ESE 531: Digital Signal Processing

Week 4

Lecture 7: February 7, 2021

Sampling

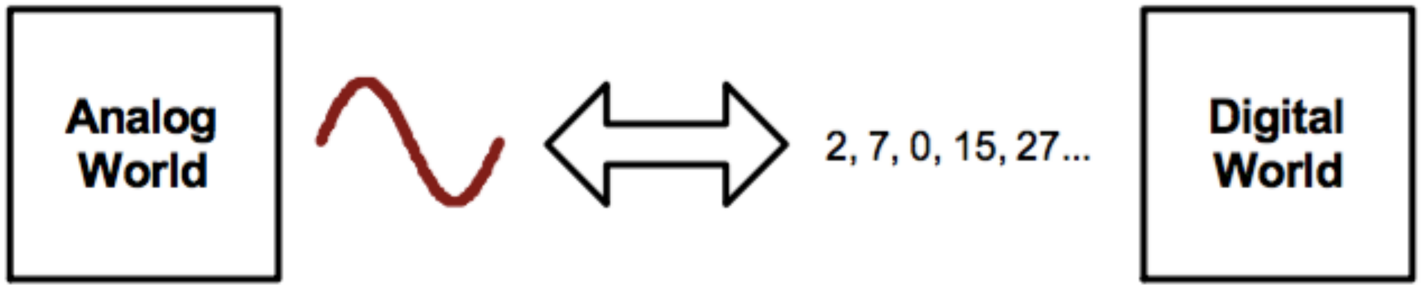
Lecture Outline

- Sampling
  - Frequency Response of Sampled Signal

Video Example

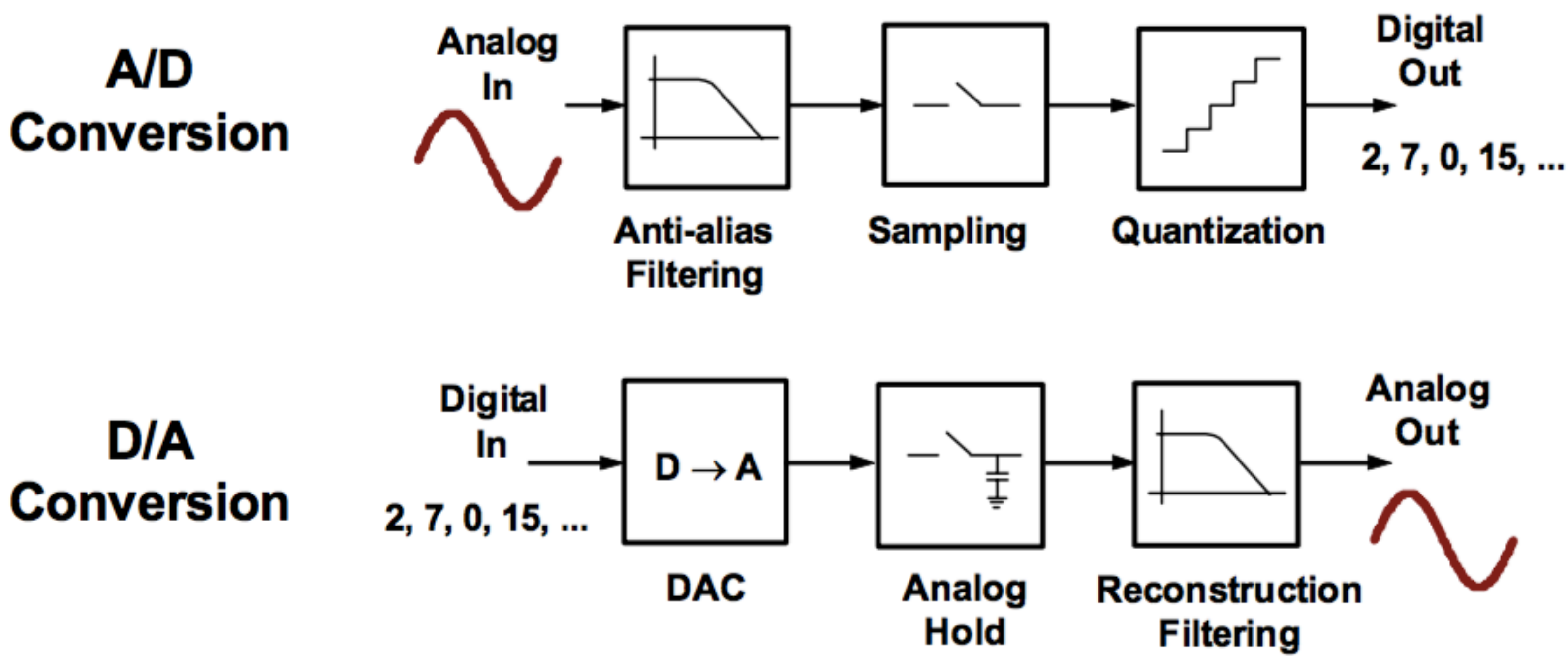
- https://www.youtube.com/watch?v=ByTsISFXUoY

