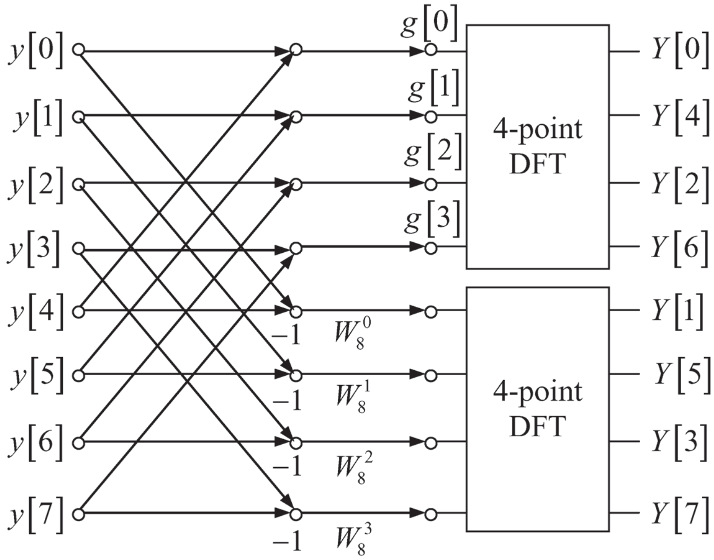
$y[0], y[1], y[2], y[3], Y[0], Y[2], Y[4], Y[6]$ and produces as output

$Y[1], Y[3], Y[5], Y[7]$. Draw or describe the associated block diagram and indicate the total cost.

Write your answers in the text block below or indicate your solutions on your file upload.

Question 3:

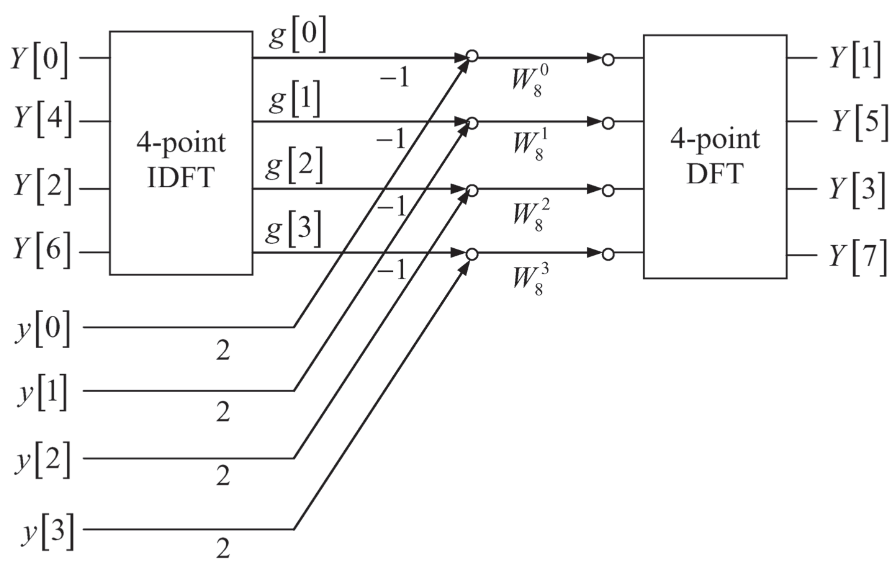
The key to this question is to note the decimation-in-frequency computation of the FFT:



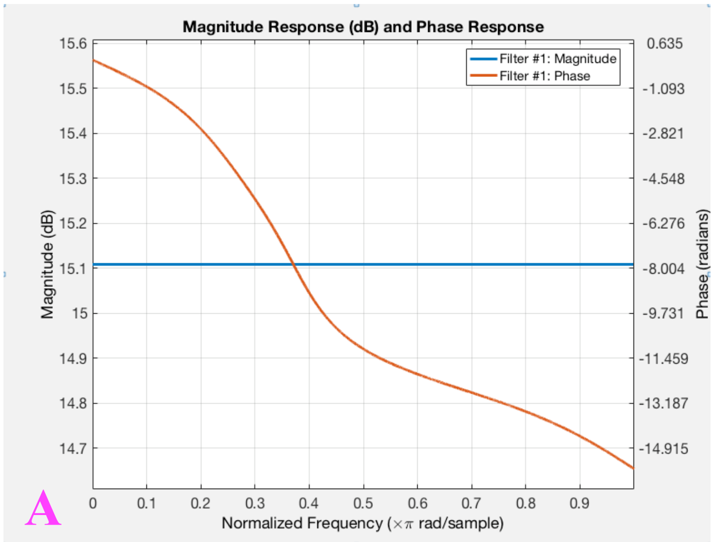
a) If Y[0] = Y[2] = Y[4] = Y[6] = 2, then the IDFT gives us g[0] = 2, g[1] = g[2] = g[3] = 0. We then can solve for y[4] = g[0] – y[0] = 2 -1 = 1, and y[5] = y[6] = y[7] = 0.

