University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

ESE531, Spring 2021 HW2: DTFT, z-transform, Sampling Sunday, February 7

Due: Monday, February 15th, 11:59PM

- **Recommended Problems for Practice:** From the book: 3.10, 3.23, 3.27, 3.40, 3.42, 3.57, 4.4, 4.8
- **Homework Problems:** All problems below must be turned in and are not optional for full credit

  1. From the book: 3.32, 3.45, 3.48, 4.21, 4.23
  2. Matlab problem 1:
     For the signal $x[n] = (0.9)^n u[n]$, compute the DTFT $X(e^{j\omega})$ using `freqz`.
     (a) Make a plot of both the magnitude and the phase versus ω over the range $[-\pi, \pi)$. This will require a shift of the `[X,W]` vectors returned from `freqz`. Submit your plot. Explain why the magnitude and phase are even and odd functions of ω respectively.
     (b) Derive formulas for the magnitude and phase of the DTFT.
     (c) Compute and plot the magnitude and phase by a direct evaluation of the formulas derived in (b) and compare to plot in (a). How do they compare?
  3. Matlab problem 2: Aliasing a Sinusoid
     Consider the formula for a continuous-time sinusoidal signal:

     $$x(t) = sin(2\pi f_0 t + \phi)$$

     We can sample x(t) at a rate $f_s = 1/T_s$ to obtain a discrete-time signal

     $$x[n] = x(t)|_{t=nT_s} = x(t)|_{t=n/f_s} = sin\left(2\pi \frac{f_0}{f_s} n + \phi\right)$$

     If we make plots of x[n] for different combinations of f_0 and f_s, the aliasing problem can be illustrated. For the following, take the sampling frequency to be $f_s = 8kHz$. Be careful to use `stem` and `plot` for discrete and continuous time respectively, and label all axes.
     (a) Make a single plot of a sample sine wave. Let the frequency of the sine wave be 300 Hz, and take samples over an interval of 10ms. The phase can be arbitrary and for ease can be set to 0. Plot and submit the resulting discrete-time signal. You should be able to see the outline of a sinusoid visually as your eyes and brain perform a reconstruction visualizing the envelope of the discrete-time signal.

(b) Now make a series of plots, just like part (a), but very the sinusoidal frequency from 100Hz to 475Hz in steps of 125 Hz. Use `subplot` to put four plots on one window. Submit your plots for increasing frequency.

(c) Make another series of plots, just like part (b), but very the sinusoidal frequency from 7525Hz to 7900 Hz, in steps of 125 Hz. Identify and explain the phenomenon you see with the apparent frequency from your visualization. Submit your plots for increasing frequency.

(d) Again make another series of plots, just like part (b), but very the sinusoidal frequency from 32,100 Hz to 32,475 Hz, in steps of 125 Hz. Try to predict in advance whether the apparent frequency will look like it's increasing or decreasing. You don't need to submit anything for this part, but feel free to tell us if you got it right!

4. Matlab problem 3: Aliasing a Chirp Signal

A linear frequency-modulated signal makes a good test for aliasing, because the frequency moves over a range. This signal is often called a "chirp," due to the audible sound it makes when played through a speaker. The mathematical definition of a chirp is

$$c(t) = cos(\pi\mu t^2 + 2\pi f_1 t + \psi) \tag{1}$$

The instantaneous frequency of this signal can be found by taking the time derivative of the phase (the argument of the cosine). The result is

$$f_i(t) = \mu t + f_1 \tag{2}$$

which exhibits a linear variation versus time.

(a) Take the parameters of the chirp to be $f_1 = 4kHz$, $\mu = 600kHz/s$, and ψ arbitrary. If the total time duration of the chirp is 50ms, determine the frequency range that is covered by the swept frequency of the chirp.

(b) Let the sampling frequency be $f_s = 8kHz$. Plot the discrete-time samples of the chirp using both `stem` and `plot` again assuming the length of the chirp is 50ms. Submit a single figure with both the `stem/plot` on it (useful functions `hold` and `figure`). You can listen to the chirp with `sound`.

(c) Let the sampling frequency be $f_s = 70kHz$. Plot the discrete-time samples of the chirp using both `stem` and `plot` again assuming the length of the chirp is 50ms. Submit a single figure with both the `stem/plot` on it.

(d) What do you notice between the plots from (b) and (c)? What was the difference of the sounds? Explain why they are different.