The Data Conversion Problem



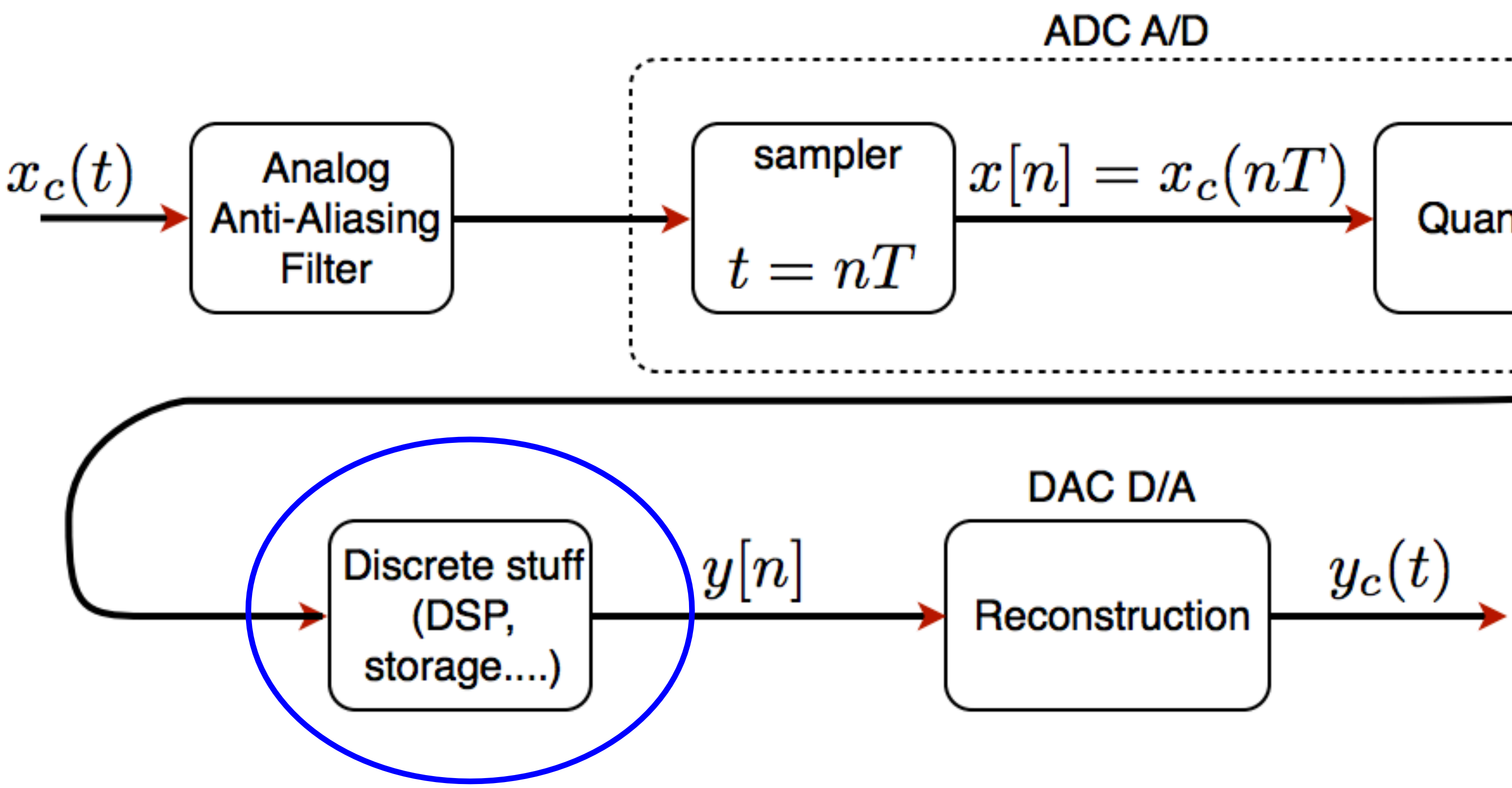
- Real world signals
  - Continuous time, continuous amplitude
- Digital abstraction
  - Discrete time, discrete amplitude
- Two problems
  - How to go discretize in time and amplitude
    - A/D conversion
  - How to "undescretize" in time and amplitude
    - D/A conversion

Overview



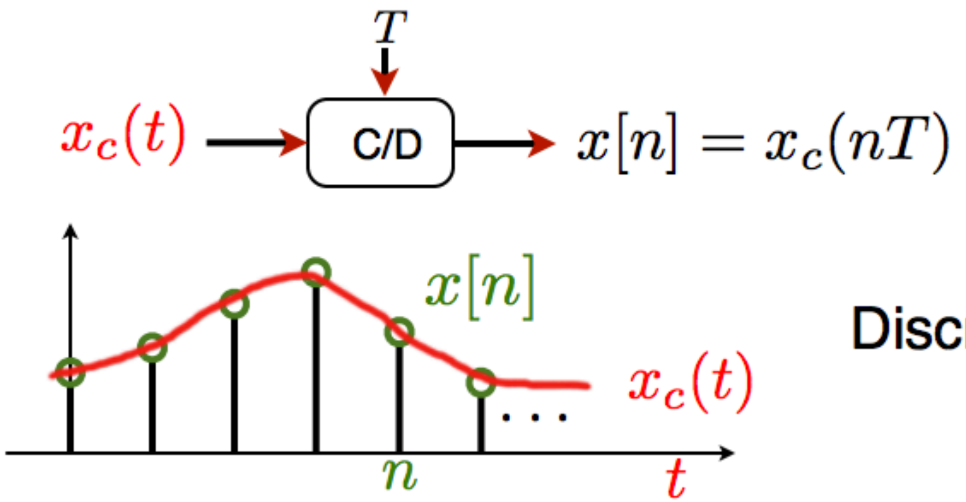
- We'll first look at these building blocks from a functional, "black box" perspective

DSP System



ADC A/D
$x_c(t)$
Analog
Anti-Aliasing
Filter
sampler
$t = nT$
$x[n] = x_c(nT)$
Quantizer
Discrete stuff
(DSP,
storage....)
$y[n]$
DAC D/A
Reconstruction
$y_c(t)$

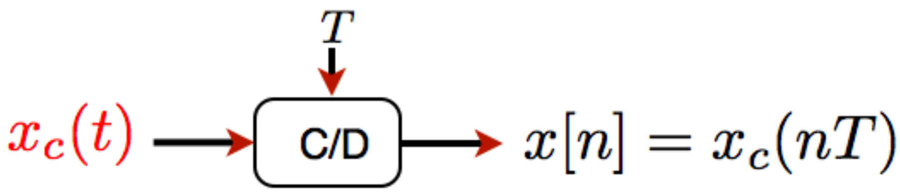
Ideal Sampling Model

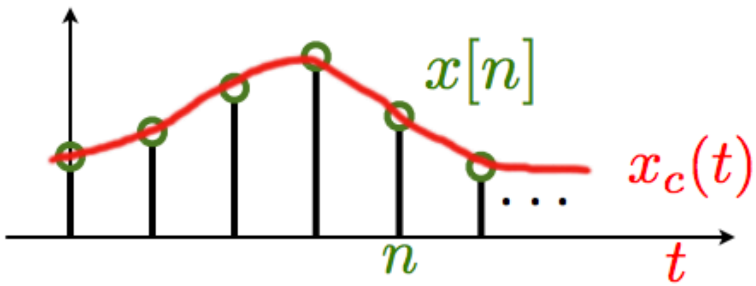


Discrete and Continuous

- Ideal continuous-to-discrete time (C/D) converter
  - T is the sampling period
  - f_s=1/T is the sampling frequency
  - Ω_s=2π/T