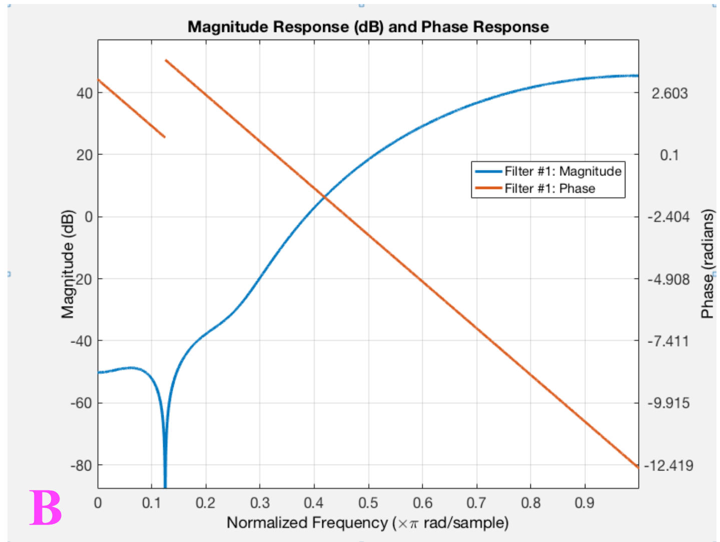
Then computing the 8-pt DFT of our time samples gives Y[1] = Y[3] = Y[5] = Y[7] = 0

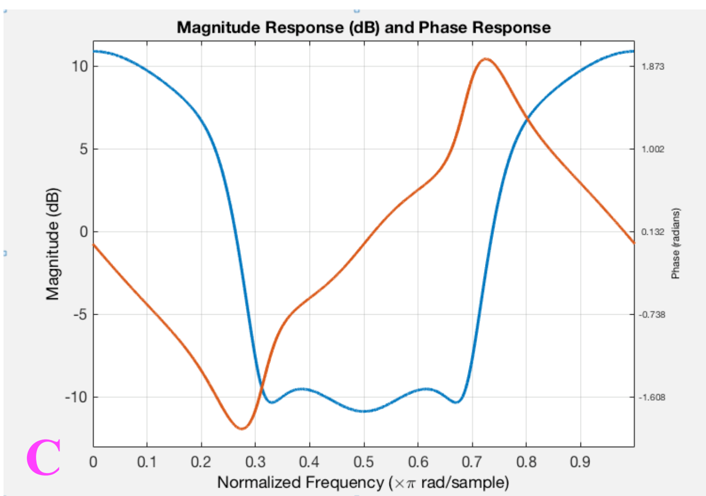
b) The plan to implement this with the given computational blocks is below:

