Students have either already taken or started taking this quiz, so be careful about editing it. If you change any quiz questions in a significant way, you may want to consider regrading students who took the old version of the quiz.

Points 100 Published

Details Questions

Q1 Pick 1 questions, 15 pts per question

Q1

Consider the following non minimum-phase system described by its transfer function $H(z)$.

$$H(z) = \frac{1-2z^{-1}}{1+\frac{1}{3}z^{-1}}$$

Specify a minimum-phase system function $H_{min}(z)$ such that the frequency-response magnitudes of the two systems are equal. I.e., $|H(e^{j\omega})| = |H_{min}(e^{j\omega})|$.

Solution:

Zero at $z = 2$ is reflected inside unit circle by multiplying by all-pass filter with magnitude of 1:

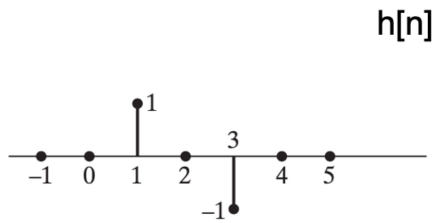
$$H_{ap}^{-1} = \frac{2-z^{-1}}{1-2z^{-1}} = \frac{2(1-\frac{1}{2}z^{-1})}{1-2z^{-1}}$$

$$H_{min} = \frac{2(1-\frac{1}{2}z^{-1})}{1+\frac{1}{3}z^{-1}}$$

Q2 Pick 1 questions, 15 pts per question

Q2

Below is the impulse response, $h[n]$, for an LTI system. Determine the group delay of the system.

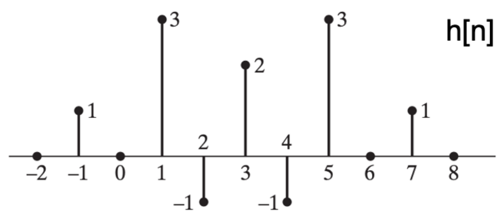


Solution:

Due to the symmetry of the impulse response, the system has generalized linear phase of $arg[H(e^{j\omega})] = \beta - n_0\omega$ where n_0 is the point of symmetry in the impulse response. Group delay is the negative derivate of the phase, which is just n_0. Therefore by inspection above group delay = 2.

Q2

Below is the impulse response, $h[n]$, for an LTI system. Determine the group delay of the system.

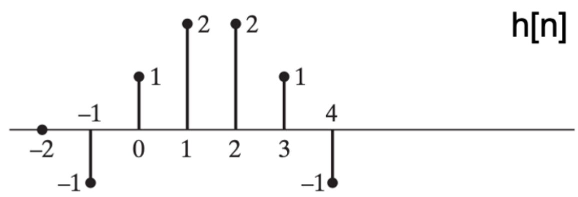


Solution:

Due to the symmetry of the impulse response, the system has generalized linear phase of $arg[H(e^{j\omega})] = \beta - n_0\omega$ where n_0 is the point of symmetry in the impulse response. Group delay is the negative derivate of the phase, which is just n_0. Therefore by inspection above group delay = 3.

Q2

Below is the impulse response, $h[n]$, for an LTI system. Determine the group delay of the system.

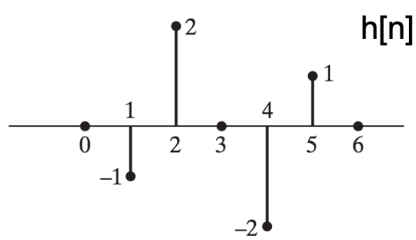


Solution:

Due to the symmetry of the impulse response, the system has generalized linear phase of $arg[H(e^{j\omega})] = \beta - n_0\omega$ where n_0 is the point of symmetry in the impulse response. Group delay is the negative derivate of the phase, which is just n_0. Therefore by inspection above group delay = 1.5.

Q2

Below is the impulse response, $h[n]$, for an LTI system. Determine the group delay of the system.



Solution:

Due to the symmetry of the impulse response, the system has generalized linear phase of $arg[H(e^{j\omega})] = \beta - n_0\omega$ where n_0 is the point of symmetry in the impulse response. Group delay is the negative derivate of the phase, which is just n_0. Therefore by inspection above group delay = 3.

