University of Pennsylvania
Department of Electrical and System Engineering
Digital Signal Processing

HW7: GLP Systems, Windowing, and DFT

Sunday, Apr. 4

Due: Monday, Apr 12, 11:59PM

- **Homework Problems:** All problems must be turned in and are not optional for full credit

1. Homework problems from the book: 5.39, 7.15, 7.45, 8.16

2. Matlab problem 1: Design an FIR filter using Windowing
   Design a length-23 linear-phase FIR low-pass filter with a band edge of $\omega_0 = 0.3\pi$ using the following windows:
   - Rectangular (`rectwin`)
   - Triangular or Bartlett (`bartlett`)
   - Hann (`hann`)
   - Hamming (`hamming`)
   - Blackman (`blackman`)

   Plot the impulse response, magnitude response, and phase response or the 5 filters. Compare the characteristics of the magnitude response of the 5 filters. Do this by comparing the squared error and transition bandwidth. The discrete squared error is defined as

   $$\epsilon = \frac{1}{N}\sum_{k=0}^{N-1} |H_d(e^{j\omega_k}) - H(e^{j\omega_k})|^2$$

   NOTE: For the problem below you need to count the number of floating point operations (flops). To approximate the number of flops, you can use the code available on the Mathworks website here: `https://www.mathworks.com/matlabcentral/fileexchange/50608-counting-the-floating-point-operations--flops-`. You can use any timing method including `clock, etime, tic, toc` to measure the execution time. Just pick one method for comparison.

3. Matlab problem 2: Calculation of the DFT. In this problem you will measure the execution time and the number of floating point operations (flops) of a direct calculation of the DFT using three different organizations of the algorithm. The definition of the DFT is given by:

   $$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk} \tag{1}$$

for

$$W_N = e^{-j2\pi/N} = cos(2\pi/N) - jsin(2\pi/N) \quad (2)$$

and $k = 0, 1, ...N - 1$.

(a) Two-Loop Program. Write a program (M-file) or a function in MATLAB to evaluate the DFT given above using two nested `for` loops with the inner loop summing over n and the outer loop indexing over k. Time the program for several lengths using the `clock` and `etime` commands. Evaluate the number of flops required for several lengths. Compare the times and flops of your DFT program with the built-in MATLAB command `fft` for the same lengths. Comment on the results.

(b) One-Loop Program. Write a DFT program using one loop which steps through each value of k and executes an inner product. Time the program and evaluate the number of flops as was done in (a). Explain the results obtained.

(c) No-Loop Program. Write a DFT program using a single matrix multiplication. Write your own DFT matrix rather than using the built-in `dftmtx`. Use the `exp` command with the exponent formed by an outer product of a vector of `n = 0:(N-1)` and a vector of `k = 0:(N-1)`. Time and evaluate the flops as you did for the previous programs.

(d) Comment on the differences and on the comparisons of the three implementations. How many flops are used in generating the complex exponentials?

4. **Recommended Problems for Practice:** From the book: 5.38, 7.27, 7.34, 8.14