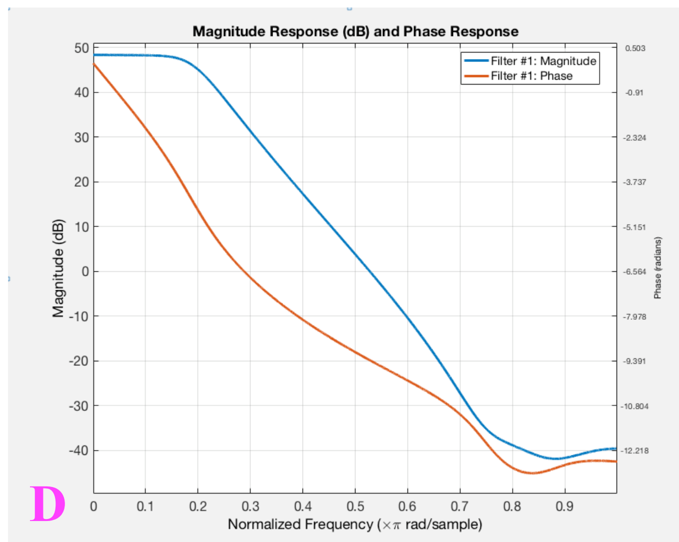


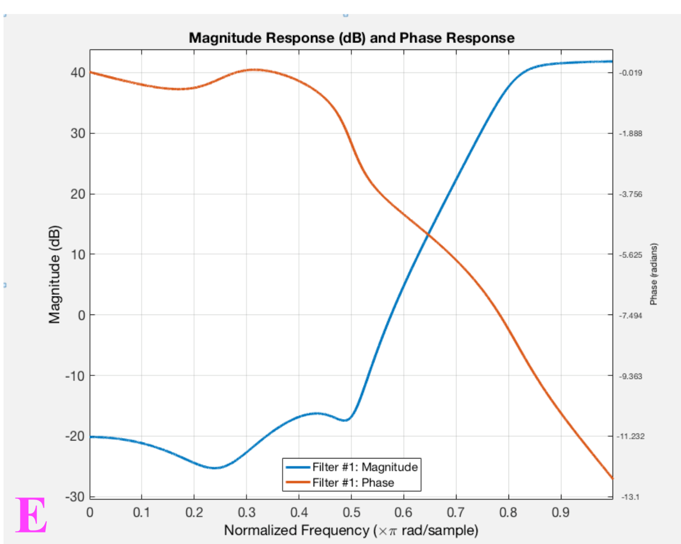
The diagram above shows 7 multipliers and 4 subtractors for a total cost of $110.

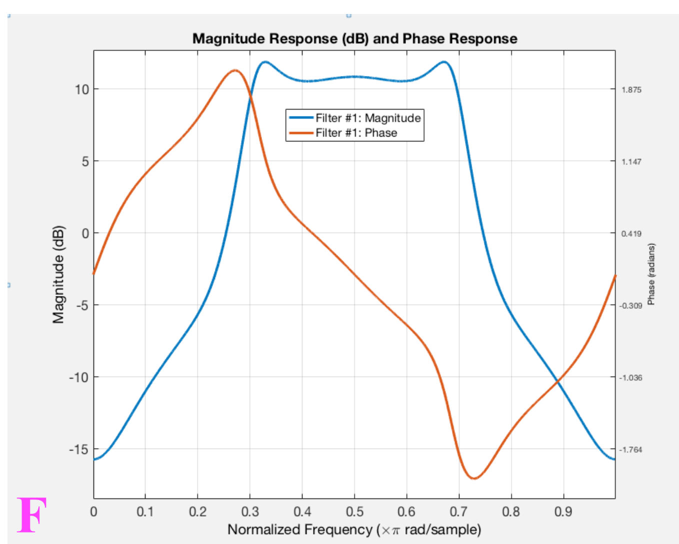

Magnitude Response (dB) and Phase Response
Filter #1: Magnitude
Filter #1: Phase
Magnitude (dB)
Phase (radians)
15.6
15.5
15.4
15.3
15.2
15.1
15
14.9
14.8
14.7
0.635
-1.093
-2.821
-4.548
-6.276
-8.004
-9.731
-11.459
-13.187
-14.915
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)
A