Q3 Pick 1 questions, 30 pts per question

Q3Soln

Solution:

a) The DTFT is given by:

$$\hat{X}(e^{j\omega}) = X(e^{j\omega}) + X(e^{j\omega})e^{-j\omega N} = X(e^{j\omega})(1 + e^{-j\omega N})$$

The DFT is samples of the DTFT:

$$\hat{X}[k] = \hat{X}(e^{j\omega})\big|_{\omega=\frac{2\pi k}{2N}}$$
$$= X(e^{j2\pi k/2N})(1+(-1)^k)$$

Therefore:

$$\hat{X}[k] = \begin{cases} 2X[\frac{k}{2}] & \text{k even} \\ 0 & \text{k odd} \end{cases}$$

b)

From figure 1:

$$\hat{Y}[k] = \hat{X}[k]H[k]$$

From figure 2:

$$W[k] = G[k]X[k]$$

$W[k]$ is a downsampled version of $\hat{Y}[k]$, therefore:

$$W[k] = \hat{Y}[2k]$$
$$G[k]X[k] = \hat{X}[2k]H[2k] = 2X[k]H[2k]$$

This simplifies to:

$$G[k] = 2H[2k]$$

$G[k]$ is two times the even samples of the 2N-point DFT of H, which we know can be found by time aliasing h[n] with N points. See the decimation-in-frequency FFT for more information. Therefore $g[n] = 2(h[n] + h[n+N])$, and system A time aliases and multiplies by 2.

For system B we need:

$$Y[k] = \begin{cases} W[\frac{k}{2}] & \text{k even} \\ 0 & \text{k odd} \end{cases}$$

Thus System B regenerates the 2N-point sequence by repeating w[n].

Q3

Let $x[n]$ be an N-point sequence such that $x[n] = 0$ for $n < 0$ and for $n > N-1$. Let $\hat{x}[n]$ be the 2N-point sequence obtained by repeating $x[n]$; i.e.,

$$\hat{x}[n] = \begin{cases} x[n] & 0 \le n \le N-1 \\ x[n-N] & N \le n \le 2N-1 \\ 0 & else \end{cases}$$

Consider the implementation of a discrete-time filter shown in Figure 1 below. This system has an impulse response $h[n]$ that is 2N points long; i.e., $h[n] = 0$ for $n < 0$ and for $n > 2N-1$.

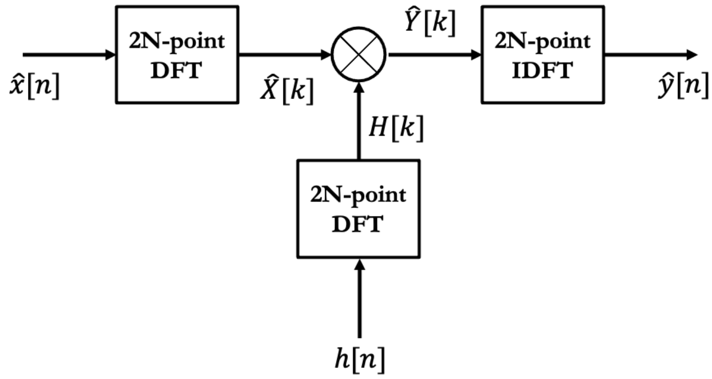


Figure 1

a) In Figure 1, what is $\hat{X}[k]$, the 2N-point DFT of $\hat{x}[n]$, in terms of $X[k]$, the N-point DFT of $x[n]$? Hint: start by writing $\hat{X}(e^{j\omega})$ in terms of $X(e^{j\omega})$.

b) The system from Figure 1 can be implemented using only N-point DFTs as indicated in Figure 2 below for appropriate choices for System A and System B. Specify System A and System B so that $\hat{y}[n]$ in Figure 1 and $y[n]$ in Figure 2 are equal for $0 \leq n \leq 2N-1$. Note that $h[n]$ and $y[n]$ in Figure 2 are 2N-point sequences and $w[n]$ and $g[n]$ are N-point sequences. Be careful with the DFT and sequence lengths in your analysis. You can specify systems A and B with equations or a functional description.