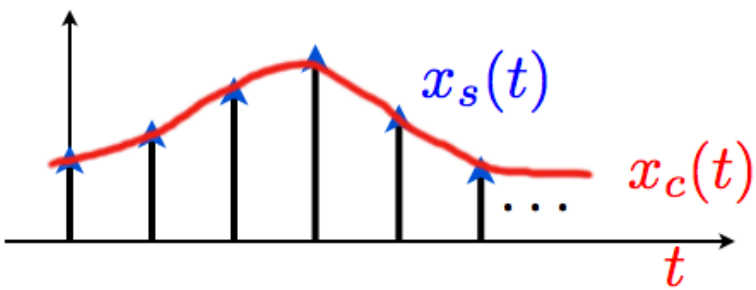
Ideal Sampling Model





Discrete and Continuous

define impulsive sampling:



Continuous

$$x_s(t) = \cdots + x_c(0)\delta(t) + x_c(T)\delta(t-T) + \cdots$$

$$x_s(t) = x_c \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

Ideal Sampling Model

$$x_s(t) = x_c \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

- Dirac delta function, δ(t)
  - Infinitely high and thin, area of 1
  - Not physical—for modeling

$$x[n] \leftrightarrow x_s(t) \leftrightarrow x_c(t)$$

- Three signals. How are they related? In time? In frequency?

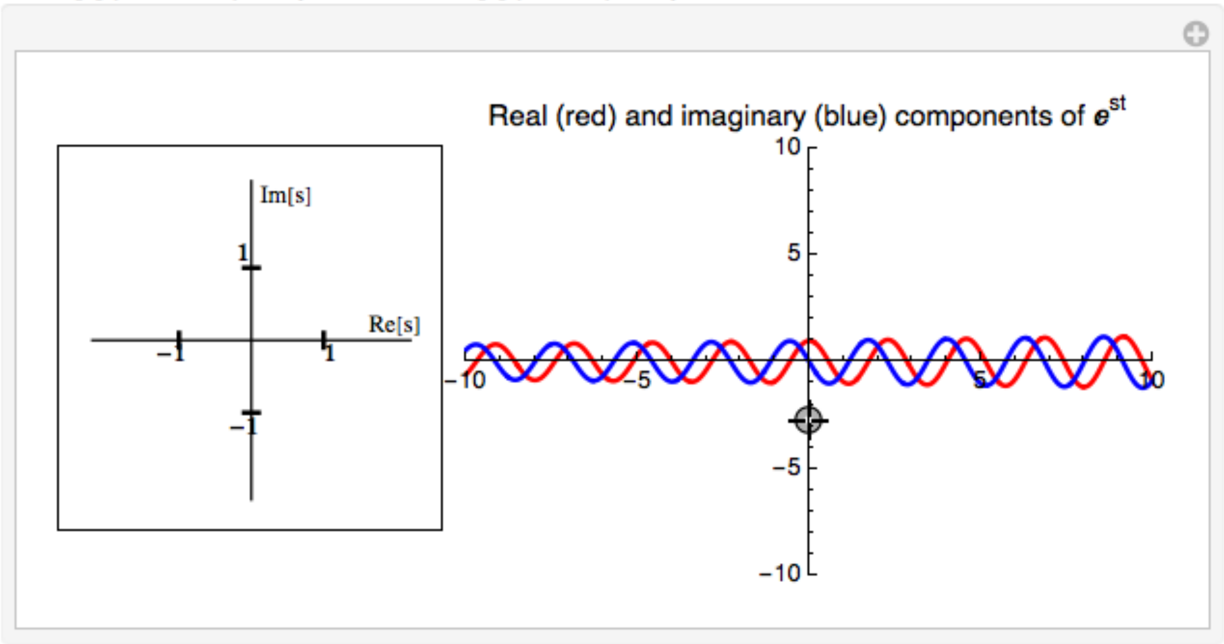
Reminder: Laplace Transform

- The Laplace transform takes a function of time, t, and transforms it to a function of a complex variable, s.
- Because the transform is invertible, no information is lost and it is reasonable to think of a function f(t) and its Laplace transform F(s) as two views of the same phenomenon.
- Each view has its uses and some features of the phenomenon are easier to understand in one view or the other.

