University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

HW4: Filter Banks and Reconstruction

Sunday, Feb. 21

Due: Monday, March 1, 11:59PM

- **Recommended Problems for Practice:** From this homework onward, relocated to end of homework to avoid confusion.
- **Homework Problems:** All problems must be turned in and are not optional for full credit

1. Homework problems from the book: 4.52, 4.58, 4.65, 4.66
2. Matlab problem 1: Reconstruction of Signals from Samples

   This problem explores various methods that can be used for reconstruction on analog signal from digital samples. Consider the case where you are given three samples of an analog signal, $x(t)$: $x(0) = 2, x(t)|_{t=1} = 1, x(2) = x(t)|_{t=2} = -1$. No other information is given. We know there is no one signal that fits these samples, but with assumptions reconstruction methods we can create an estimate for the analog signal. You will try to fit the three data points with a sine wave, a polynomial, and then with ideal and non-ideal low pass filtering.

   (a) Fitting a Sine Wave
      i. Assume that the three samples correspond to a sinusoidal waveform of the form:
      $$x(t) = Acos(\omega t + \phi)$$
      You have $x(0), x(1)$, and $x(2)$. Set up the relevant equations to solve for A, ω, and ϕ and find a solution to the system of equations.
      ii. Can you always solve these equations given 3 samples?
      iii. Find another solution to the equations in part (a) and plot the sinusoids from both solutions you found on a very fine grid with $\Delta t = .01s$.

   (b) Linear and Polynomial Interpolation
      i. Using MATLAB, connect the samples from (a) with straight lines. Plot the result on a fine grid with spacing, $\Delta t = .01s$. Note: there a MATLAB function that does this automatically.
      ii. Convolve the three samples with an impulse response that is triangular, but first insert four zeros between each of them, and use an impulse response 0.2, 0.4, 0.6, 0.8, 1.0, 0.8, 0.6, 0.4, 0.2. Show that this result is identical to linear interpolation if we assume that the samples at $t = -1$ and $t = +3$ are zero.

iii. Using MATLAB, fit a second-degree polynomial to the three data points (see `polyfit` and `polyval`). Plot the polynomial on a fine grid for $-5 \leq t \leq 5$. Is this curve realistic in a practical sense? Does it do a good job in extending the signal values beyond the range $0 \leq t \leq 2$?

(c) Ideal Low-Pass Filtering
There are no ideal low-pass filters available in reality. However, we can calculate the waveform that would result from an ideal low-pass filter as follows: An ideal low-pass operation corresponds to a multiplication of the spectrum of a signal by a rectangular function in the frequency domain. This corresponds to a convolution with the inverse Fourier transform, which is a sinc function in the time domain. As applied to point samples, this amounts to sinc interpolation:

$$x_r(t) = \sum_{k=-\infty}^{\infty} x(t_k) \frac{sin(\pi(t - kT_s)/T_s)}{\pi(t - kT_s)/T_s}$$

where the samples $x(t_k)$ are taken at $t_k = kT_s$.

i. Write a sinc interpolator based on the equation above. Assume that only a finite number of the signal samples will be nonzero and that the signal need only be reconstructed over a finite time interval. Interpolate a single-point sample of value 1 at $t = 0$. Plot the result from $-5 \leq t \leq 5$.

ii. Now interpolate the three-point case given in (a). Plot the result from $-5 \leq t \leq 5$ and compare the result to that obtained by fitting a sine-wave.

- **Recommended Problems for Practice:** From the book: 4.41, 4.46, 4.59, 4.62