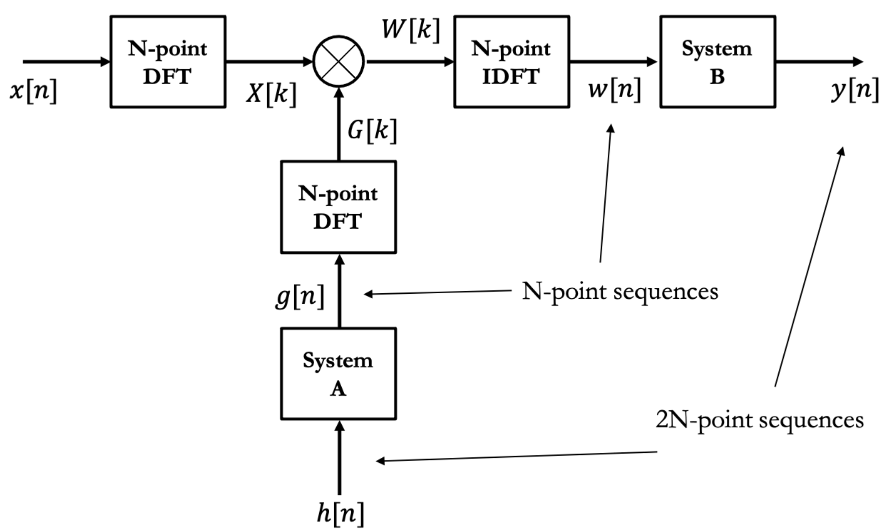


Figure 2

Q4 Pick 1 questions, 30 pts per question

Q4

Let $x[n]$ and $h[n]$ be two finite duration sequences of length $L = 6$; i.e., $x[n] = h[n] = 0$ for $n < 0$ and $n \geq 6$. Let $X_6[k]$ and $H_6[k]$ denote 6-point DFT's of $x[n]$ and $h[n]$, respectively. The 6-point inverse DFT of the product $Y_6[k] = X_6[k]\, H_6[k]$, denoted $y_6[n]$, produces the following values:

n	0	1	2	3	4	5
$y_6[n]$	21	20	19	18	17	16

Let $X_8[k]$ and $H_8[k]$ denote the 8-point DFT's of the aforementioned sequences $x[n]$ and $h[n]$. The 8-point inverse DFT of the product $Y_8[k] = X_8[k]\, H_8[k]$, denoted $y_8[n]$, produces the following values:

n	0	1	2	3	4	5	6	7
$y_8[n]$	12	15	14	14	16	16	15	9

Given $y_6[n]$ and $y_8[n]$, find the linear convolution of $x[n]$ and $h[n]$. I.e., determine the numerical values of $y[n] = x[n] * h[n]$.

Solution:

Because $x[n]$ and $h[n]$ are both length 6, the linear convolution will have length, L=2*6-1=11. Therefore the 6-point and 8-point IDFTs, $y_6[n]$ and $y_8[n]$ are time aliased versions of $y[n] = x[n] * h[n]$, such that $y_6[n] = y[n] + y[n+6]$ and $y_8[n] = y[n] + y[n+8]$. With these two relationships you can derive the following equations:

$$y_6[0] = y[0] + y[6] = 21$$
$$y_6[1] = y[1] + y[7] = 20$$
$$y_6[2] = y[2] + y[8] = 19$$
$$y_6[3] = y[3] + y[9] = 18$$
$$y_6[4] = y[4] + y[10] = 17$$
$$y_6[5] = y[5] = 16$$

and

$$y_8[0] = y[0] + y[8] = 12$$
$$y_8[1] = y[1] + y[9] = 15$$
$$y_8[2] = y[2] + y[10] = 14$$
$$y_8[3] = y[3] = 14$$
$$y_8[4] = y[4] = 16$$
$$y_8[5] = y[5] = 16$$
$$y_8[6] = y[6] = 15$$
$$y_8[7] = y[7] = 9$$

From these equations the linear convolution $y[n]$ can be derived as:

y[n] = [6 11 13 14 16 16 15 9 6 4 1]