S-Plane

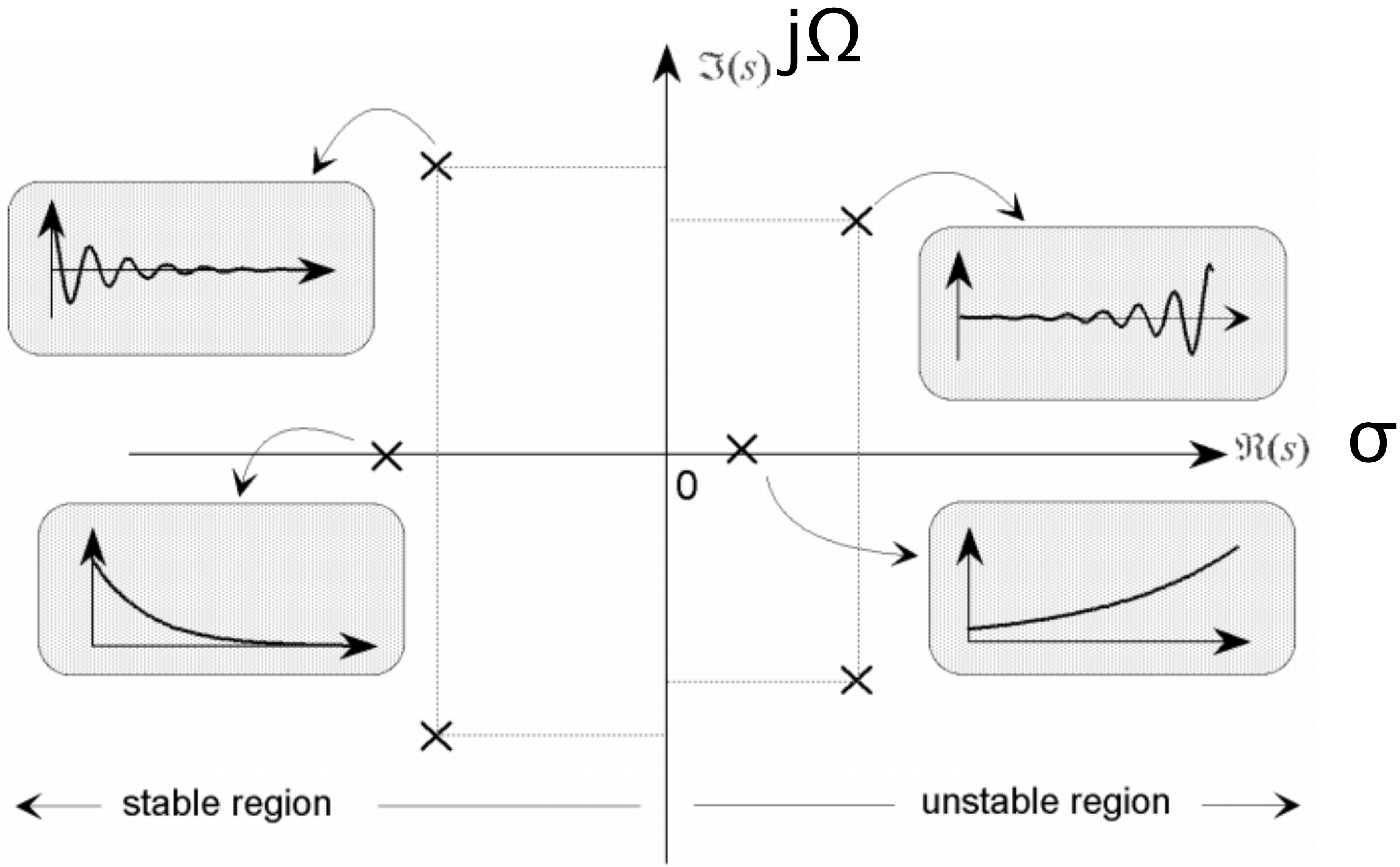
- $s=\sigma+j\Omega$

- Wolfram Demo

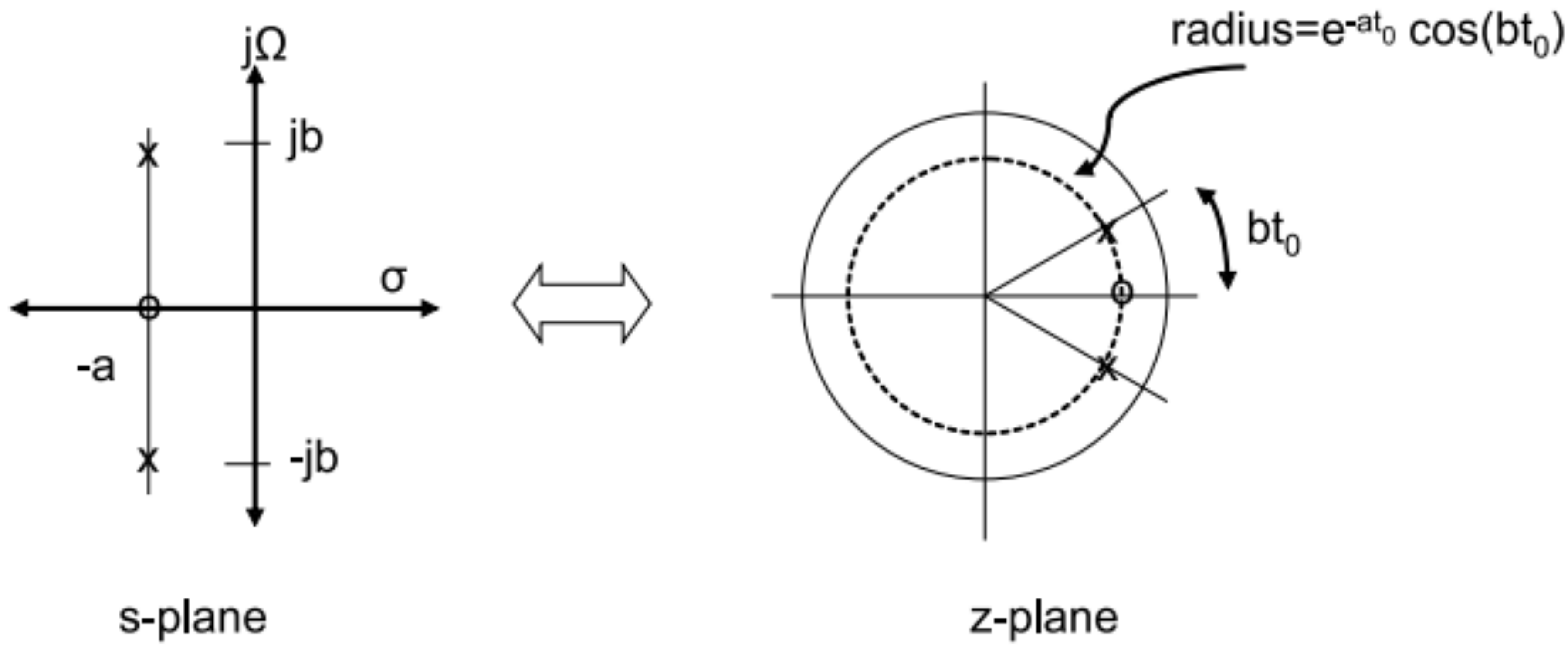


- See piazza for more information on using this demo:
  - https://piazza.com/class/kjloarx1214170?cid=25