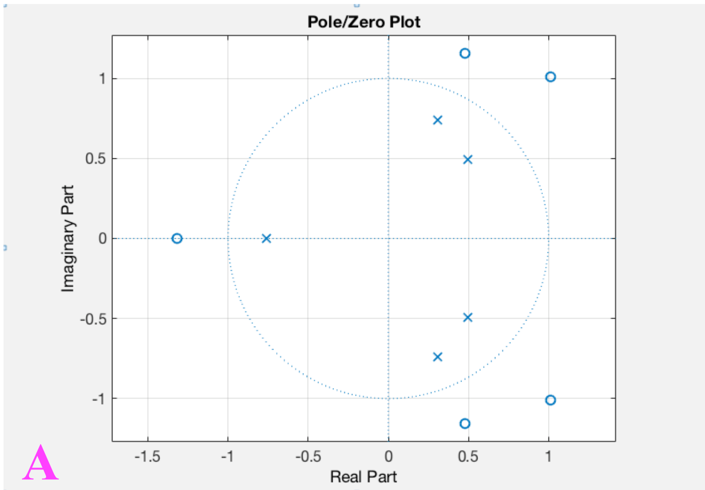

Magnitude Response (dB) and Phase Response
Filter #1: Magnitude
Filter #1: Phase
Magnitude (dB)
Phase (radians)
40
20
0
-20
-40
-60
-80
2.603
0.1
-2.404
-4.908
-7.411
-9.915
-12.419
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)
B


Magnitude Response (dB) and Phase Response
Magnitude (dB)
Phase (radians)
10
5
0
-5
-10
1.873
1.002
0.132
-0.738
-1.608
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)
C


Magnitude Response (dB) and Phase Response
Filter #1: Magnitude
Filter #1: Phase
Magnitude (dB)
Phase (radians)
50
40
30
20
10
0
-10
-20
-30
-40
0.503
-0.91
-2.324
-3.737
-5.151
-6.564
-7.978
-9.391
-10.804
-12.218
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)
D

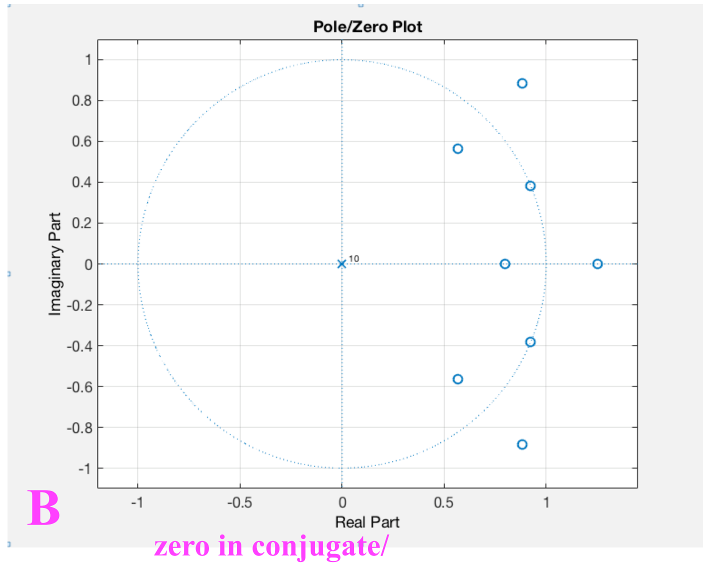

Magnitude Response (dB) and Phase Response
Filter #1: Magnitude
Filter #1: Phase
Magnitude (dB)
Phase (radians)
40
30
20
10
0
-10
-20
-30
-0.019
-1.888
-3.756
-5.625
-7.494
-9.363
-11.232
-13.1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)
E


Magnitude Response (dB) and Phase Response
Filter #1: Magnitude
Filter #1: Phase
Magnitude (dB)
Phase (radians)
10
5
0
-5
-10
-15
1.875
1.147
0.419
-0.309
-1.036
-1.764
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
Normalized Frequency (×π rad/sample)
F