Q4

Let $x[n]$ and $h[n]$ be two finite duration sequences of length $L = 6$; i.e., $x[n] = h[n] = 0$ for $n < 0$ and $n \geq 6$. Let $X_6[k]$ and $H_6[k]$ denote 6-point DFT's of $x[n]$ and $h[n]$, respectively. The 6-point inverse DFT of the product $Y_6[k] = X_6[k]H_6[k]$, denoted $y_6[n]$, produces the following values:

n	0	1	2	3	4	5
$y_6[n]$	21	21	21	21	21	21

Let $X_9[k]$ and $H_9[k]$ denote the 9-point DFT's of the aforementioned sequences $x[n]$ and $h[n]$. The 9-point inverse DFT of the product $Y_9[k] = X_9[k]H_9[k]$, denoted $y_9[n]$, produces the following values:

n	0	1	2	3	4	5	6	7	8
$y_9[n]$	9	12	15	18	20	21	15	10	6

Given $y_6[n]$ and $y_9[n]$, find the linear convolution of $x[n]$ and $h[n]$. I.e., determine the numerical values of $y[n] = x[n] * h[n]$.

Solution:

See above for solution methodology.

Q4

Let $x[n]$ and $h[n]$ be two finite duration sequences of length $L = 7$; i.e., $x[n] = h[n] = 0$ for $n < 0$ and $n \geq 7$. Let $X_7[k]$ and $H_7[k]$ denote 7-point DFT's of $x[n]$ and $h[n]$, respectively. The 7-point inverse DFT of the product $Y_7[k] = X_7[k]H_7[k]$, denoted $y_7[n]$, produces the following values:

n	0	1	2	3	4	5	6
$y_7[n]$	20	19	18	17	18	19	20

Let $X_{11}[k]$ and $H_{11}[k]$ denote the 11-point DFT's of the aforementioned sequences $x[n]$ and $h[n]$. The 11-point inverse DFT of the product $Y_{11}[k] = X_{11}[k]H_{11}[k]$, denoted $y_{11}[n]$, produces the following values:

n	0	1	2	3	4	5	6	7	8	9	10
$y_{11}[n]$	10	11	11	11	13	15	20	15	12	7	6

Given $y_7[n]$ and $y_{11}[n]$, find the linear convolution of $x[n]$ and $h[n]$. I.e., determine the numerical values of $y[n] = x[n] * h[n]$.

Solution:

See above for solution methodology.

Q4

Let $x[n]$ and $h[n]$ be two finite duration sequences of length $L = 7$; i.e., $x[n] = h[n] = 0$ for $n < 0$ and $n \geq 7$. Let $X_7[k]$ and $H_7[k]$ denote 7-point DFT's of $x[n]$ and $h[n]$, respectively. The 7-point inverse DFT of the product $Y_7[k] = X_7[k]H_7[k]$, denoted $y_7[n]$, produces the following values:

n	0	1	2	3	4	5	6
$y_7[n]$	20	20	20	20	20	20	20

Let $X_{10}[k]$ and $H_{10}[k]$ denote the 10-point DFT's of the aforementioned sequences $x[n]$ and $h[n]$. The 10-point inverse DFT of the product $Y_{10}[k] = X_{10}[k]H_{10}[k]$, denoted $y_{10}[n]$, produces the following values:

n	0	1	2	3	4	5	6	7	8	9
$y_{10}[n]$	11	13	17	14	15	16	20	15	12	7

Given $y_7[n]$ and $y_{10}[n]$, find the linear convolution of $x[n]$ and $h[n]$. I.e., determine the numerical values of $y[n] = x[n] * h[n]$.

Solution:

See above for solution methodology.

Q5 Pick 1 questions, 10 pts per question

Q5

How are the DTFT and the DFT of a discrete-time sequence related? Be specific and explain clearly. Graphs and equations may be helpful in explaining.

Solution:

The DFT is uniform samples of the DTFT over the range 0 to 2pi.

Q5

How are the DFT and FFT of a discrete-time sequence related? Be specific and explain clearly. Graphs and equations may be helpful in explaining.

Solution:

They are the same. The FFT is just a faster computation of the DFT, because it requires less multiplications.

Q5

How are the DTFT and Z-transform of a discrete-time sequence related? Be specific and explain clearly. Graphs and equations may be helpful in explaining.

Solution:

The DTFT is the Z-transform evaluated at $z = e^{j\omega}$.

Q6 0 pts

Upload a single file (,pdf preferred) of your answers with work here for grading and partial credit.

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