S-plane and stability



S-Plane Mapping to Z-Plane



Frequency Domain Analysis

- How is x[n] related to $x_s(t)$ in frequency domain?

$$x[n] = x_c(nT)$$

$$x_s(t) = x_c \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

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$$x_s(t) = x_c \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$x_s(t)$:C.T

$$X_s(j\Omega) = \sum_n x_c(nT) e^{-j\Omega nT}$$

$x[n]$:D.T

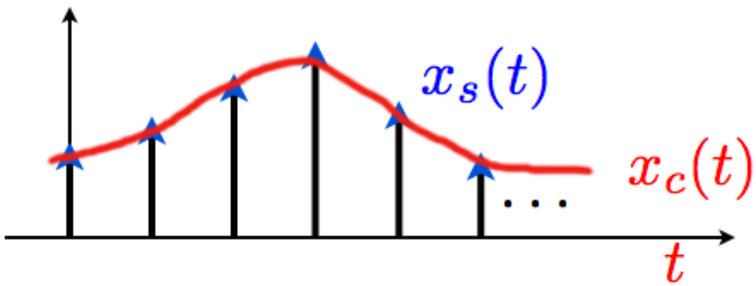
$$X(e^{j\omega}) = \sum_n x[n] e^{-j\omega n}$$

$$\omega = \Omega T$$

$$X(e^{j\omega}) = X_s(j\Omega)\big|_{\Omega=\omega/T}$$

$$X_s(j\Omega) = X(e^{j\omega})\big|_{\omega=\Omega T}$$

Frequency Domain Analysis



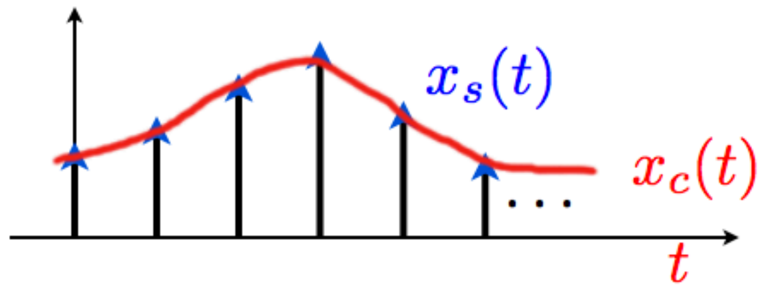
Continuous

$$x_s(t) = \cdots + x_c(0)\delta(t) + x_c(T)\delta(t-T) + \cdots$$

$$x_s(t) = x_c \underbrace{\sum_{n=-\infty}^{\infty} \delta(t-nT)}_{\triangleq s(t)}$$

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

Frequency Domain Analysis



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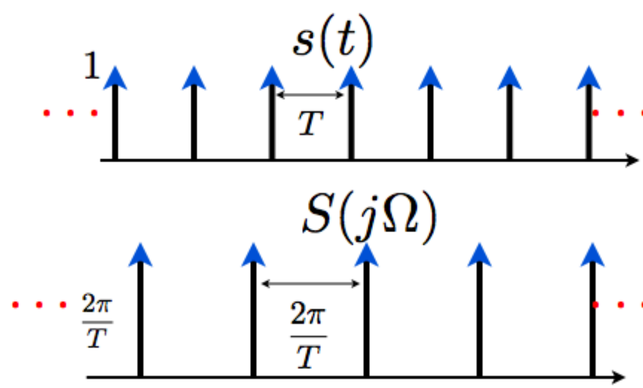
$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$s(t) \leftrightarrow S(j\Omega)$$

$$S(j\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - \frac{2\pi}{T}k)$$

$\frac{2\pi}{T} = \Omega_s$



Frequency Domain Analysis

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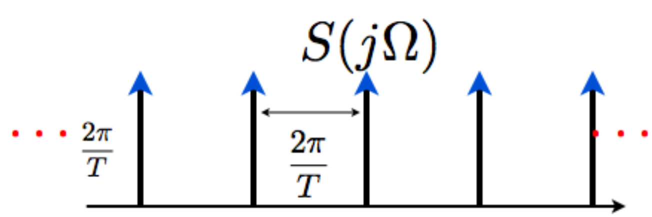

Frequency Domain Analysis

$$x_s(t) = s(t) \cdot x_c(t)$$

$$\begin{aligned} X_s(j\Omega) &= \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega) \\ &= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s)) \quad \Big| \quad \Omega_s = \frac{2\pi}{T} \end{aligned}$$

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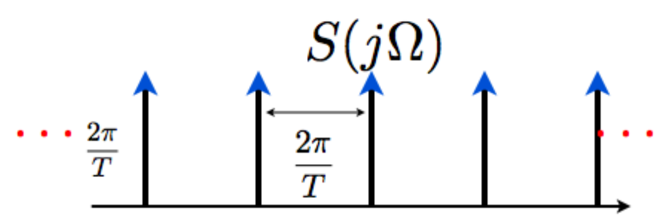
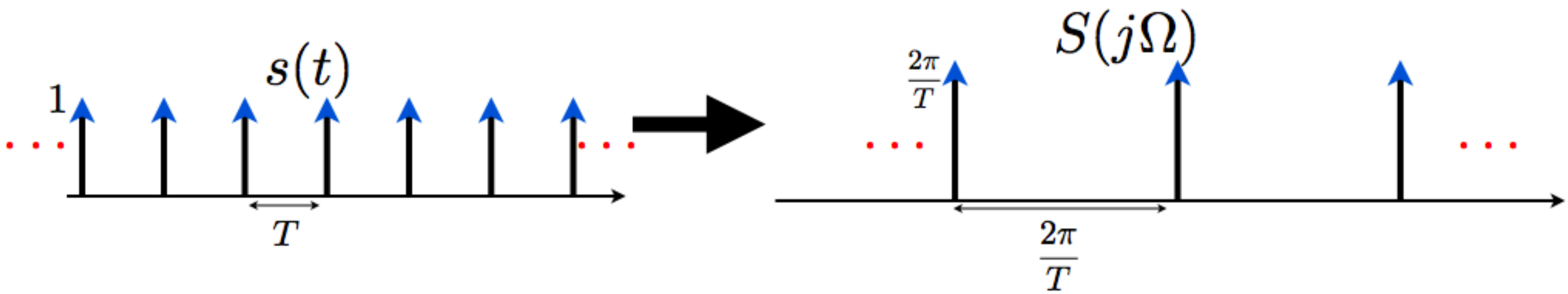
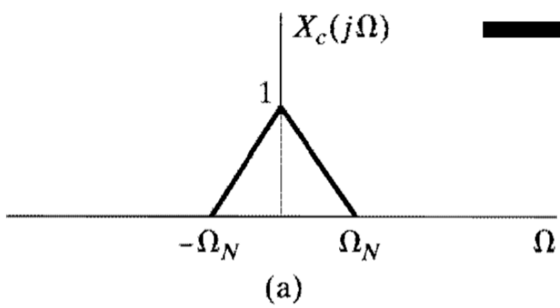


