University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

ESE531, Spring 2020 HW6: Frequency Response LTI Systems Sunday, Mar. 8

Due: Sunday, March 22, 11:59PM

- **Recommended Problems for Practice:** From the book: 5.4, 5.7, 5.10, 5.13, 5.24
- **Homework Problems:** All problems must be turned in and are not optional for full credit
  1. Homework problems from the book: 5.23, 5.33, 5.34, 5.39, 5.42
  2. Matlab problem 1: Group Delay
     The group delay is defined as the negative derivative of the phase of the frequency response. However, computation of the group delay is best done without explicitly evaluating the derivative with respect to ω. The M-file below exploits the fact that multiplying by n in the time domain will generate a derivative in the frequency domain. Furthermore this function is configured for the case where the signal $x[n]$ starts at an index other than $n = 0$.

```
 function [gd, w] = gdel(x, n, Lfft)
%GDEL compute the group delay of x[n]
%
% x:  Signal x[n] at the times (n)
% n:  Vector of time indices
% Lfft:  Length of the FFT used
% gd:  Group delay values on [-pi, pi)
% w:  List of frequencies over [-pi, pi)
%
% NOTE: Group delay of B(z)/A(z) = gdel(B) - gdel(A)
%
X = fft(x, Lfft);
dXdw = fft(n.*x, Lfft); % --- transform of nx[n]
gd = fftshift(real(dXdw./X )); % --- when X==0, gd=infinity
w = (2*pi/Lfft)*[0:(Lfft-1)] - pi;
```

     Test the group delay function with a shifted unit impulse signal. Define a unit impulse sequence $\delta[n - n_o]$ of lenght 128, over the range $-64 \leq n \leq 63$. Pick $n_o = \pm 5$, and then make a plot of the signal with the time axis correctly labeled, to show that the impulse is located at $n = n_o$. In addition, compute and plot the group delay to verify the proper value is obtained. Submit your signal and group delay plots.

3. Matlab problem 2: Causal First-Order System
   Using the Matlab function `filter`, generate the impulse response of the causal system:

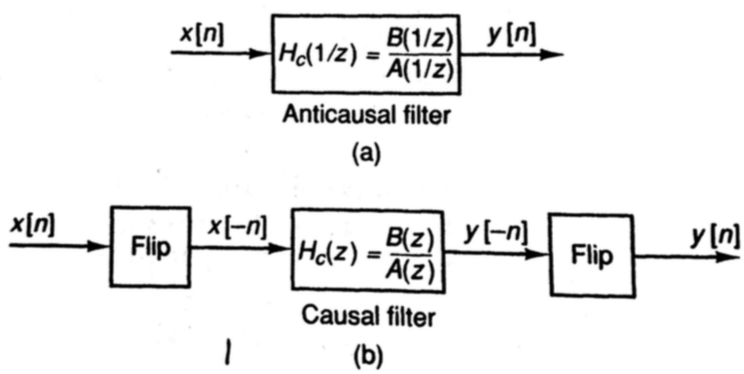
$$H_C(z) = \frac{1}{1 - 0.77z^{-1}} \quad ROC = \{z : |z| > 0.77\} \tag{1}$$

   (a) Plot the impulse response of the signal over the range $-64 \leq n \leq 63$.

   (b) Plot the frequency response magnitude and group delay. To generate the frequency response, compute the FFT form a finite section of the impulse response.

   (c) Repeat a and b for a pole closer to the unit circle; try 0.95 instead of 0.77. Describe the differences between the two cases.

4. Matlab problem 3: Anticausal First-Order System
   For an anticausal filter, the impulse response is zero for $n > 0$. Anticausal filtering can be accomplished in a three-step process: time reverse the input, filter with a causal fitler, and the time-reverse the output. The signal can be time-reversed using either `fliplr` or `flipud`. Specifically, the two systems shown below are identical from an input/output point of view. If $H_C(z)$ corresponds to a causal filter, then $H_a(z) = H_c(1/z)$ will correspond to an anticausal filter, and vice versa.



   For the anticausal filter

$$H_u(z) = \frac{1}{1 - 0.95z} = H_c(1/z) \quad ROC = \{z : |z| < 1/0.95\} \tag{2}$$

   (a) Generate and plot the impulse response over the range $-64 \leq n \leq 63$ by time-reversing.

   (b) Calculate and plot the frequency response magnitude and group delay.

   (c) Discuss how the impulse response, frequency response magnitude, and group delay for this filter relate to those for the causal filter in Matlab problem 2.

   (d) Do the same relationships hold when the pole is at $z = 1/0.77$.

5. Matlab problem 4: Higher-Order System
   Given the general form of a transfer function

$$H(z) = b_0 \frac{\prod_{l=1}^{M} 1 - z_l z^{-1}}{\prod_{k=1}^{N} 1 - p_k z^{-1}}$$

specified by it's poles, zeros and constant: $p_1 = 0.9$, $p_{2,3} = 0.6718 \pm j0.6718$, $z_1 = -1$, $z_{2,3} = \pm j$, and $b_0 = 1/77$.

   (a) Use `pzmap` to create a pole-zero plot. Label all axes. See the Mathworks website for help on all matlab functions.

   (b) Write a function `zp2tf` for converting the pole-zero description into the description by the transfer function given as

$$H(z) = \frac{\sum_{l=0}^{M} b_l z^{-l}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

   (Hint: the `poly` function will be useful. Make sure that you obtain a polynomial real coefficients.)

   (c) Apply your program to the example above and use the `filter` function with the transfer function representation from (b) to derive the impulse response.

   (d) Using the impulse response, calculate and plot 100 samples of the step response of the system. (Hint: What is the input for the step response output?) Try different input sequences (of length 100) to see how the filter responds. Some examples are $[1, -1, 1, -1, ...]$ and $[1, 0, -1, 0, 1, 0, -1, ...]$.

   (e) Find an input sequence of length 3, such that the corresponding output sequence is proportional to $(0.9)^n$ for $n \geq 3$.