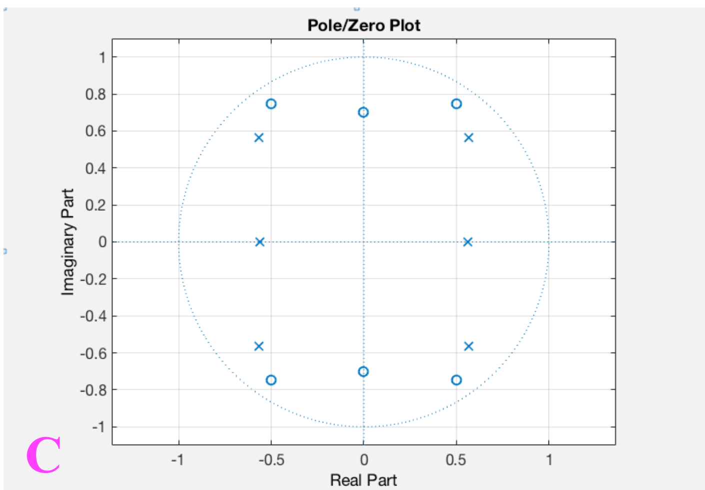
A

pole-zero conjugate/ reciprocal pairs ->All pass filter



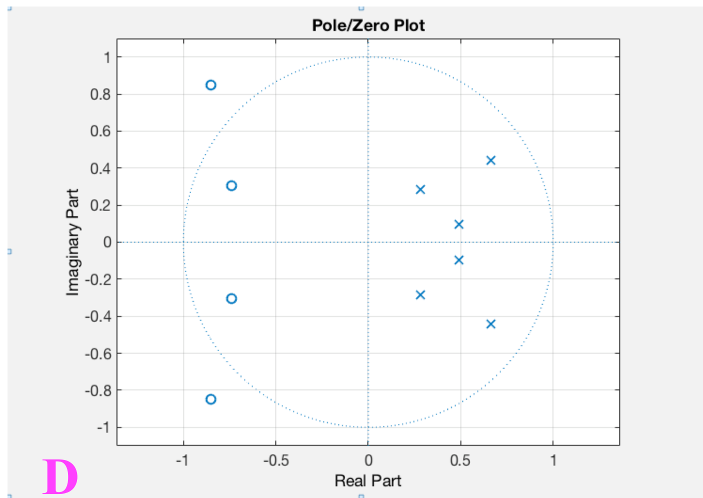
B

zero in conjugate/ reciprocal groups -> GLP system



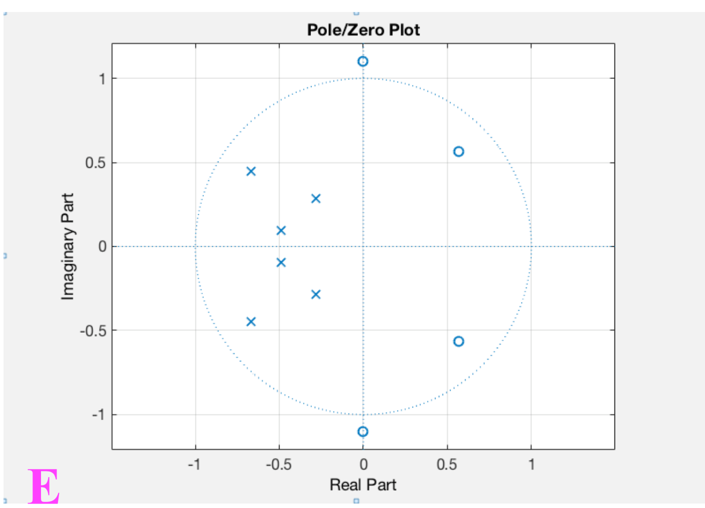
C

zeros in mid-band -> Band stop filter



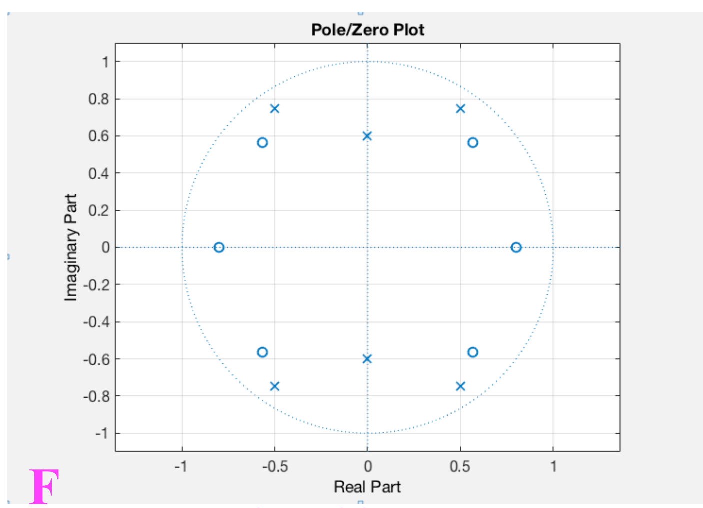
D

zeros at high freq -> Low pass filter



E

zeros at low freq -> High pass filter



F

zeros in low/high-band, poles in mid-band -> Band stop filter