Frequency Domain Analysis



and $\Omega_s > 2\Omega_N$



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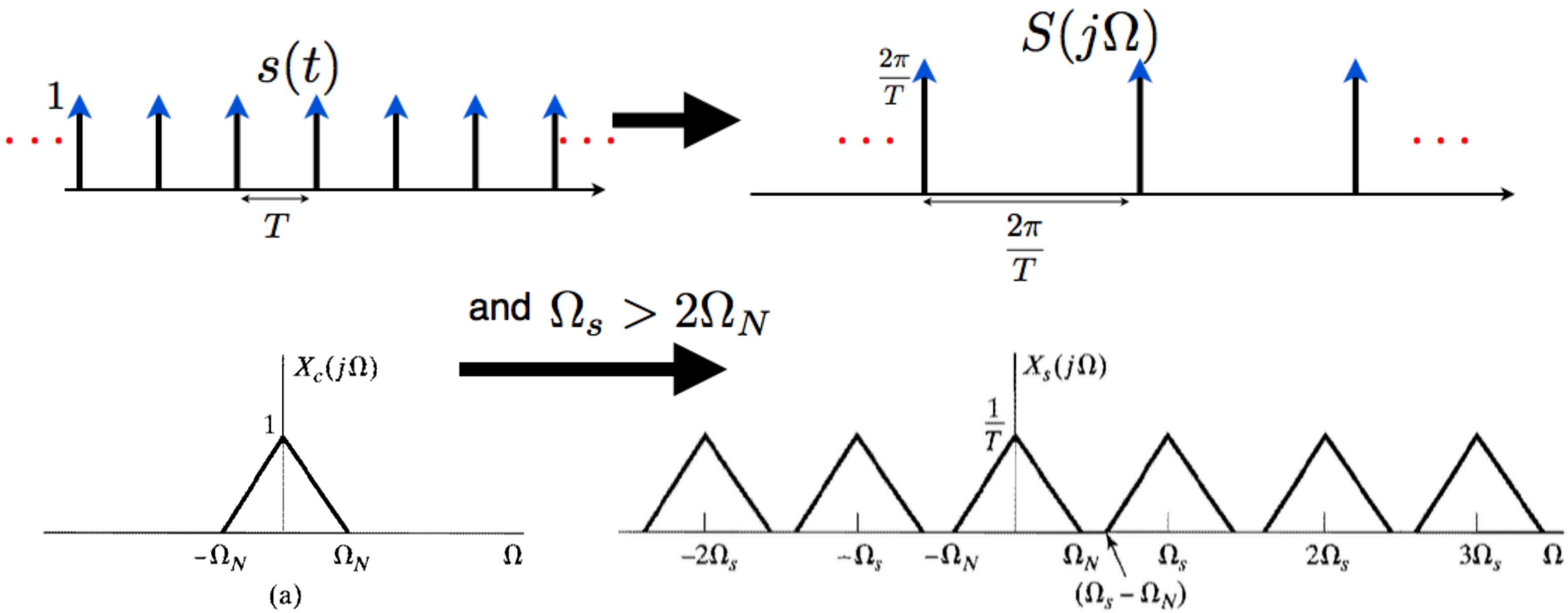
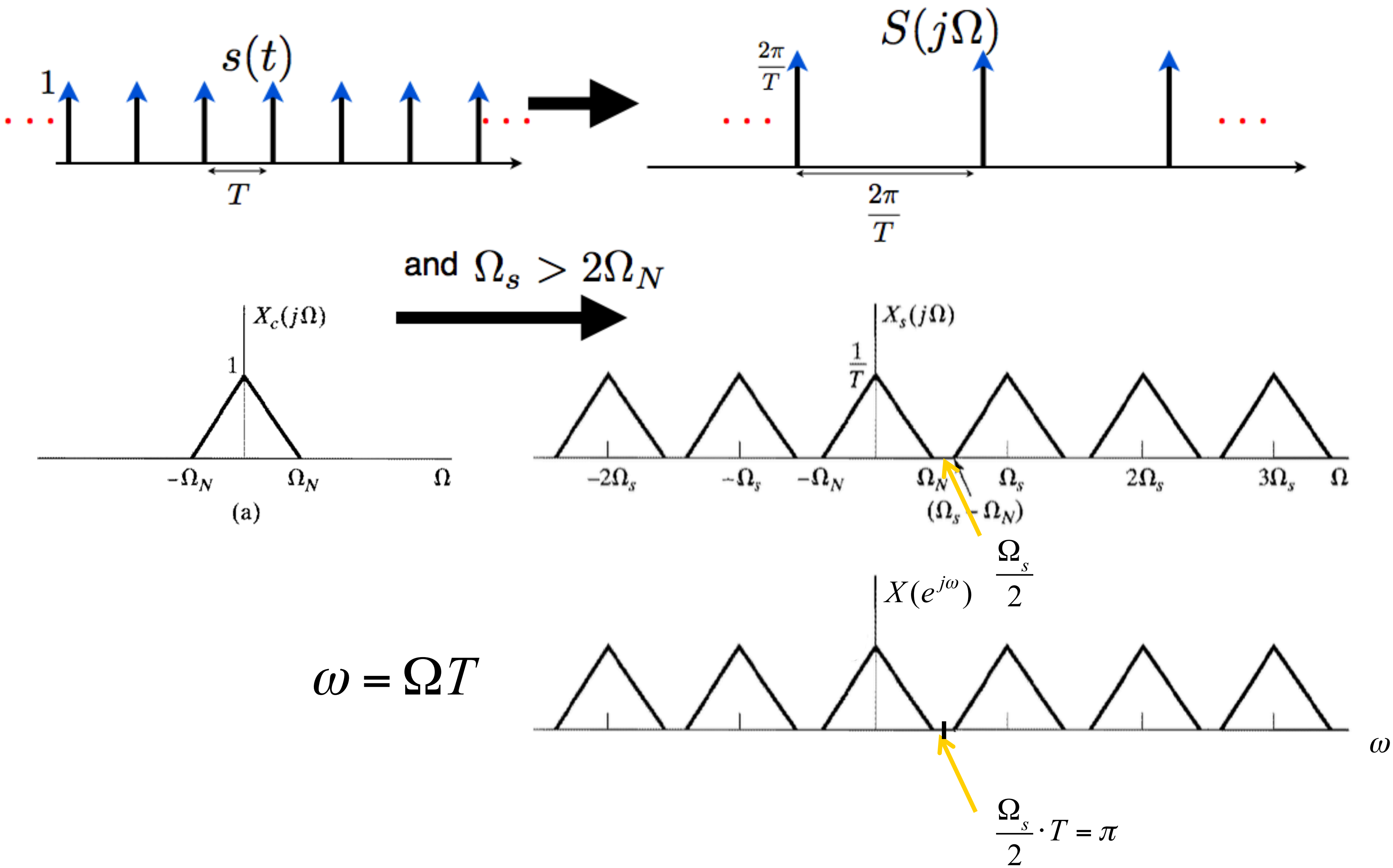
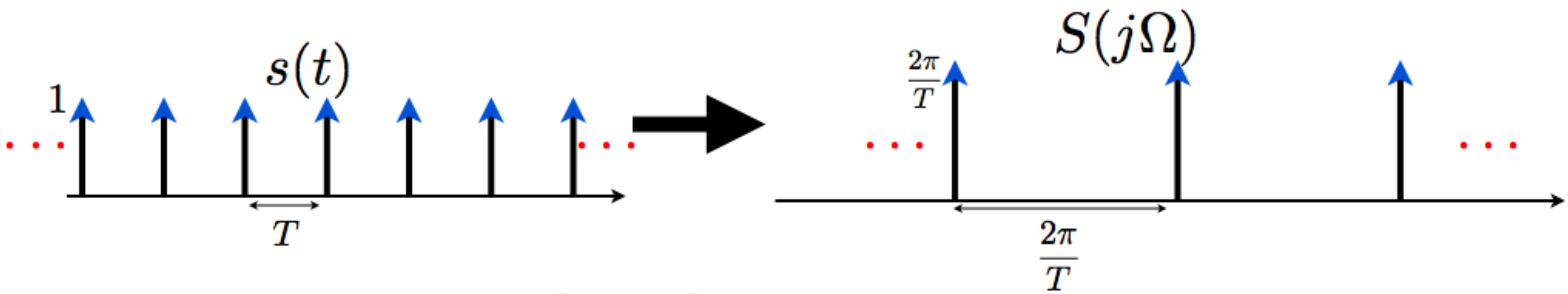


Frequency Domain Analysis



Frequency Domain Analysis w/ Aliasing



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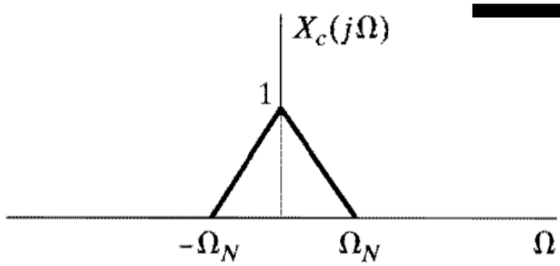
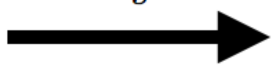
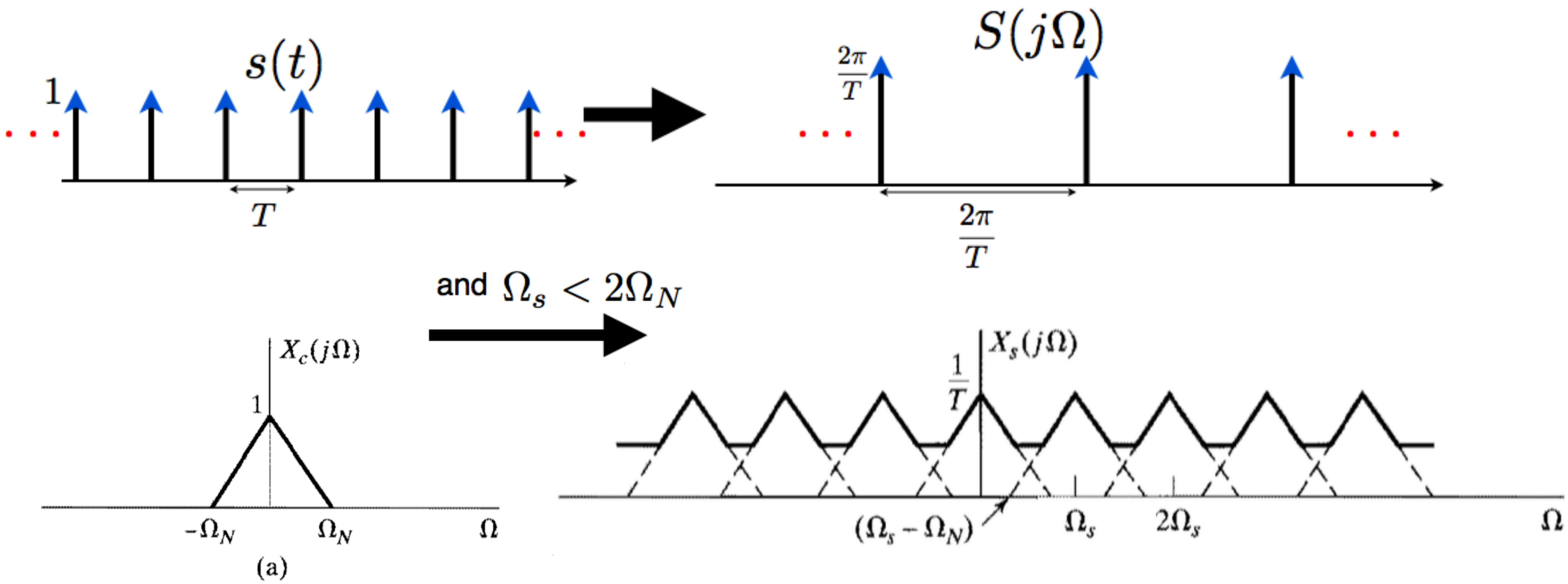


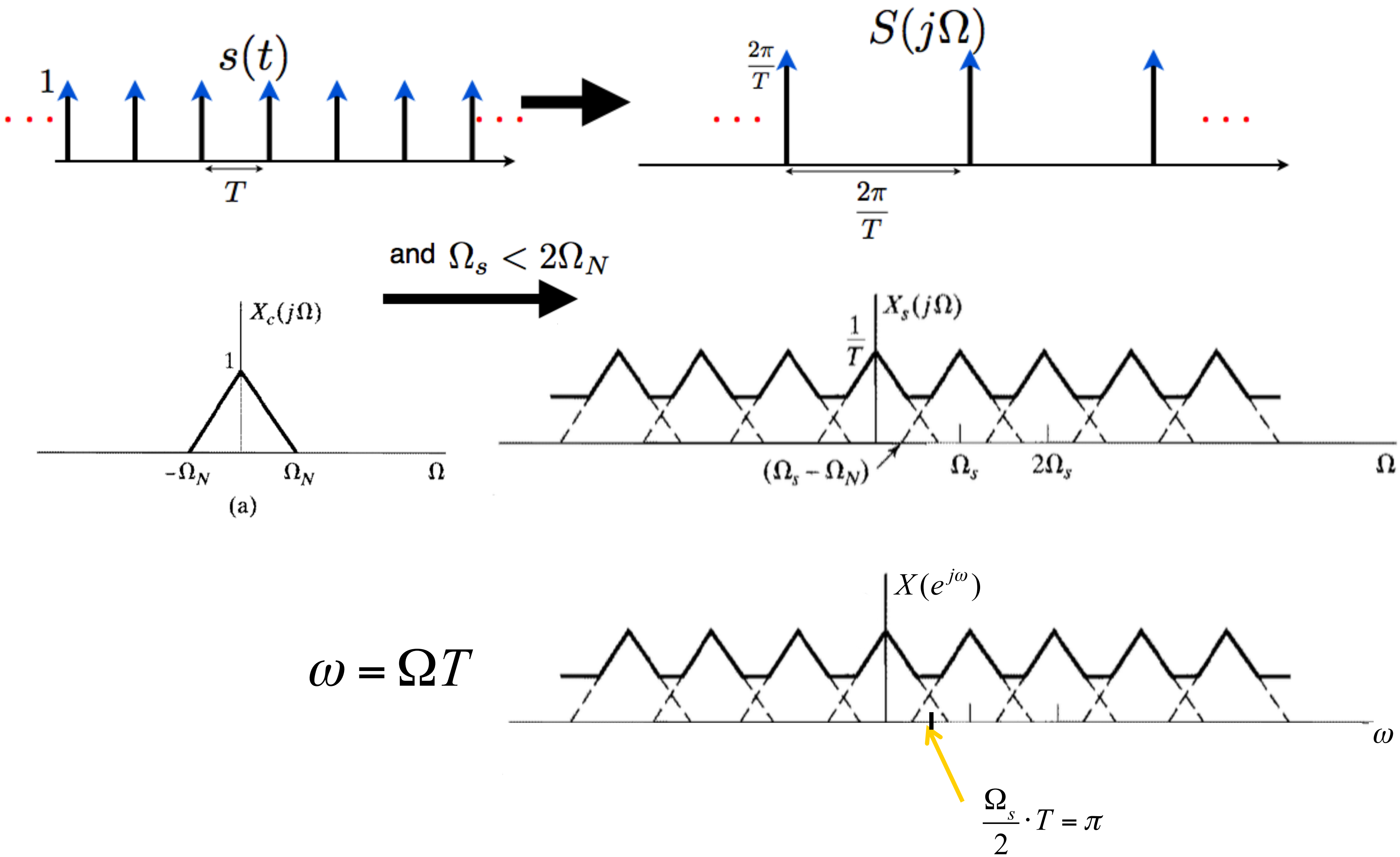
(a)



Frequency Domain Analysis w/Aliasing



Frequency Domain Analysis w/Aliasing



Example: Cosine Input

- Sample the continuous-time signal $x_c(t)=\cos(4000\pi t)$ with sampling period T=1/6000 s (f_s=6kHz)

$$\begin{aligned} X_s(j\Omega) &= \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega) \\ &= \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c(j(\Omega - k\Omega_s)) \quad \Big| \quad \Omega_s = \frac{2\pi}{T} \end{aligned}$$

Example: Cosine Input

- Sample the continuous-time signal $x_c(t)=\cos(16000\pi t)$ with sampling period $T=1/6000$ (f_s=6kHz)

Big Ideas

- Sampling
  - Ideal sampling modeled as impulsive sampling
  - Sample at Nyquist rate for recovery of unique bandlimited signal (i.e. avoid aliasing)
- Frequency Response of Sampled Signal
  - Sampled signal is period replicated input CT signal

Admin

- HW 2 due Monday 2/15 at midnight
  - Double check that your submission by viewing it!
- Recitation videos in Canvas and notes on course webpage
- Make sure you have access to Matlab
  - Let us know ASAP if you need help!